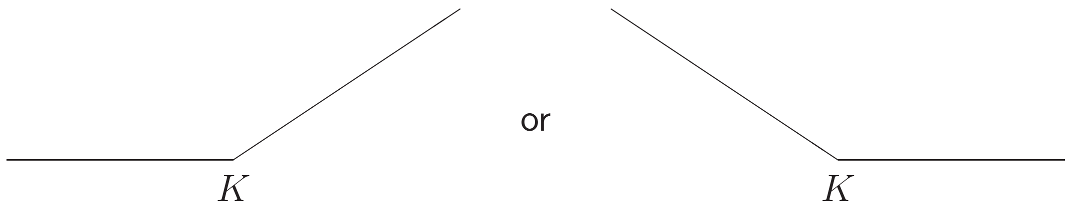
Exotic Options: An Overview

Exotic options: Options whose characteristics vary from standard call and put options



Exotic options also encompass options which involve combined or multiple underlying assets

Exotic options are important for a number of reasons:

1. Expand the available range of risk-management opportunities
2. Create unique pricing and hedging problems
3. Provide insights into previously little-considered risk dimensions
4. Increase focus on risk-management frameworks and practices

Evolution: The emergence of exotic options is driven by the overall evolution of risk management itself

Key factors include:

1. Uncertainty/Volatility in asset markets

   Example: At the onset of hostilities in the Middle East between Kuwait and Iraq, the oil markets were affected by enormous price disruptions

2. Increased focus on financial risk management

   Example: Q-Cap allows for more precise hedging than standard options, thereby eliminating the possibility of overhedging (for a 23% cost saving)

3. Demand for highly customized risk-reward profiles

   Example: CHOYS, a two factor spread option on the yields of two notes of different maturities

4. Development of option pricing and hedging technology

Product Developers: In 1991, the following 17 banks were identified by industry experts as innovative in terms of product development undertaken from a London base

	Head office
Barclays Bank	England
Chase Manhattan Bank	U.S.
Chemical Bank	U.S.
Citicorp Investment Bank	U.S.
Credit Suisse First Boston	U.S.
First National Bank of Chicago	U.S.
Goldman Sachs International	U.S.
Hambros Bank	England
Morgan JP Securities	U.S.

	Head office
Midland Montagu	England
Morgan Stanley	U.S.
National Westminster Bank	England
Noimura Bank International	England
Solomon Brothers International	U.S.
Societe Generale	France
Swiss Bank Corporation	Switzerland
Union Bank of Switzerland	Switzerland

Key Players: Rankings in September 1993 and September 1994 for the ten most popular second-generation derivatives, including Asian (average) options, spread options, lookback options, barrier options, quanto options, and compound options

September 1993		
Bankers Trust	Union Bank of Switzerland	Solomon Brothers
JP Morgan	Mitsubishi Finance	Barclays Bank
Credit Suisse Financial Products	Merrill Lynch	Morgan Stanley
General Re Financial Products	Swiss Bank Corporation	Goldman Sachs
Societe Generale		

September 1994		
Bankers Trust	Union Bank of Switzerland	Solomon Brothers
Swiss Bank Corporation	Morgan Stanley	JP Morgan
Goldman Sachs	Merrill Lynch	Societe Generale
Credit Suisse Financial Products		

User Groups: Who are the users of exotic options?

1. Investors and asset managers

   On the buy side to enhance the yield of their assets

2. Derivatives dealers

   Derivatives dealers are interested in option premiums

3. Nondealer financial institutions

   Commercial banks or insurance companies use exotic options to deal with their asset and liability mismatches

4. Corporations

   Corporations aim to generate cost-effective funding and to create more appropriate hedge structures for their risk exposures

Example of Cost-Effective Funding: Benetton's innovative funding campaign in July 1993

Five-year L200 billon Eurolira bond issue with knock-out warrants

1. Bond: (i) 4.5% annual coupon

   (ii) yield to maturity of 10.54%

2. Warrant: (i) priced at L17,983

   (ii) 63 warrants per bond

   (iii) conditional put strike price of L21,543

   (iv) knock-out price of L24,353

   (v) call strike price of L29,973

   (vi) exercisable after three years

Example of Cost-Effective Funding (continued)

What does the knock-out warrant provide?

1. Exposure to the upside performance of the ordinary share
2. While minimizing the downside risk that comes with holding an ordinary share through a conditional money-back feature
3. At a 4% discount to the share price

How does it work? When an investor buys a warrant, he is

1. Buying an ordinary share
2. Selling the dividend cash flow associated with it
3. Buying a conditional put option at the strike price of L21,543
4. Selling a call option at the strike price of L29,973

Example of Cost-Effective Funding (continued)

What happens on expiration?

1. Closing price of the share at maturity is less than or equal to L21,543

   AND

   Share price has not at any time equaled or exceeded L24,353

   One warrant will entitle the holder to receive L21,543

2. At maturity the share price exceeds L21,543

   OR

   Closing share price has at any time prior to maturity equaled or exceeded L24,353

   One warrant will entitle the holder to receive either (i) one ordinary share (or its cash equivalent)

   or (ii) L29,973

Example of Hedging of Corporate Risk: DM-based company switching to US-based component supplier

Costings were based on an exchange rate of 1.5000 DM/$

Any strengthening of the dollar will give rise to a material loss

Total purchases for the year are projected to be $20 million, to be paid fairly evenly over the period

To hedge this exposure, three alternatives have been suggested:

1. Forwards

   Problem: If the total purchases fall short of the projections, then they will have purchased more dollars than required and be at risk to a fall in the dollar

2. (A strip of) regular options

   Problem: expensive and cumbersome

3. Average rate options (AVROs)

Example of Hedging of Corporate Risk: (continued)

AN AVRO can be used to solve this problem as follows:

1. Twelve monthly payments for a total of $20 million
2. Projected total DM cost is 20 million $\times$ 1.5000
3. The risk is that the actual total DM cost turns out to be higher than projected
4. The company should purchase a dollar call/DM put AVRO
   - (i) A strike of 1.5000
   - (ii) On an amount of $20 million
   - (iii) With 12 monitoring dates on the 12 projected payment dates

Key Applications: Exotic options can be used to serve the following objectives:

1. Yield enhancement

   Example: Use of “best of” options to capture the return of that index which will have the best performance at the end of a period

2. Proprietary trading/positioning

3. Structured protection

   Example: A gold mining company feels that protection against rising interest rates is only necessary when gold prices are falling

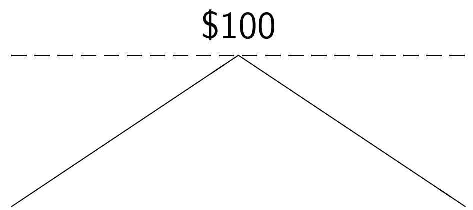
   It purchases an interest rate cap with an up-barrier in gold price

4. Premium reduction strategies

Example of Proprietary Positioning: Straddle vs double barrier box

A client wants to take a position on the European currencies converging (volatility of exchange rate declines)

Short a straddle: Simultaneously sell a call and a put, each struck at $100



The one-year straddle costs $11.58

The client makes all the $11.58 if the underlying stays at or near $100 at the end of the year

However, the client will lose big if at the end of the year the underlying is far from $100

The client makes $11.58 to potentially lose an unlimited amount

Example of Proprietary Positioning (continued)

Buy a double barrier box: Simultaneously buy (i) a double barrier call struck at $88

(ii) a double barrier put struck at $112

each with barriers at $88 and $112

If the underlying has not touched either $88 or $112 throughout the year, the client gets $24

If the underlying touches either of these values, the double barrier pays nothing

The client spends $4.30 to potentially earn $24

Example of Premium Reduction Strategies: "Is an expensive hedge better than no hedge at all? We would rather be unhedged than expensively hedged." — a German corporate treasurer

U.K. retailer Kingfisher survived the ERM breakdown relatively unscathed by using foreign currency basket options

Traditionally, a corporate treasurer with a series of currency exposures could either hedge all the exposures separately or find a proxy currency or currencies through which to hedge the overall exposure

1. Hedging exposures individually is expensive

2. Finding proxies is difficult and imprecise

Basket options get around these problems

They are a simple, inexpensive way to collect a series of identifiable foreign exchange risk positions and then hedge them with a single transaction

Building Blocks: We subdivide exotic-option instruments into a number of categories

1. Payoff modified options

   Payoff under the contract is modified from the conventional return (either zero or difference between strike price and asset price)

   Examples: digital, contingent-premium, power options

2. Time/Volatility-dependent options

   Options where the purchaser has the right to nominate a specific characteristic as a function of time

   Useful when there is some event which occurs in the short term which will then potentially affect outcomes further in the future

   Examples: chooser, compound, forward start options

Building Blocks (continued)

3. Correlation-dependent/Multifactor options

   A pattern of pay-offs based on the relationship between multiple assets (not just the price of single assets)

   Examples: basket, exchange, quanto, rainbow options

4. Path-dependent options

   Payoffs are a function of the particular continuous path that asset prices follow over the life of the option

   Examples: average rate, average strike, barrier, one-touch, lookback options

Statistical Concepts: For a continuous random variable X with PDF $f(x)$, we have

1. Mean $= \mu_X = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$
2. Variance $= \sigma_X^2 = \mathrm{Var}(X) = E\left\{\left[X - E(X)\right]^2\right\} = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x)\, dx$

If Y is a random variable defined by $Y = g(X)$ for some function g, then

$$E(Y) = E\big[g(X)\big] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

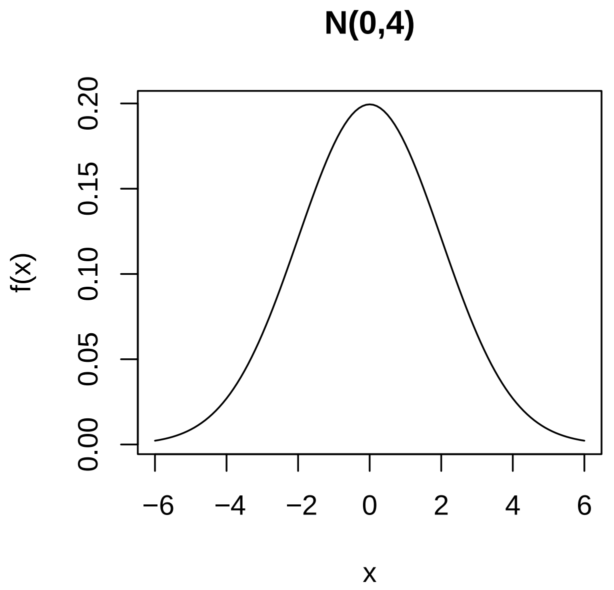
It is easy to verify that $\mathrm{Var}(X) = E(X^2) - \left[E(X)\right]^2$, where $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

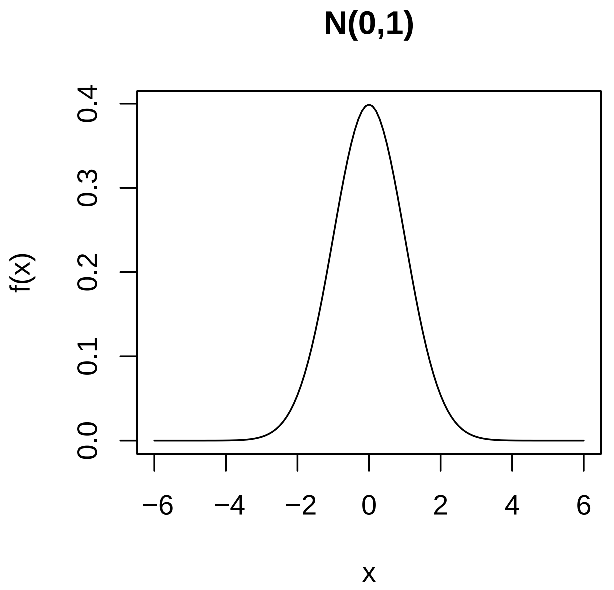
Normal distribution: If X has a $N(\mu, \sigma^2)$ distribution, then the PDF of X is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad \text{(a bell-shaped function)}$$

so that $E(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$

Statistical Concepts (continued)





Standardization: $Z = \dfrac{X - \mu}{\sigma}$ has a $N(0,1)$ distribution with PDF $n(z)$ and CDF $N(z)$

$$n(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \qquad \text{and} \qquad N(z) = \int_{-\infty}^{z} n(t)\,dt = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$$

The standard normal CDF is widely tabulated

Statistical Concepts (continued)

Brownian motion: A [Markov] process with continuous time and continuous paths such that

1. $B(0) = 0$ [convention]

2. $B_{t_1} - B_{t_0}, \ldots, B_{t_n} - B_{t_{n-1}}$ $(t_0 < t_1 \cdots < t_n)$ are independent [independent increments]

3. The distribution of $B_t - B_s$ only depends on $t - s$ [stationary increments]

4. B_t has a $N(0, t)$ distribution [standard Gaussian]

5. $t \longrightarrow B_t$ is continuous [continuous paths]

Black-Scholes Model: The underlying asset price S_t follows the geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$

Equivalently

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t\right] \qquad \text{or} \qquad X_t = \ln\left\{\frac{S_t}{S_0}\right\} = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t$$

X_t is normally distributed with mean $(\mu - \sigma^2/2)t$ and variance $\sigma^2 t$

Historical volatility: Sample asset prices $S_0, S_1, \ldots, S_n$ at regular intervals of length t

Compute asset returns $x_j = \ln(S_j/S_{j-1})$ and estimate $v^2 = \sigma^2 t$ using $\hat{v}^2 = \frac{1}{n}\sum_{j=1}^{n} x_j^2$ (MLE)

Estimate annualized volatility with $\hat{\sigma} = \hat{v}/\sqrt{t}$ (e.g., $\hat{\sigma} = \hat{v}\sqrt{248}$ if $t = 1$ trading day)

Implied volatility: That value of σ in the Black-Scholes formula (later) such that observed option price equals Black-Scholes price

Valuation: Exotic options (along with standard ones) can be priced using the following methods

1. Partial differential equations

   Use arbitrage-free arguments to derive second-order PDEs satisfied by option prices

   Apply appropriate boundary conditions to solve the PDEs for option pricing formulas

   For example, the value of a European call option satisfies the PDE

$$-rC + \frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S\frac{\partial^2 C}{\partial S^2} = 0$$

   with the boundary condition $C(S,T) = (S-K)^+$

   PDEs can be solved analytically or numerically using finite-difference methods

   Finite-difference approximations of the partial derivatives are used to rewrite the PDEs as finite-difference equations, which can solved by building a lattice on which approximate values of the desired variables are obtained

Valuation (continued)

2. Risk-neutral valuation (evaluating expected payoffs)

   Under risk neutrality, the expected return of the underlying asset μ must equal $r - q$ (where $r =$ risk-free rate, domestic rate; $q =$ dividend rate, foreign rate)

   The values of European options can be obtained by discounting the expected payoffs of the options at maturity by the risk-free rate r

$$\text{value} = e^{-rT} E\big[\text{payoff}_T\big]$$

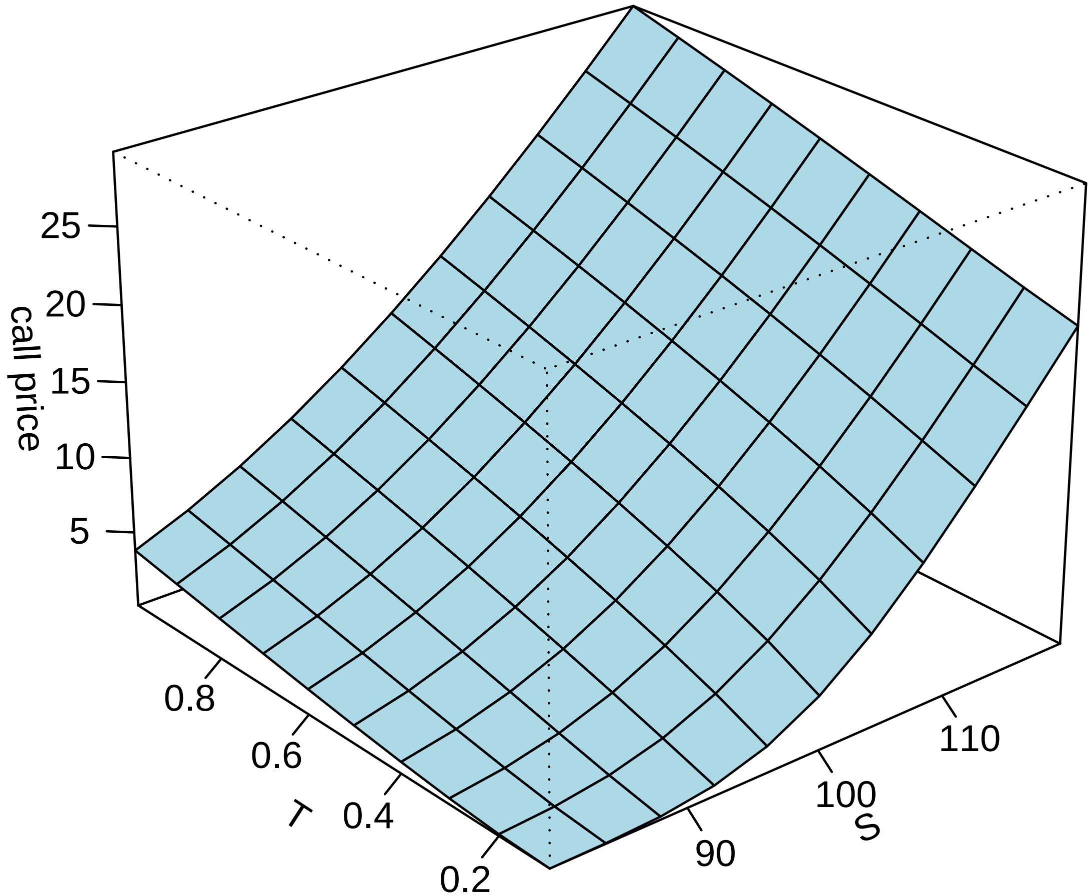
   For example, the price of a European call option is

$$e^{-rT} E\big[(S_T - K)^+\big] = e^{-rT} \int_{\ln(K/S_0)}^{\infty} (S_0 e^x - K) f(x)\, dx$$

   which evaluates to the Black-Scholes formula

$$C = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

Valuation (continued)



Call option price C decreases with passage of time but increases with moneyness (as S increases)

Valuation (continued)

3. Monte Carlo simulations

   Generate large numbers of numerically simulated realizations of random walks followed by the underlying asset prices (essentially normally distributed variables)

   Evaluate the terminal payoff (payoff_T) of each sample path

   Average over all simulated values of payoff_T and discount by risk-free rate r

   Compare with risk-neutral valuation: value $= e^{-rT} E\left[\text{payoff}_T\right]$

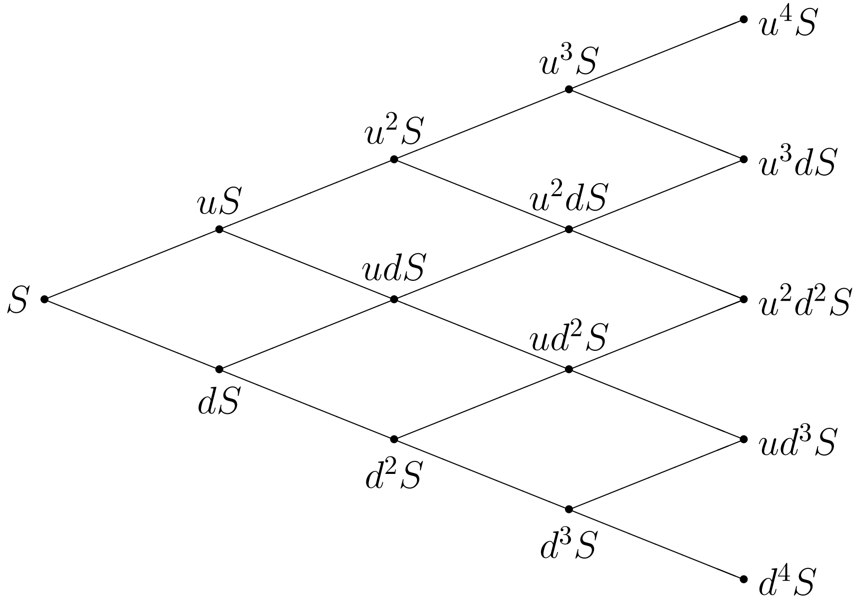
   Simulations are increasingly used to price path-dependent derivatives because products have become more complex in nature and it is difficult to obtain closed-form solutions for many of them

   Variance-reduction techniques are often needed to efficiently reduce the standard error of the price estimates

Valuation (continued)

4. Lattice- and tree-based methods

   We concentrate on the recombining tree model, where the total number of upward moves and that of downward moves determine a path completely



<u>n-step tree</u>

$$\Delta = T/n$$

$$u = e^{\sigma\sqrt{\Delta}}$$

$$d = 1/d$$

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

At the terminal date evaluate the payoff and work backwards to get the value at any node as

$$p \times \text{value}_u + (1-p) \times \text{value}_d$$

Price Sensitivities: The sensitivity parameters are important in managing an option position

Delta: The delta measures how fast an option price changes with the price of the underlying asset

$$\Delta = \frac{\partial C}{\partial S} = e^{-qT} N(d_1)$$

It represents the hedge ratio, or the number of options to write/buy to create a risk-free portfolio

Charm, given by $\partial \Delta / \partial T$, is used as an ad hoc measure of how delta may change overnight

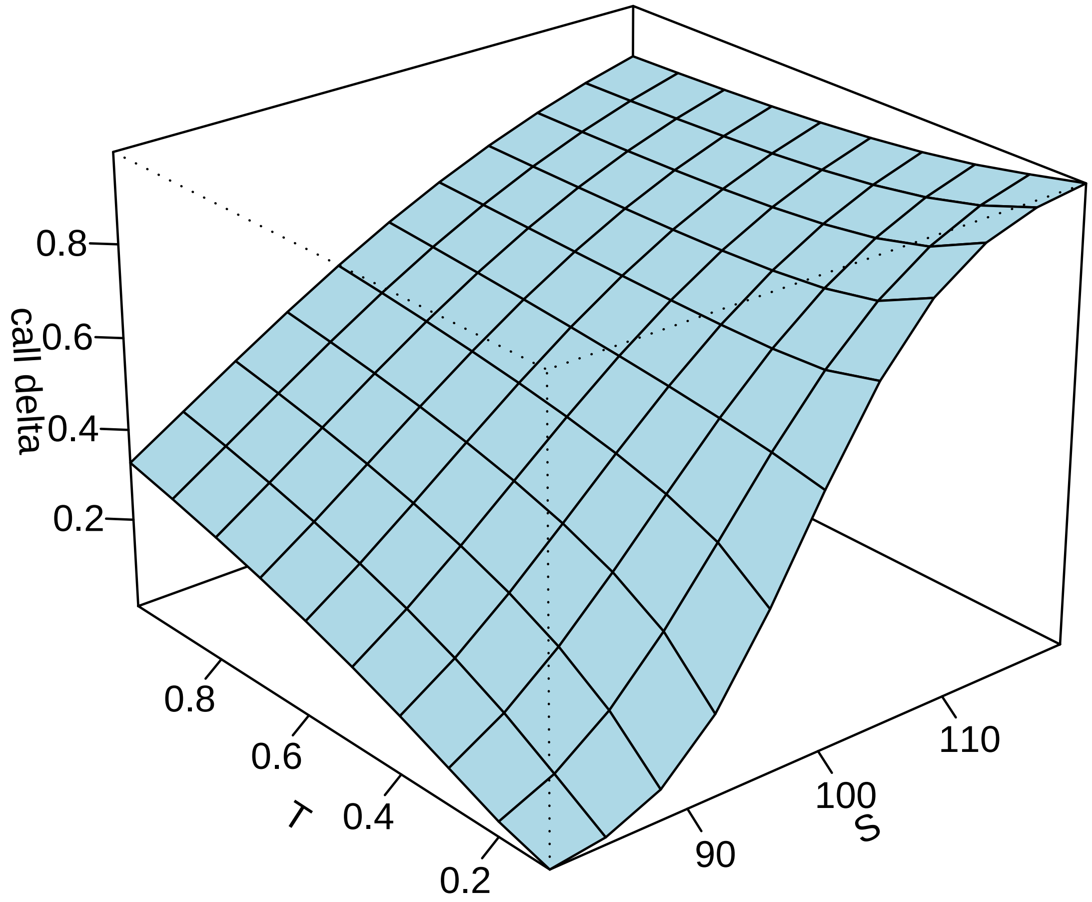
Delta hedging is a trading strategy to make the delta of a portfolio neutral to fluctuations of the underlying asset price

For example, the portfolio which is long/short one unit of a European call option and short/long Δ units of the underlying asset is delta-neutral

Long/short a call: $S \uparrow \Rightarrow \Delta \uparrow$ so delta hedge by selling/buying more underlying asset

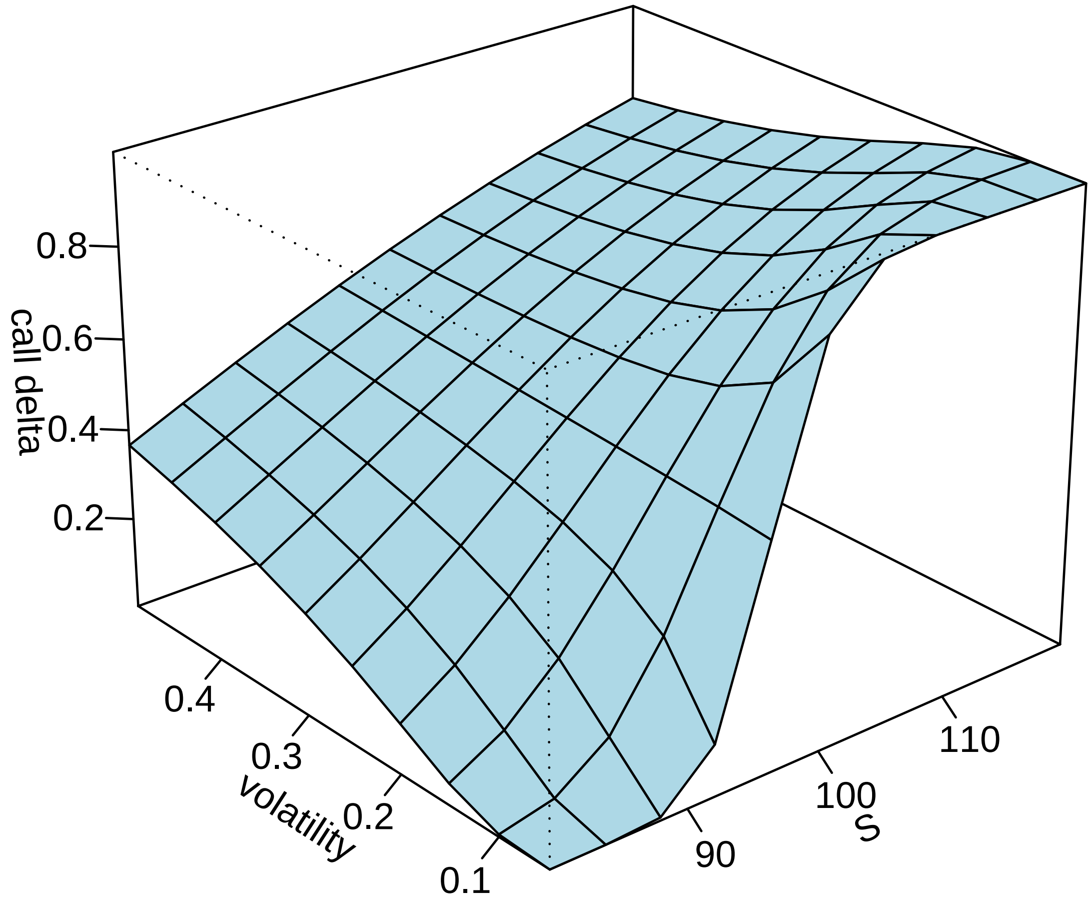
Delta hedging does not protect an option position against variations in time remaining to maturity

Price Sensitivities (continued)



Long an ITM/ATM/OTM call: $T \downarrow \Rightarrow \Delta$ rises/holds/falls so sell/—/buy more underlying asset

Price Sensitivities (continued)



Effect of σ on Δ similar to effect of T (intuition: in the Δ formula there are terms $\sigma^2 T$ and $\sigma\sqrt{T}$)

Price Sensitivities (continued)

Gamma: The gamma measures how fast the option's delta changes with the price of its underlying asset

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{e^{-qT} n(d_1)}{S\sigma\sqrt{T}}$$

It is an indication of the vulnerability of the hedge ratio (large Γ equates to greater risk in option)

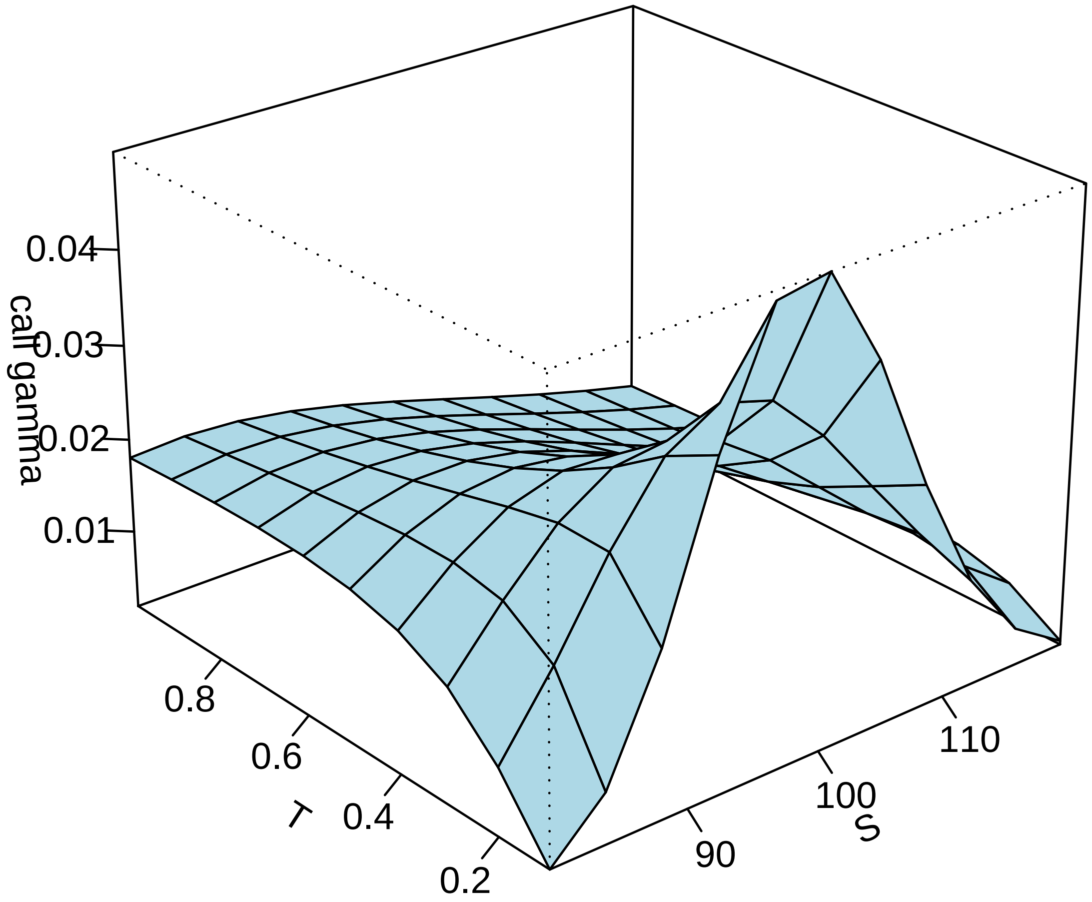
Γ is highest for an ATM option and decreases either side when the option gets ITM or OTM

Γ of an ATM option rises significantly when σ decreases and the option approaches maturity

For delta-neutral strategies, a positive Γ allows profits when market conditions change rapidly while a negative Γ produces losses

Two related risk measures are speed, given by $\partial\Gamma/\partial S$ and color, given by $\partial\Gamma/\partial T$

Price Sensitivities (continued)



Effect of σ on Γ (not shown) similar to effect of T

Price Sensitivities (continued)

Theta: The theta measures how fast an option price changes with time to expiration

$$\Theta = \frac{\partial C}{\partial T} = -qSe^{-qT}N(d_1) + rKe^{-rT}N(d_2) + \frac{\sigma S e^{-qT} n(d_1)}{2\sqrt{T}}$$

A large Θ indicates high exposure to the passage of time

Θ is highest for ATM options with short maturity

Vega: The vega measures how fast an option price changes with volatility

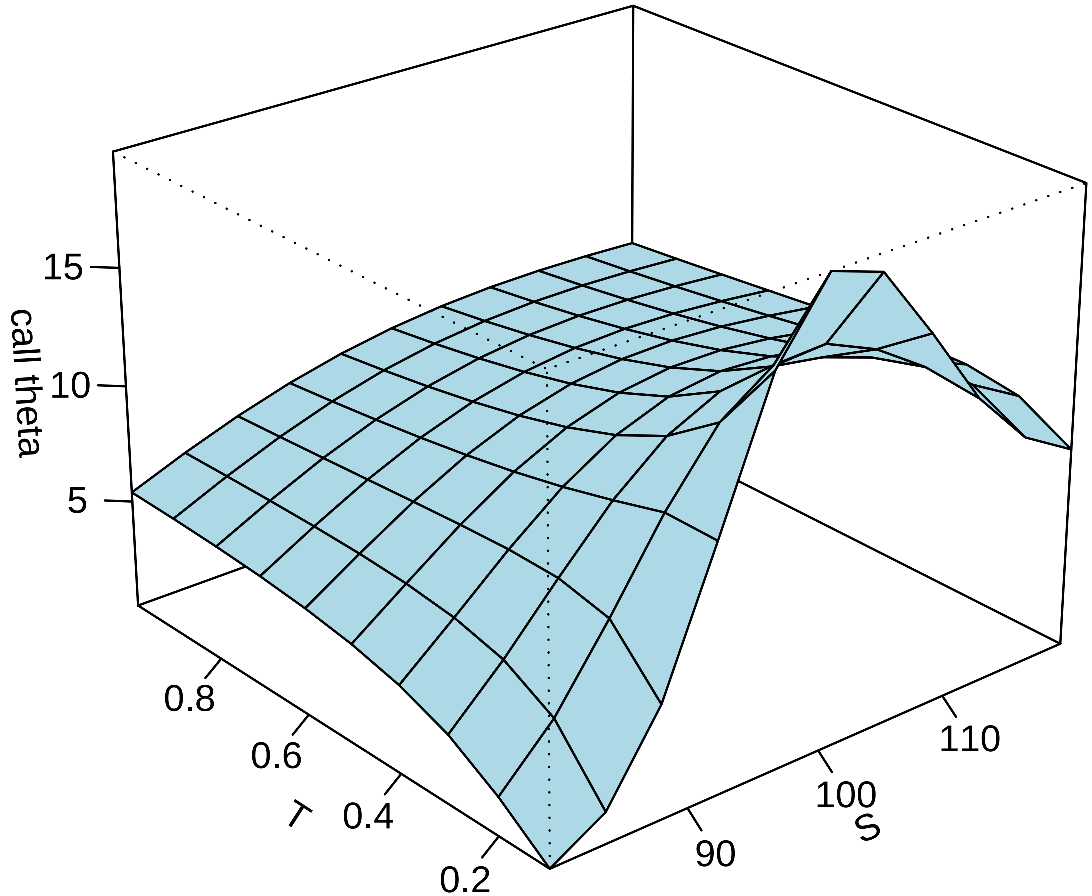
$$V = \frac{\partial C}{\partial \sigma} = Se^{-qT}n(d_1)\sqrt{T} = \sigma T S^2 \Gamma$$

Positive V means option price is an increasing function of volatility

Buying/Selling options is equivalent to buying/selling volatility

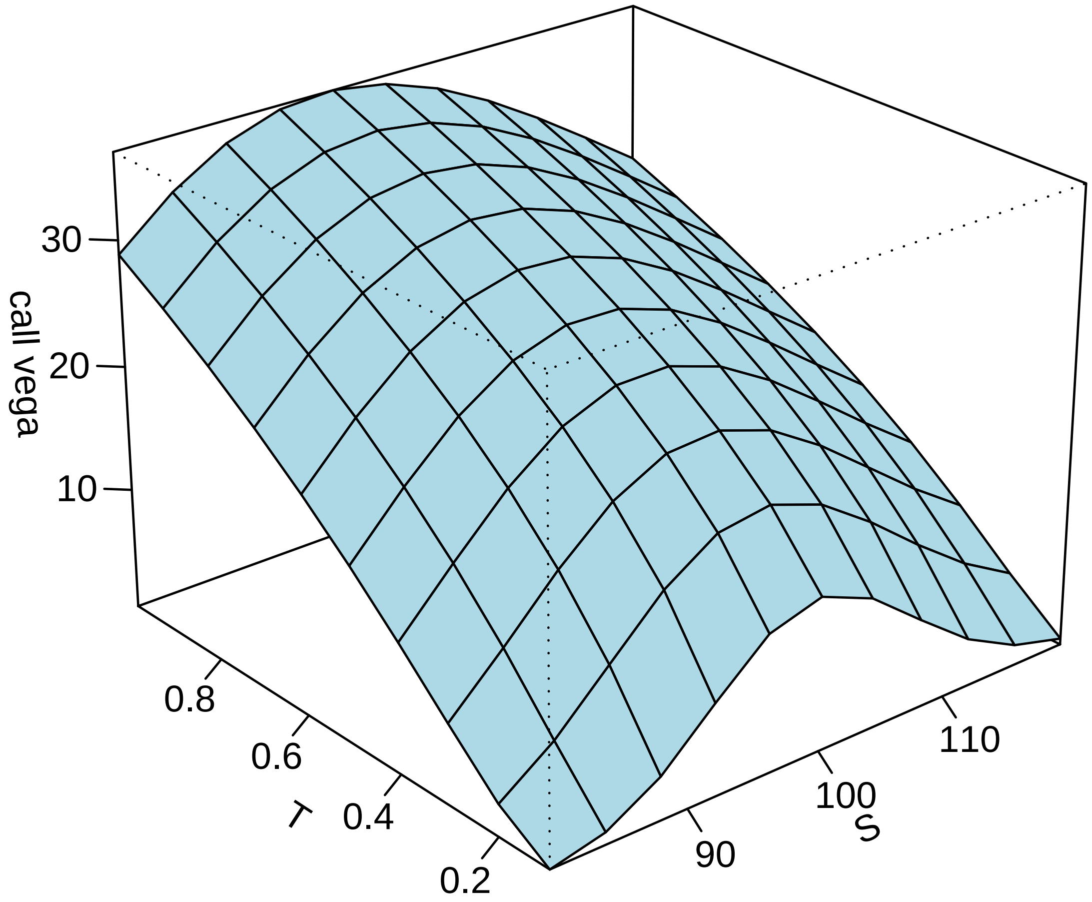
V is highest for ATM long-term options

Price Sensitivities (continued)



The option buys loses Θ while the option writer “gains” it

Price Sensitivities (continued)



V is related directly to Γ via $V = \sigma T S^2 \Gamma$

Summary: New generations of risk-management products, like traditional derivatives, continue to permit the separation, unbundling and ultimate redistribution of price risks in financial markets

The increased capacity to delineate risks more precisely should, theoretically, improve the capacity of markets to redistribute and reallocate risks based on the economic value of the market risk, thus facilitating a more efficient allocation of economic resources