

Eurodollar Futures: The Basics

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9 December 1981. Chicago Mercantile Exchange launches Three-Month Eurodollar futures. It goes badly. Within months, financial commentators cluck over “sluggish activity and lack of liquidity...so disappointing that some retail commodity speculators are all but ready to bury the contracts.”¹

That was then.

Thirty years on, Three-Month Eurodollar futures reign as the most innovative, flexible, highly traded, and widely used of all interest rate derivatives. This user’s guide lays out their basics: how they work, how they trade, how they relate to their companion money markets.

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¹ Reier, Sharon, “The disappointing debut of Eurodollar futures”, *Institutional Investor*, May 1982.

Eurodollar Futures Contract Terms

Exhibit 1 summarizes the terms of the Three-Month Eurodollar futures contract, known by its CME Globex ticker symbol as GE. The following passages examine the details of:

- the trading unit (the underlying reference from which the contract derives);
- the schedule of delivery months and last trading days;
- the contract price mechanism;
- tick sizes (minimum price increments) for trading; and
- notional contract size.

Exhibit 1 – CME Three-Month Eurodollar Futures Contract Specifications

(All times of day are Chicago Time unless otherwise noted.)

Trading Unit	Interest on a Eurodollar interbank deposit having approximately \$1 mln principal value for a term of three months, for spot settlement on 3 rd Wednesday of contract Delivery Month	
Delivery Months	Nearest 40 months in March Quarterly cycle plus nearest 4 months not in March Quarterly cycle	
Price Basis	IMM price points: 100 points minus three-month London interbank offered rate for spot settlement on 3 rd Wednesday of Delivery Month (eg, a 6.33 percent rate equals 93.67 points). One interest rate basis point (0.01 price points) equals \$25 per contract.	
Price Increments	<p><i>Nearest Delivery Month</i> $\frac{1}{4}$ of one interest rate bp (0.0025 price points) = \$6.25 per contract.</p> <p><i>All other Delivery Months</i> $\frac{1}{2}$ of one interest rate bp (0.005 price points) = \$12.50 per contract.</p>	
Last Trading Day	Second London business day before 3 rd Wednesday of Delivery Month	
Delivery Standard	Three-month US dollar BBA LIBOR [®] as set on Last Trading Day for spot (T+2) settlement on 3 rd Wednesday of Delivery Month, rounded to nearest 1/100 th of one bp.	
Delivery Method	Cash settlement, by mark-to-market to Final Settlement Price. Final Settlement Price = Price Basis evaluated at Delivery Standard. Final settlement occurs on Last Trading Day.	
Trading Hours	<p>CME Globex: 5pm to 4pm, Sun-Fri. Open outcry: 7:20am to 2pm, Mon-Fri.</p> <p>Trading of expiring contracts terminates at 11am London time on Last Trading Day.</p>	
Position Accountability	10,000+ contracts	
Reportable Positions	850+ contracts	
Block Trade Thresholds	Nearest 20 March Quarterly Delivery Months and all Delivery Months not in March Quarterly cycle:	Farthest 20 March Quarterly Delivery Months:
<i>RTH (7am to 4pm)</i>	4,000+ contracts	1,000+ contracts
<i>ATH (4pm to 12am)</i>	1,000+ contracts	250+ contracts
<i>ETH (12am to 7am)</i>	2,000+ contracts	500+ contracts
Designated Contract Market	CME Three-Month Eurodollar futures are listed with and subject to the rules and regulations of CME.	

Source: CME Group

Trading Unit

The underlying cash-market reference for any GE contract is *interest on a Eurodollar interbank deposit* having approximately \$1 million principal value, with a term to maturity of *three months*, for *spot settlement* on the 3rd Wednesday of the contract Delivery Month. They look obvious, but the components of this definition hold subtleties worth scrutiny.

Eurodollar Interbank Deposit

...is a US dollar deposit booked by one bank with another bank (or bank branch) domiciled outside the US. In principle, “domicile” could be anywhere outside the US. In practice, it is reasonable to assume “domicile” is London, since GE futures expire with reference to the three-month London interbank offered rate.

Standard Spot Settlement

...for a three-month US dollar London interbank deposit is T+2. The settlement date (when borrower receives lender’s monies) occurs two London business days after the trade date (when borrower and lender agree to the deposit’s amount and rate of interest). An additional requirement is that the settlement date must be a New York bank business day. If not, then the deposit settles on the next following day for which banks are open for business in both jurisdictions. Three months later the interbank deposit matures. As with the settlement date, this too must be a bank business day in both London and New York.²

Example 1

A three-month interbank placement is undertaken on Monday, 18 February, for settlement on the third Wednesday of the month, 20 February. Borrower repays lender principal plus interest on 20 May. If 20 May happens not to be a London and New York bank business day, then the placement matures on the next following London and New York bank business day.

How Long is Three Months?

In regular years the interval from 20 February to 20 May spans 89 days. In leap years it covers 90 days, owing to February’s extra 29th day. By contrast, the three months from 20 May to 20 August span 92 days.

Clearly, not all three-month intervals are created equal. Given the conventions that determine settlement and maturity dates for London interbank placements, and given the peculiarities of bank holiday calendars in the UK and the US, “three months” may run as short as 86 days or as long as 95 days. The day count matters, because it bears directly upon the interest amount borrower pays to lender.

Interest

...on a US dollar London interbank deposit is always calculated on an actual/360 basis. The rate per annum to which lender and borrower agree is assumed to accrue so that the actual number of days between settlement date and maturity date is applied to a 360-day year.

Example 2

On Monday, 18 February, lender agrees to extend \$1 million to borrower at a rate of 5 percent per year for three months, for standard spot settlement. Two London business days later, on Wednesday, 20 February, lender remits \$1 million to borrower.³ Assuming February has its usual 28 days (as in 1991, or 2002, or 2013), borrower repays lender \$1 million principal plus interest 89 days later, on Monday, 20 May. The interest amount is:

$$\$12,361.11 = (\$1 \text{ million}) \times (5 \text{ percent per year}) \times (89 \text{ days} / 360 \text{ days per year})$$

² As a general principle, if this is not a London-NY business day, then the deposit’s maturity date may get pushed back to the next following London-NY business day, or pulled forward to the first preceding London-NY business day, depending on whether the maturity date falls near a month end. For more detail, see **Appendix: A Concise Guide to US Dollar BBA LIBOR®** on page 31. For the bank deposit rate that stands as underlying reference for a GE futures contract, the deposit maturity date almost always gets pushed back to the next following London-NY business day.

³ Reminder: For settlement to occur on Wednesday, 20 February, this date must be a bank business day in New York as well as London.

Example 3

If the loan in Example 2 is undertaken in a leap year (eg, 1980 or 2008), then borrower repays lender principal plus interest 90 days later, on Tuesday, 20 May. The interest amount is:

$$\$12,500.00 = (\$1 \text{ million}) \times (5 \text{ percent per year}) \times (90 \text{ days} / 360 \text{ days per year})$$

Delivery Month and Last Trading Day

Quarterlies

At any given time the exchange lists for trading 40 Quarterly GE futures delivery months -- one for every March, June, September, and December reaching a decade forward. Each GE contract ceases trading and expires on the second London business day (typically the Monday) before the third Wednesday of its delivery month. When the nearby Quarterly expires, a new fortieth Quarterly is listed for trading on the next following exchange business day (typically the Tuesday before the third Wednesday of the month).

Example 4

GE futures for delivery in March 2013 are scheduled to cease trading and expire on Monday, 18 March 2013. On the following exchange business day, Tuesday, 19 March 2013, trading is scheduled to commence in newly-listed GE futures for delivery ten years hence, in March 2023.

Monthly Serials

Monthly Serial GE futures are identical to Quarterlies except for their delivery months, which are any months other than March, June, September, or December.⁴ At any given time the Exchange lists four Monthly Serials. For instance on Tuesday, 14 December 2010, Monthly Serials are listed for delivery in January, February, April, and May 2011. See Exhibit 2.

Example 5

January 2011 Monthly Serial GE futures expired on Monday, 17 January 2011, the second London business day before the third Wednesday of the month.⁵ On the next exchange business day, Tuesday, 18 January 2011, a new Monthly Serial contract was listed for trading, for delivery in July 2011.

Color Coding

For ease of identification the 40 Quarterly delivery months are grouped into ten foursomes. Each foursome is identified by its own unique color code. In order of proximity to expiration, these are White, Red, Green, Blue, Gold, Purple, Orange, Pink, Silver, and Copper.

Exhibit 2 illustrates for Tuesday, 14 December 2010. The White year comprises the nearest four Quarterly delivery months – in this example, March, June, September, and December 2011. The Red delivery year comprises the fifth through eighth nearest Quarterly delivery months – here, March, June, September, and December 2012 -- and so on.

⁴ A common point of confusion is the distinction between Monthly Serial GE futures and Monthly Serial options on Quarterly GE futures. Apart from the superficial similarity of expiring in months other than March, June, September, or December, they have no bearing upon each other. The underlying reference futures contract for any Monthly Serial GE option is the GE future for delivery in the Quarterly month next following the option's expiration month. For instance, October or November Monthly Serial GE options always exercise into GE futures for delivery the next following December, while January or February Monthly Serial GE options always exercise into GE futures for delivery the next following March, and so forth. The Exchange lists no options on Monthly Serial GE futures.

⁵ Note that expiration and final settlement of Jan 2011 GE futures goes through on schedule, even though Monday, 17 January, is Martin Luther King Day, a US federal and bank holiday. How so? As long as (a) Monday, 17 January, is a London bank business day (which it is), and (b) the following Wednesday, 19 January, is a bank business day in both London and New York (which it is), the BBA LIBOR[®] fix that determines the final settlement price for Jan11 GE futures proceeds, as usual, on Monday for spot (T+2) settlement on Wednesday.

Exhibit 2 -- GE Futures Prices, Volumes, and Open Interest, Tuesday, 14 December 2010

(Pink shaded rows denote Monthly Serial delivery months)

Delivery Month	Delivery Year	Color Code	Open Price	High Price	Low Price	Last Price	Daily Settlement Price	Change vs Previous Settlement	Volume	Open Interest
JAN	2011		99.6625	99.6700	99.6625		99.6650	0.0025	20,510	52,421
FEB	2011		99.625	99.630	99.625		99.630	0.005	6,973	15,390
MAR	2011	White	99.590	99.595	99.575	99.585 B	99.585		202,417	1,162,173
APR	2011				99.530 A		99.530	-0.005		52
MAY	2011				99.485 A		99.490	-0.005		
JUN	2011		99.450	99.465	99.425	99.440 B	99.440	-0.010	255,389	1,004,741
SEP	2011		99.330	99.330	99.265	99.290 B	99.290	-0.030	261,751	905,925
DEC	2011		99.175	99.175	99.080	99.110 B	99.110	-0.055	281,458	749,445
MAR	2012	Red	98.995	99.000	98.875	98.910 B	98.910	-0.075	304,204	808,964
JUN	2012		98.780	98.790	98.635	98.675 B	98.675	-0.090	254,698	484,111
SEP	2012		98.560	98.565	98.385	98.430 B	98.430	-0.110	231,410	368,117
DEC	2012		98.310	98.320	98.110	98.160 B	98.165	-0.125	203,612	278,529
MAR	2013	Green	98.065	98.075	97.835	97.890 B	97.895	-0.145	128,190	192,472
JUN	2013		97.800	97.820	97.550	97.615 A	97.610	-0.170	118,058	156,108
SEP	2013		97.550	97.565	97.270	97.340 A	97.335	-0.190	90,247	183,058
DEC	2013		97.285	97.300	96.985	97.050 B	97.050	-0.210	96,968	130,327
MAR	2014	Blue	97.040	97.055	96.725	96.790 B	96.790	-0.230	33,864	109,916
JUN	2014		96.795	96.810 B	96.465	96.530 A	96.530	-0.245	28,034	108,825
SEP	2014		96.545	96.570	96.215	96.290 A	96.280	-0.260	28,817	73,963
DEC	2014		96.300	96.340	95.970	96.040 A	96.040	-0.270	31,830	73,600
MAR	2015	Gold	96.100	96.145	95.770	95.845 A	95.840	-0.275	21,371	52,429
JUN	2015		95.910	95.955	95.580	95.660 A	95.655	-0.275	19,987	38,923
SEP	2015		95.765	95.790	95.405	95.490 A	95.485	-0.275	17,788	27,423
DEC	2015		95.585	95.620	95.250	95.335 A	95.330	-0.275	18,987	19,244
MAR	2016	Purple	95.455	95.480 B	95.130	95.255 A	95.200	-0.275	5,339	14,179
JUN	2016		95.335	95.340	95.015 A	95.135 A	95.090	-0.265	3,077	13,024
SEP	2016		95.225	95.250	94.930	95.030 A	94.990	-0.260	2,471	9,888
DEC	2016		95.135	95.185 B	94.845	94.935 A	94.905	-0.250	3,997	9,599
MAR	2017	Orange	95.080	95.085	94.825 A	94.885 A	94.855	-0.250	2,364	7,347
JUN	2017		95.020	95.020	94.740	94.825 A	94.795	-0.250	1,477	3,417
SEP	2017		94.955	94.955	94.710	94.775 A	94.750	-0.245	1,584	8,181
DEC	2017		94.900	94.900	94.645 A	94.705 A	94.680	-0.245	645	4,659
MAR	2018	Pink	94.680	94.700	94.615 A	94.675 A	94.655	-0.240	352	1,632
JUN	2018				94.575 A	94.635 A	94.620	-0.235	28	1,659
SEP	2018				94.540 A	94.600 A	94.590	-0.230	21	1,880
DEC	2018		94.545	94.555 B	94.495 A	94.555 A	94.550	-0.225	11	1,821
MAR	2019	Silver			94.460 A	94.520 A	94.520	-0.220	410	1,199
JUN	2019				94.410 A	94.470 A	94.475	-0.215		884
SEP	2019				94.375 A	94.435 A	94.440	-0.215	3	1,225
DEC	2019				94.330 A	94.390 A	94.395	-0.215		1,576
MAR	2020	Copper			94.290 A	94.350 A	94.355	-0.215	205	679
JUN	2020				94.240 A	94.300 A	94.305	-0.215		435
SEP	2020				94.185 A	94.245 A	94.250	-0.215		623
DEC	2020				94.140 A	94.200 A	94.205	-0.215	5	5
Totals									2,678,552	7,080,068

B ~ Bid Price. A ~ Asking Price. Source: CME Group.

Nearby vs Front

The “nearby” is simply the GE futures contract that expires next. It can be either a Quarterly or a Monthly Serial. Context matters, however. Market practitioners sometimes use “nearby” as a short-hand reference for the next expiring Quarterly.

“Lead” or “front” refers to the most actively traded Quarterly. In the heyday of open outcry trading, the “front” designation served to aid the exchange’s Eurodollar futures pit committee and trading floor operations staff in deciding which contract delivery month would take pride of place within the then massive Eurodollar futures pit. Although most volume is now mediated through the CME Globex electronic trading platform, the “front” label lives on as a convenient nickname for the most actively traded delivery month.

The Exchange has full discretion in determining which delivery month is “most actively traded.” For many years, front status has been evaluated at least once each quarter on the basis of a comparison of contract trading volumes – specifically the 12-day moving average of daily trading volumes for each Quarterly delivery month. Exchange staff monitor this measure during the two-month interval that begins five weeks after the latest expiration of a Quarterly contract, and that ends with expiration of the nearby Quarterly.

Price = 100 Minus Rate

GE futures prices are quoted in “100 minus rate” terms.⁶ Price is expressed on the basis of 100 points, with each point representing one percent (ie, 100 basis points) per annum of contract interest rate exposure.

At Futures Expiration – Final Settlement Price

“Delivery” on expiring futures takes place not by physical delivery and acceptance of a three-month London interbank deposit, but rather by cash settlement. The contract is fulfilled by a mark-to-market to its final settlement price. The final settlement price is determined as 100 minus three-month US dollar BBA LIBOR[®] on the second London bank business day preceding the third Wednesday of the contract delivery month.⁷

The British Bankers’ Association publishes BBA LIBOR[®] to five decimal places of accuracy. The exchange rounds the published value to four decimal places before applying it to establish the final settlement price of the expiring GE futures contract. This ensures the futures final settlement price will produce a final margin mark-to-market for which the minimum increment is 25 cents per contract. (Without rounding, the final mark-to-market would be measured out in unmanageably tiny increments of 2 ½ pennies per contract.) In the case of a tie value, ie, a published value ending in 0.00005, the exchange rounds up to four decimal places.

This interest rate, so rounded, is then subtracted from 100 to determine the contract final settlement price.

Example 6

A published three-month US dollar BBA LIBOR[®] of 3.14149 percent would be rounded by the exchange to 3.1415 percent before application to futures final settlement. A published value of 3.14145 percent also would get rounded up to 3.1415 percent. In either case the resultant futures final settlement price would be 96.8585, equal to 100 minus 3.1415 percent.

⁶ When introduced in 1981, Three-Month Eurodollar futures were listed for trading under the auspices of what was then the International Money Market (IMM) division of Chicago Mercantile Exchange. The “100 minus rate” contract pricing engine, now in wide use at futures exchanges around globe, came thus to be known as the “IMM Index.”

⁷ For more, see **Appendix: A Concise Guide to US Dollar BBA LIBOR[®]** on page 31.

Before Futures Expiration – Contract Price and Contract Interest Rate

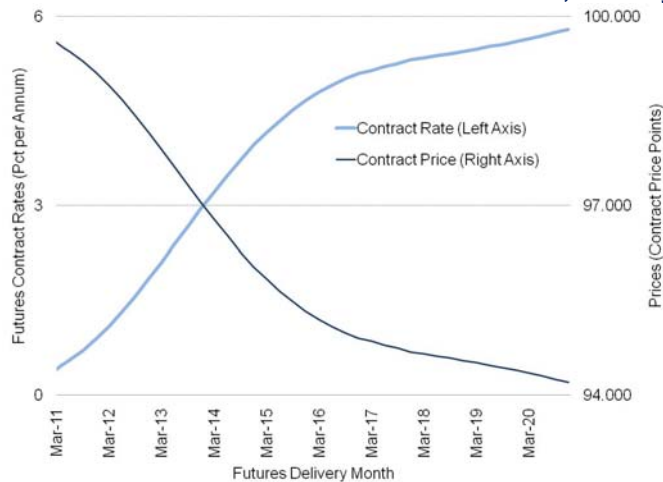
The contract interest rate (equal to 100 minus contract price) is approximately the market consensus expectation of the three-month London interbank offered rate for settlement on the third Wednesday of the contract delivery month.⁸

Example 7

If a GE futures contract is priced at 93.670 five months prior to expiration, the implication is that the market consensus expectation calls for three-month US dollar deposits to be offered in the London interbank market at a rate of approximately 6.330 percent per annum for spot settlement five months hence.

Two features of this scheme are noteworthy. One is that it ensures a loose but intuitively appealing imitation of the inverse relationship between a bond's price and its interest rate. As Exhibit 3 demonstrates, if the contract's reference interest rate rises, the contract price falls, and vice versa. A change of one interest rate basis point is always equal in magnitude, and opposite in sign, to a futures price change of 0.01 price points.

Exhibit 3 -- GE Futures Prices and Contract Rates, Tuesday, 14 December 2010



Source: CME Group

The second feature is that, practically speaking, the futures price is bounded by a minimum of zero and a maximum of 100 points. As a purely theoretical matter these bounds are not strict. Futures price might exceed 100, but only if market participants broadly expect the corresponding three-month US dollar interbank deposit rate to be set at negative levels. Similarly, futures price might drop below zero, but only if the contract's reference three-month interest rate were generally expected to exceed 100 percent per year.

⁸ To see why the two are "approximately" equal rather than exactly equal, see **Almost a Forward Rate, but Not Quite: Convexity Bias** on page 22.

Contract Size and Price Increments

One Basis Point = \$25

The notional scale of the GE futures contract is defined in terms of the dollar value of one interest rate basis point per annum, or DV01. One basis point (0.01 contract price points,⁹ or one “tick”) is always worth \$25. Crucially, this applies regardless of the length of time until the contract expires.

Minimum Price Increment = ½ Tick or ¼ Tick

With one exception, the minimum price movement for trading GE futures is one half of one interest rate basis point (0.005 contract price points or ½ tick), equal to \$12.50 per contract.

The exception is the nearby contract¹⁰ for which the minimum price movement is one quarter of one interest rate basis point (0.0025 contract price points or ¼ tick), equal to \$6.25 per contract. Here, “nearby” applies uniformly to Quarterly and Monthly Serial delivery months. On the eve of the nearby contract’s last day of trading, the next expiring contract, whether Quarterly or Monthly Serial, begins trading in ¼ tick minimum price increments.

Example 8

May 2015 Monthly Serial GE futures become the nearest expiring contract on Monday, 13 April 2015, the last day of trading in the April 2015 Monthly Serial. The minimum price increment for the May 2015 contract graduates from ½ tick to ¼ tick as of 5pm Chicago time on Sunday, 12 April 2015, when the CME Globex trading session commences for the Monday, 13 April, trade date. The May 2015 contract continues to trade in ¼ tick price increments until it too ceases trading and expires on Monday, 18 May 2015, at which time nearby status passes to the June 2015 Quarterly.

Notional Contract Size = \$1 Million ... More or Less

A popular rule of thumb says the value of one basis point of GE futures contract interest is \$25, because the size of the notional interbank deposit that serves as the contract’s underlying reference is \$1 million. The following equivalence formalizes the point:

$$\$25 \text{ per bp} = (\$1 \text{ million}) \times (0.01 \text{ percent per year}) \times (90 \text{ days} / 360 \text{ days per year})$$

If taken too literally, however, this rule of thumb may confuse rather than enlighten. That’s because it describes the structure of the GE futures contract backwards.

In fact, the contract’s notional bank placement turns out to be *approximately* \$1 million because the contract terms explicitly fix the DV01 at \$25. To see why this matters, recall that three-month intervals vary widely in duration.¹¹ With this in mind we can rearrange the equivalence above to show how the notional size of the contract reference bank deposit depends on the day count for the bank deposit’s term, subject to the requirement that the DV01 must be \$25:

$$\$1 \text{ million} = (\$25 \text{ per bp}) / \{ (0.01 \text{ percent per year}) \times (90 \text{ days} / 360 \text{ days per year}) \}$$

If the three-month interval beginning on the third Wednesday of a contract’s delivery month spans 90 days, then the size of the contract’s notional interbank placement will indeed be \$1 million.

⁹ To see why this is “crucial,” see **Almost a Forward Rate, but Not Quite: Convexity Bias** on page 22.

¹⁰ See **Nearby vs Front** on page 6.

¹¹ See **How Long is Three Months?** on page 3.

But as Column (2) of Exhibit 4 demonstrates, if the day count is 92 days instead of 90, then the notional bank placement's principal value must be around \$978,261 for the DV01 to remain \$25:

$$\$978,260.87 = (\$25 \text{ per bp}) / \{ (0.01 \text{ percent per year}) \times (92 \text{ days} / 360 \text{ days per year}) \}$$

Similarly, if the day count were 89 days rather than 90, the interbank placement's principal value would need to be around \$1,011,236 to keep the DV01 pinned at \$25.

Conversely, instead of assuming that the DV01 is fixed at \$25, one could maintain the assumption that the size of the interbank placement is fixed. If so, then as the day count for the placement's three-month interval lengthens, the DV01 rises, and the number of GE futures required to hedge the placement's interest rate exposure rises correspondingly. Columns (3) and (4) illustrate this for a bank deposit size fixed at \$1 billion.

In sum, to say a GE futures contract represents an interbank deposit with principal value of roughly \$1 million remains a useful heuristic. But the more basic point is that contract notional size is determined not by the nominal size of the underlying reference bank placement, but rather by the fact that *one basis point of GE contract interest is defined to be worth \$25, at all times, in all places.*

Exhibit 4 – GE Futures Notional Interbank Deposit Size, Interest Rate Sensitivity, and Day Count

(1)	(2)	(3)	(4)
Length of Reference Bank Deposit's 3-Month Interval (Days)	Deposit Size Required for DV01 to Equal \$25 (Dollars)	DV01 for Deposit = \$1 Million (Dollars per Basis Point)	Number of GE Futures Required to Hedge Deposit = \$1 Billion (1000 x (3) / \$25)
86	1,046,511.63	23.89	956
87	1,034,482.76	24.17	967
88	1,022,727.27	24.44	978
89	1,011,235.96	24.72	989
90	1,000,000.00	25.00	1,000
91	989,010.99	25.28	1,011
92	978,260.87	25.56	1,022
93	967,741.94	25.83	1,033
94	957,446.81	26.11	1,044
95	947,368.42	26.39	1,056

Source: CME Group

TRADING EURODOLLAR FUTURES

How GE futures trade is as important as the contract mechanism. The following sections spell out some of the more important rule of the road:

- the workings of GE Pack and Bundle trades;
- other GE combinations available to market participants in standardized form;
- how trade prices for such combinations get transformed into prices of component GE contracts;
- the algorithms by which CME Globex matches buyers and sellers to effect transactions; and
- the process by which the Exchange establishes daily settlement prices for GE futures.

Bundles and Packs

Many hedging and trading strategies call for the purchase or sale of GE futures in “strips,” or sequences of consecutive contract delivery months. Executing each of a strip’s component contracts, one by one, can be time-consuming and cumbersome. Worse, it may slow up position entry or exit, saddling the user with significant slippage in fast-moving markets.

GE Bundles and Packs provide a handy alternative. In a Bundle or Pack transaction, one buys or sells a strip of futures in a single trade, saving time and effort, and eliminating much inconvenience.

Packs

A Pack is the simultaneous sale or purchase of one each of a series of four GE futures with consecutive Quarterly delivery months. A Pack may be structured so that the nearest of its member contracts is any of the first 37 Quarterly delivery months. The most heavily traded Packs tend to be those corresponding to the ten color-coded delivery years, White through Copper, shown in Exhibit 2.¹² For instance, a Green Pack transaction on 14 December 2010 would entail simultaneous purchase or sale of one each of the four contracts for the 9th through 12th nearest delivery months -- in this case, one each of the March, June, September, and December 2013 futures.

Bundles

A Bundle is the simultaneous sale or purchase of one each of a series of eight or more GE futures with consecutive Quarterly delivery months. Bundles are listed in yearly tenors, from two to 10 years. (There are no one-year Bundles, because Packs effectively play this role.) As with Packs, the nearest member contract in a Bundle may be any Quarterly delivery month, as long as there is a sufficient number of ensuing Quarterly delivery months to accommodate the Bundle’s tenor. The most highly traded Bundles tend to be those for which the nearest contract in the Bundle sequence is the nearby Quarterly. See Exhibit 5.

Exhibit 5 – Popular GE Futures Bundles

Bundle Tenor (Years)	Bundle = 1 Each of the Following Quarterly Contracts:	DV01 (Dollars)	DV per 1/4 Tick (Dollars)
2	Nearest 8	200	50
3	Nearest 12	300	75
4	Nearest 16	400	100
5	Nearest 20	500	125
6	Nearest 24	600	150
7	Nearest 28	700	175
8	Nearest 32	800	200
9	Nearest 36	900	225
10	Nearest 40	1,000	250
5-Year Forward	Back 20 (Purple to Copper Years)	500	125

Source: CME Group

¹² See **Color Coding** on pages 4-5.

Bundle and Pack Prices

The price of any Bundle or Pack is quoted as the average change among its member GE contracts from their respective previous daily settlement prices. Prices are quoted in increments of ¼ tick.

For any Pack, the DV01 is \$100, and the dollar value of a ¼ basis point (ie, ¼ tick) interest rate change is always \$25. The third column of Exhibit 5 summarizes the interest rate sensitivity of highly traded Bundles in terms of their DV01s. These range from \$200 for the Two-Year Bundle to \$1,000 for the 10-Year Bundle. The fourth column shows the same in terms of the dollar value of a ¼ basis point (¼ tick) rate move.

How Bundle and Pack Prices Become GE Contract Prices

Once a Bundle or Pack is transacted, the price change at which it trades is assigned to each of its member contracts, in increments of 1 tick.¹³ If the Bundle or Pack trade price is an integer number of ticks (eg, -7 ticks on the day), then each constituent contract gets booked at a trade price equal to its previous daily settlement price adjusted by the Bundle or Pack price (eg, previous daily settlement price minus 7 ticks).

If the Bundle or Pack trade price involves a fractional portion (¼ or ½ or ¾) of a tick, then the exchange uses the following method to assign prices to the member contracts:

- (1) Initially, each member contract is assigned a price change equal to the integer (non-fractional) portion of the Bundle or Pack price.
- (2) The individual contract price changes are then adjusted, one by one, until their average value equals the Bundle or Pack price. If the Bundle or Pack price is positive, the adjustment pushes the individual contract price change up to the next higher tick. If the Bundle or Pack price is negative, the adjustment pushes the individual contract price change down to the next lower tick.
- (3) Importantly, the adjustment process described in (2) starts with the most distant delivery month among the contracts in the Bundle or Pack, then works forward until the average of all contract price changes matches the Bundle or Pack price.

With the prices of its constituent contracts thus determined, the Bundle or Pack is promptly unbundled (or unpacked, as the case may be). From then on, each member contract is marked to market and otherwise treated as if it were a stand-alone GE futures position.

Examples 9

The Two-Year Bundle trades at +2.25 ticks. To accommodate the ¼ tick portion of the Bundle price, the Bundle's two most deferred contracts get booked at trade prices equal to their previous daily settlements plus 3 ticks, while the six nearest contracts get booked at trade prices equal to their previous daily settlements plus 2 ticks. The average price increment among the Bundle's eight member contracts is as it should be:

$$+2.25 \text{ ticks} = \{ (6 \text{ nearest contracts} \times +2 \text{ ticks}) + (2 \text{ most deferred contracts} \times +3 \text{ ticks}) \} / 8 \text{ contracts}$$

Suppose the Ten-Year Bundle trades at -5.75 ticks. Among its 40 member contracts, the 30 most distant delivery months get booked at a net price change of -6 ticks versus their previous daily settlement prices, while the nearest 10 delivery months are assigned a net price change of -5 ticks.

$$-5.75 \text{ ticks} = \{ (10 \text{ nearest contracts} \times -5 \text{ ticks}) + (30 \text{ most deferred contracts} \times -6 \text{ ticks}) \} / 40 \text{ contracts}$$

¹³ To learn how CME Globex matches buyer and seller to make the Bundle or Pack transaction in the first place, see **CME Globex Trade Matching Algorithms for GE Futures** on page 19.

Assume the Purple Pack trades at +0.5 ticks. The third and fourth delivery months within the Pack get booked at a net price change of +1 tick versus their respective previous daily settlement prices. The first and second delivery months get booked at prices unchanged from their previous daily settlements --

$$+0.5 \text{ ticks} = \{ (2 \text{ nearest contracts @ unchanged}) + (2 \text{ most deferred contracts} \times +1 \text{ tick}) \} / 4 \text{ contracts}$$

Calendars, Flies, Condors, and Other Combinations

In addition to Packs and Bundles, eight other popular combination strategies are standardized on CME Globex, permitting market participants to transact them directly in spread form, instead of having to leg into or out of them. These include calendar spreads, butterflies, double butterflies, condors, month-Pack spreads, Pack spreads, Pack butterflies, and Bundle spreads.

For the first four – calendar spreads, butterflies, double butterflies, and condors -- the building blocks are individual GE contracts. (Examples 10 through 14, interspersed throughout, make reference to market conditions on Tuesday, 14 December 2010, shown in Exhibit 2.)

Calendar Spread

...consists of two GE contracts with different delivery months. When traded on CME Globex, the ratio for purchase of a calendar spread is always +1:-1. Thus, buying 1 calendar spread means:

- (a) buying 1 nearer delivery month (Leg1) and
- (b) selling 1 farther delivery month (Leg2).

The calendar spread price is quoted as (Leg1 price) minus (Leg2 price). For any calendar spread in which at least one leg is either a Monthly Serial and/or the nearby Quarterly delivery month, the minimum price movement for trading is one quarter of one spread basis point (0.0025 contract price points or ¼ tick), equal to \$6.25 per spread.

Example 10

Consider a Jan11-Mar11 calendar spread. At opening price levels on Tuesday, 14 December 2010, the Jan11 contract is 99.6625, and Mar11 is 99.59. At these levels, the Jan11-Mar11 spread would be fairly valued at 7.25 ticks, equal to 99.6625 minus 99.59.

For any other calendar spread the minimum price movement is one half of one calendar spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Example 11

At daily settlement price levels, the Jun11-Jun13 two-year calendar spread would be fairly valued at 183.0 ticks, equal to 99.44 minus 97.61.

Butterfly

... comprises three Quarterly GE contracts with equally distributed delivery months. CME Globex routinely enables trading in butterflies in three standardized configurations: three-month (eg, Jun13-Sep13-Dec13), six-month (eg, Jun13-Dec13-Jun14), nine-month (eg, Jun13-Mar14-Dec14), and one-year (eg, Jun13-Jun14-Jun15). In all instances, the ratio for purchase of a butterfly is +1:-2:+1. So buying 1 butterfly means:

- (a) buying 1 of the nearest delivery month (Leg1),
- (b) selling 2 of the second nearest delivery month (Leg2), and
- (c) buying 1 of the farthest delivery month (Leg3).

The butterfly spread price is quoted as (Leg1 price) minus (2 x Leg2 price) plus (Leg3 price). Minimum price movement is always one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Double Butterfly

... combines four Quarterly GE contracts with equally distributed delivery months. It's useful to imagine a double butterfly as a calendar spread between two conventional butterflies, in which the second and third legs of the nearer butterfly also serve duty as the first and second legs, respectively, of the more distant butterfly. Accordingly, the double butterfly spread ratio is always +1:-3:+3:-1. That is, buying 1 double butterfly entails:

- (a) buying 1 of the nearest delivery month (Leg1),
- (b) selling 3 of the second nearest delivery month (Leg2),
- (c) buying 3 of the third nearest delivery month (Leg3), and
- (d) selling 1 of the farthest delivery month (Leg4).

CME Globex customarily permits trading in three variants: three-month (eg, Jun13-Sep13-Dec13-Mar14), six-month (eg, Jun13-Dec13-Jun14-Dec14), and one-year (eg, Jun13-Jun14-Jun15-Jun16). Price is always quoted as (Leg 1 price) minus (3 x Leg 2 price) plus (3 x Leg 3 price) minus (Leg 4 price). Minimum price movement is one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Condor

... is a combination of four Quarterly GE contracts with equally distributed delivery months. As with double butterflies, three species of condor are routinely available for trading on CME Globex, distinguished from one another by breadth of wing span: three-month (eg, Jun13-Sep13-Dec13-Mar14), six-month (eg, Jun13-Dec13-Jun14-Dec14), and one-year (eg, Jun13-Jun14-Jun15-Jun16). The spread ratio is always +1:-1:-1:+1. So buying a condor means:

- (a) buying 1 of the nearest delivery month (Leg1),
- (b) selling 1 of the second nearest delivery month (Leg2),
- (c) selling 1 of the third nearest delivery month (Leg3), and
- (d) buying 1 of the farthest delivery month (Leg4).

Price is quoted as (Leg1 price) minus (Leg2 price) minus (Leg3 price) plus (Leg4 price). Minimum price movement is one half of one spread basis point (0.005 price points or ½ tick), equal to \$12.50 per spread.

Examples 12

At daily settlement price levels, the Mar11-Jun11-Sep11 three-month butterfly would be fairly valued at -0.5 ticks, equal to 99.585 minus (2 x 99.44) plus 99.29. At opening price levels, fair value in the Jun11-Jun12-Jun13-Jun14 one-year double butterfly would be -28.5 ticks, equal to 99.45 minus (3 x 98.78) plus (3 x 97.80) minus 96.795.

The remaining four standardized combinations – month-Pack spreads, Pack spreads, Pack butterflies, and Bundle spreads – employ GE Packs and Bundles as their elementary building blocks.

Month-Pack Spread

...consists of a position in a Pack combined with a countervailing position in a single GE futures contract with a Quarterly delivery month. The four GE contracts in the Pack leg are always the next four Quarterly delivery months immediately following the delivery month for the contract on the singleton leg. The spread ratio is always (+4 singletons):-1 pack). In other words, buying a month-Pack spread entails:

- (a) buying 4 of the (nearer) single GE delivery month and
- (b) selling 1 (more distant) Pack.

Pack Spread

...is similar to the futures calendar spread described above, except built with Packs instead of individual futures contracts. Each of the spread's two Packs corresponds to one of the color-coded delivery years exemplified in Exhibit 2. Thus, the futures delivery months represented in the Pack on one leg are always removed from the futures delivery months in the Pack on the other leg by an integer multiple of one year, ranging from one (eg, White-Red, Silver-Copper) to nine (uniquely, White-Copper). The spread ratio is always +1:-1. Buying 1 Pack spread means:

- (a) buying 1 Pack of GE contracts with nearer delivery months (Leg 1) and
- (b) selling 1 Pack of GE contracts with more distant delivery months (Leg 2).

Pack Butterfly

...is simply a butterfly spread where each of the legs is a Pack. As with the Pack Spread, each of the Packs in the butterfly must correspond to one of the ten color-coded GE futures delivery years. CME Globex permits trading in two species of Pack butterfly: one-year (eg, Green-Blue-Gold) and two-year (eg, Green-Gold-Orange). As with the futures butterfly, the spread ratio for purchase of a Pack butterfly is always +1:-2:+1. So buying 1 Pack butterfly means:

- (a) buying 1 Pack of GE contracts with four nearest delivery months (Leg 1),
- (b) selling 2 Packs of GE contracts with next four nearest delivery months (Leg 2),
- (c) buying 1 Pack of GE contracts with the most distant four delivery months (Leg 3).

Bundle Spread

...is a calendar spread between two Bundles. The Bundle on one leg must comprise *at least two* GE futures delivery months that are not represented in the Bundle on the other leg. Moreover, each leg must have the same number of constituent GE contracts as the other leg. For instance, a Two-Year Bundle can be paired only with another Two-Year Bundle to make a Bundle spread. The Bundle spread ratio is always +1:-1. Buying 1 Bundle spread means:

- (a) buying 1 Bundle of GE contracts for nearer delivery (Leg 1) and
- (b) selling 1 Bundle of GE contracts for more distant delivery (Leg 2).

Prices of month-Pack spreads, Pack spreads, Pack butterflies, and Bundle spreads are quoted in the same terms as for the component Packs and Bundles. In each case, the spread price is quoted as the net change on the day among the legs of the spread,¹⁴ and the minimum price movement is one quarter of one spread basis point (0.0025 contract price points or ¼ tick).

Examples 13

Assume that, just prior to the daily 2pm CT daily settlement, Red, Green, and Blue Packs are priced at -10 ticks, -18 ticks, and -25.25 ticks, respectively, in their characteristic "net-change-on-the-day" terms. The Red-Blue 2-year Pack spread would be fairly valued at a net change of +15.25 ticks, equal to (-10 ticks) minus (-25.25 ticks). The Red-Green-Blue one-year Pack butterfly would be fairly priced at a net change of +0.75 ticks, equal to (-10 ticks) minus (2 x -18 ticks) plus (-25.25 ticks).

¹⁴ For any month-Pack spread, the price of the GE futures contract on the spread's singleton leg is expressed not in the usual 100-minus-rate futures price format, but rather in terms of the net-change-on-the-day convention that applies to Packs.

How Combination Prices Become GE Contract Prices

Like Packs and Bundles, the other eight combination strategies described above are standardized so as to facilitate position entry or exit in one trade rather than many. Another important similarity is that, from the standpoint of the exchange, they are ephemeral. Like a Pack or a Bundle, a calendar spread or butterfly or condor is not marked-to-market as such. It loses its identity at the moment CME Globex assigns prices to its constituent GE contracts. From then on, each of the spread's member GE contracts is marked-to-market and otherwise treated as if it were a distinct GE futures position.

The chief difference between Packs and Bundles versus the other combination strategies is how CME Globex translates their trade prices into prices for booking the member GE contracts. As described earlier,¹⁵ the price of a Pack or Bundle is apportioned to its constituent contracts on the basis of (a) each contract's net price change versus its latest daily settlement price and (b) the length of term to expiry for each contract relative to the other contracts in the Pack or Bundle.

By contrast the price of a calendar spread, butterfly, double butterfly, condor, or month-Pack spread is distributed to its component GE contracts essentially on the basis of current market conditions in those contracts. Before we explain the process, an item of terminology requires introduction.

C-Last Price

The CME Last (or C-Last) price for a GE contract for a given delivery month is simply the contract's latest trade price, or actionable price indication, or settlement price. Specifically, the C-Last price is the most recent of:

- (a) latest CME Globex transaction price, or
- (b) CME Globex bid price that betters the bid side of the market, or
- (c) CME Globex asking price that betters the ask side of the market, or
- (d) latest daily settlement price.

In this context, a bid that betters the market is understood to be a bid to buy at a higher price than the incumbent C-Last price. Similarly, a better ask price is an offer to sell at a price below the preceding C-Last price.

Calendar Spread = (+1 Leg1):(-1 Leg2)

To assign prices to a calendar spread's legs, CME Globex first checks which one has traded more recently during the current CME Globex trading session and assigns the fresh trade price to that leg. This is then combined with the transaction price of the calendar spread to derive the price of the other leg:

If Leg1 has traded more recently than Leg2, then
Leg1 is assigned its latest trade price and
Leg2 price = (Leg 1 price) minus (calendar spread price).

If Leg2 has traded more recently than Leg1, then
Leg2 is assigned its latest trade price and
Leg1 price = (Leg 2 price) plus (calendar spread price).

If the latest trades in both legs occurred simultaneously, then Leg1 takes precedence: Leg1 is assigned its latest trade price, and Leg 2's price is derived as (Leg 1 price) minus (calendar spread price).

¹⁵ See **How Bundle and Pack Prices Become GE Contract Prices** on page 11.

Finally, if there have been no trades in either of the Leg1 or Leg2 contracts during the current CME Globex trading session, then the calendar spread's legs are assigned on the basis of Leg1's latest daily settlement price:

Leg1 price = Leg1's latest daily settlement price, and
 Leg2 price = (Leg1 price) minus (calendar spread price).

Examples 14

Suppose the Jun11-Jun13 two-year calendar spread trades at 165.5 ticks. Suppose that, of the two legs, the latest trade has occurred in the Jun13 GE contract at a price of 97.80. For the buyer, CME Globex confirms the transaction as the sale of the Jun13 at its latest trade price of 97.80 and the purchase of the Jun11 GE contract at an imputed price of 99.455, equal to Jun13 price plus calendar spread trade price, 97.80 plus 1.655. Conversely, the seller receives confirmation of a sale of Jun11 at 99.455 and a purchase of Jun13 at 97.80.

Suppose instead that the latest transaction in the Jun13 GE contract, at 97.80, coincides with a trade in the Jun11 GE contract at 99.43. The nearer Jun11 takes precedence. The buyer of the calendar spread receives confirmation of purchase of Jun11 at its latest trade price, 99.43, and sale of Jun13 at a calculated price of 97.775, equal to Jun11 trade price minus calendar spread trade price, or 99.43 minus 1.655, and conversely for the seller.

Butterfly = (+1 Leg1):(-2 Leg2):(+1 Leg3)

To assign prices to the legs of a butterfly, CME Globex books Leg1 and Leg2 at their respective C-Last prices, then derives Leg3's price as

Leg3 price = (butterfly price) minus (Leg1 price) plus (2 x Leg2 price)

Double Butterfly = (+1 Leg1):(-3 Leg2):(+3 Leg3):(-1 Leg4)

To map a double butterfly price into its component contract prices, CME Globex sets Leg1, Leg2, and Leg3 at their respective C-Last prices, then computes Leg4's price as

Leg4 price = (Leg1 price) minus (3 x Leg2 price) plus (3 x Leg3 price)
 minus (double butterfly price)

Condor = (+1 Leg1):(-1 Leg2):(-1 Leg3):(+1 Leg4)

Likewise, to bust a condor price into its component contract prices, CME Globex books Leg1, Leg2, and Leg3 at their respective C-Last prices, and sets Leg4's price as

Leg4 price = (condor price) minus (Leg1 price) plus (Leg2 price) plus (Leg3 price)

Examples 15

The Mar11-Jun11-Sep11 three-month butterfly trades at -1 tick. Assume C-Last prices for the Mar11 and Jun11 contracts are 99.585 and 99.44, respectively. CME Globex confirms to the buyer the purchase of one Mar11 contract at 99.585, the sale of two Jun11 contracts at 99.44, and the purchase of one Sep11 contract at an imputed price of 99.285, equal to -0.01 minus 99.585 plus (2 x 99.44), and conversely to the seller.

A Jun11-Jun12-Jun13-Jun14 one-year double butterfly trades at -27 ticks. Assume C-Last prices for the Jun11, Jun12, and Jun13 contracts are, respectively 99.45, 98.78, and 97.80. CME Globex confirms to the buyer the purchase of one Jun11 contract at 99.45, the sale of three Jun12 contracts at 98.78, the purchase of three Jun13 contracts at 97.80, and the sale of one Jun14 contract at an imputed price of 96.78, equal to (Leg1 price) minus (3 x Leg2 price) plus (3 x Leg3 price) minus (double butterfly price), or 99.45 minus (3 x 98.78) plus (3 x 97.80) minus (-0.27).

Month-Pack Spread = +4 Leg1 GE:-1 Leg2 Pack

In distributing a month-Pack spread price to its components, CME Globex always accords precedence to the singleton GE contract on Leg1. With this in mind, it is convenient to imagine the price attribution process working in four steps:

- (a) Leg1 GE contract is set to its C-Last price.
- (b) Leg1 GE price is re-expressed in Pack-equivalent terms, as (contract C-Last price) minus (contract previous daily settlement price)
- (c) Leg2 Pack price is then derived as (Leg1 Pack-equivalent price) minus (month-Pack spread price).
- (d) Leg2 Pack price, as determined in Step (c), is then translated into price levels for each of the Pack's four member contracts, as described earlier.¹⁶

CME Globex confirms to the buyer the purchase of four Leg1 GE futures contracts at the price established in Step (a) and the sale of one each of the Pack's four constituent contracts as determined in Step (d). The seller of the spread receives the obverse confirmation.

Example 16

Consider the purchase of a month-Pack spread comprising purchase of four Dec12 GE contracts and sale of a Pack of one each of the Mar13, Jun13, Sep13, and Dec13 contracts. The spread trades at +4.5 ticks. CME Globex apportions this spread price to the component contracts as follows:

- (a') The Dec12 contract on Leg1 gets its C-Last price. Assume this is 99.11.
- (b') The Dec12 contract price is re-expressed in Pack-equivalent terms. Assuming the contract's previous daily settlement price is 99.165, its Pack-equivalent price is -5.5 ticks, equal to 99.11 minus 99.165.
- (c') Leg2 Pack price is derived as -10 ticks, equal to Leg1's Pack-equivalent price minus the month-Pack spread price, or (-5.5 ticks) minus (+4.5 ticks).
- (d') Each of the Pack's four member contracts (Mar13, Jun13, Sep13, and Dec13) gets booked at its respective previous daily settlement price minus 10 ticks.

Understanding the futures contract price assignment process for the remaining three combinations -- Pack spreads, Pack butterflies, and Bundle spreads -- calls for a modified definition of C-Last Price:

Pack/Bundle C-Last Price

Unlike the C-Last Price for a futures contract, the Pack/Bundle C-Last Price makes no reference to a previous daily settlement price.¹⁷ Within any CME Globex trading session, the C-Last Price for a given Pack or Bundle is the most recent of:

- (a) latest CME Globex transaction price, or
- (b) CME Globex bid price that betters the bid side of the market, or
- (c) CME Globex asking price that betters the ask side of the market.

If there is no C-Last Price for a given Pack or Bundle during the current CME Globex trading session, CME Globex calculates a synthetic price for the Pack or Bundle using futures C-Last Prices. For each GE contract in the Pack or Bundle, it obtains the differential between the contract's C-Last Price and the contract's previous daily settlement price. It then computes the arithmetic average of these differentials. The average value gets rounded to the nearest ¼ tick, with tie values rounded towards zero. For instance, an average of -2.125 ticks gets rounded up to -2 ticks, whereas an average of +5.625 ticks gets rounded down to +5.5 ticks.

¹⁶ See **How Bundle and Pack Prices Become GE Contract Prices** on page 11.

¹⁷ Technically, daily settlement prices are made only for futures contracts, not for Packs, Bundles, or any other combinations or spreads.

One set of ground rules applies to contract price assignments for Pack spreads, Pack butterflies, and Bundle spreads. The nearby legs of the combination always take precedence, in the sense that they are always set to their respective Pack/Bundle C-Last Prices. The most deferred leg of the combination is always derived on the basis of the price(s) assigned to the other leg(s) and the combination trade price. Finally, with prices thus established for all Packs or Bundles involved in the combination, each Pack or Bundle price is then distributed to its respective member GE contracts, as described earlier.¹⁸

Pack Spread = (+1 Leg1 Pack):(-1 Leg2 Pack)

Leg1 Pack gets assigned its Pack C-Last Price, as defined above. Leg2 Pack price is then derived as the difference between Leg1 Pack price and the Pack spread price –

$$\begin{aligned} \text{Leg1 Pack price} &= \text{Leg1 Pack C-Last Price, and} \\ \text{Leg2 Pack price} &= (\text{Leg1 Pack price}) \text{ minus (Pack spread price).} \end{aligned}$$

Pack Butterfly = (+1 Leg1 Pack):(-2 Leg2 Packs):(+1 Leg3 Pack)

The Packs on Leg1 and Leg2 are given their respective Pack C-Last Prices. CME Globex then uses these and the Pack butterfly price to impute the price of the Leg3 Pack –

$$\begin{aligned} \text{Leg1 Pack price} &= \text{Leg1 Pack C-Last Price} \\ \text{Leg2 Pack price} &= \text{Leg2 Pack C-Last Price} \\ \text{Leg3 Pack price} &= (\text{Pack butterfly price}) \text{ minus (Leg1 Pack price)} \\ &\quad \text{plus (2 x Leg2 Pack price)} \end{aligned}$$

Bundle Spread = (+1 Leg1 Bundle):(-1 Leg2 Bundle)

Leg1 Bundle is set to its Bundle C-Last Price, after which the Leg2 Bundle price is calculated as the difference between the Leg1 Bundle price and the Bundle spread price –

$$\begin{aligned} \text{Leg1 Bundle price} &= \text{Leg1 Bundle C-Last Price, and} \\ \text{Leg2 Bundle price} &= (\text{Leg1 Bundle price}) \text{ minus (Bundle spread price).} \end{aligned}$$

Examples 17

The Red-Green one-year Pack spread trades at +8 ticks. Assume the Red Pack C-Last price is -10 ticks. Because it is the nearer of the two Packs, it gets booked at this price. The imputed price at which the Green Pack gets booked is -18 ticks, equal to the Leg1 price minus the spread price, or (-10 ticks) minus (+8 ticks). The buyer of the Pack spread receives buy confirmations for one each of the four GE contracts in the Red Pack, at prices equal to their respective previous daily settlement prices minus 10 ticks, and sell confirmations for one each of the four GE contracts in the Green Pack, at prices equal to their respective previous daily settlements minus 18 ticks. The seller of the Pack spread gets the obverse set of confirmations.

The Red-Blue-Purple 2-year Pack butterfly trades at +14.75 ticks. The relatively nearby Red and Blue Packs get booked at their Pack C-Last prices. Suppose these are, respectively, -10 ticks and -25.5 ticks. The most deferred component of the butterfly, the Purple Pack, gets booked at the derived price of -26.25 ticks, equal to (butterfly price) minus (Red Pack price) plus (2 x Blue Pack price), or (+14.75 ticks) minus (-10 ticks) plus (2 x -25.5 ticks). For each Pack the assigned price is then distributed to the Pack's member futures contracts.

¹⁸ As before, see **How Bundle and Pack Prices Become GE Contract Prices** on page 11.

CME Globex Trade Matching Algorithms for GE Futures

CME Globex matches every GE futures trade according to one of two matching algorithms, depending upon the contract or spread or combination being transacted. Before taking a closer look at these, two bits of nomenclature are worth establishing.

Resting Orders versus Aggressor Orders

A resting order is a posted actionable bid or offer at a given price for a given number of futures contracts (or for a given number of Bundles or Packs or other defined strategies). An aggressor order is either a sell order that entails hitting resting bid orders, or a buy order that entails lifting resting offered orders.

TOP Order

...is the first order to improve the market – ie, to achieve a higher resting best bid or a lower resting best offer -- at a given prevailing price level. TOP order designation ensures that any order that seizes the initiative in improving its side of the market will be filled before other resting orders on the same side of the market.¹⁹ Both the bid side and the offered side of the market may simultaneously have TOP orders. At any given moment, however, no more than one buy order and no more than one sell order can hold TOP designation.

On either side of the market, there may not always be a TOP order. Suppose for example that an incoming resting buy order improves the market and becomes TOP bid. If that order is subsequently cancelled, TOP status does not automatically pass to another resting bid already in the order book. Rather, TOP designation will be conferred upon the next arriving buy order that improves upon the prevailing best bid price. The same applies on the offered side of the market.

Contracts, Calendar Spreads, Butterflies, Double Butterflies, Condors, Pack Spreads, and Pack Butterflies

...are matched by the “Pro Rata Allocation with TOP Price” (or A) algorithm. In essence, the A algorithm distributes an incoming aggressor order pro rata to resting orders at the best price, with deference to the TOP order if there is one. Specifically:

- (a) When there is a TOP order, the aggressor order is assigned to it first.
- (b) Any unfilled portion of the aggressor order is then matched to other resting orders at the best price, on a pro rata basis. At any given resting price level each resting order's pro-rated percentage is calculated by dividing its order quantity by the total quantity of all resting orders at that price level. Any unfilled portion of the aggressor order remaining after Step (a) is then multiplied by each resting order's pro-rated percentage.
- (c) The quantity allocated to each resting order is the result from Step (b) rounded down to the nearest integer number of contracts. For any resting order to which the rounded-down allocation would be less than two contracts, the quantity assigned is zero.
- (d) Any unfilled portion of the aggressor order that remains after application of Steps (b) and (c) is then assigned on a FIFO basis.

If there is no TOP order, the process starts with Step (b).

¹⁹ Among other of the Exchange's futures contracts, TOP order status may be governed by a minimum, whereby an order that improves the market is designated as TOP only if its size meets or exceeds a specified minimum threshold. Likewise, for some contracts, TOP order status is subject to a maximum, whereby the TOP order enjoys priority in getting filled only up to a specified amount. In many instances, both a minimum and a maximum apply.

Exhibit 6 illustrates how the algorithm would work where a hypothetical aggressor sell order for 633 contracts meets five resting orders at the best bid, as described in Columns 1 and 2. ("Contract" is used loosely to signify either an individual GE futures contract or any of the combinations listed above.)

Exhibit 6

Algorithm A – Pro Rata Allocation with TOP Price. Aggressor Order = 633 Contracts.

(Number of Contracts, Unless Otherwise Noted)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Resting Orders at Best Bid, by Time Priority	Order Size	TOP Allocation	Resting Orders after TOP Allocation = (2)-(3)	Pro Rata Allocation Percentage	Pro Rata Allocation = (4)x(5)	Resting Orders after Pro Rata Allocation = (4)-(6)	FIFO Allocation	Total Allocation = (3)+(6)+(8)
1 – TOP	150	150						150
2	5		5	5 / 3005		5	2	2
3	1000		1000	1000 / 3005	160	840		160
4	500		500	500 / 3005	80	420		80
5	1500		1500	1500 / 3005	241	1259		241
Total		150			481		2	633

Source: CME Group

The five resting orders at the best price signify total depth at the best bid equal to 3,155 contracts. Proceeding as above --

- The highest-priority resting order, for 150 contracts, holds TOP status (Column 1). It gets matched first, in this case in its entirety (Column 3).
- The remaining 483 contracts in the aggressor order are distributed pro rata across the 3,005 contracts that remain among resting orders at the best price level (Column 5). Each pro rata allocation is rounded down to the nearest integer number of contracts. Note that the allocation to the second resting order in terms of time priority is less than two contracts and is therefore rounded down to zero (Column 6).
- The remaining 2 contracts in the aggressor order are distributed FIFO. The second resting order in terms of time priority, for 5 contracts, absorbs this assignment (Column 8).

Column 9 summarizes the resultant distribution of the aggressor sell order among the resting bid orders.

Packs, Bundles, Month-Pack Spreads, and Bundle Spreads

...are matched by the "FIFO with LMM" (or T) algorithm. The T algorithm first allocates any aggressor order to designated lead market makers (LMMs), after which the remaining unmatched portion of the aggressor order gets matched on a FIFO basis.

Exhibit 7 illustrates with another hypothetical example. As in Exhibit 6, an aggressor sell order for 633 contracts meets five resting orders at the best bid, as specified in Columns 1 and 2. (Note that "contract" denotes a combination – a Pack, Bundle, month-Pack spread, or Bundle spread – rather than an individual GE futures contract.) Moreover, the resting orders in Exhibit 7 include bids posted by a cadre of lead market makers, each of whom is assumed to be entitled to a 15 percent share of any aggressor order; each such share allocation gets rounded down to the nearest integer number of contracts.

Accordingly, each of the three LMMs is allocated 94 contracts, equal to 15 percent of 633 contracts (Column 3). The remaining 351 contracts in the aggressor order are then assigned on a FIFO basis (Column 5). Column 6 summarizes how the algorithm distributes the aggressor sell order among the resting bid orders.

Exhibit 7

Algorithm T – FIFO with LMM. LMM Allocation = 15%. Aggressor Order = 633 Contracts.
(Number of Contracts, Unless Otherwise Noted)

(1) Resting Orders at Best Bid, by Time Priority	(2) Order Size	(3) LMM Allocation = 0.15 x 633	(4) Resting Orders after LMM Allocation = (2)-(3)	(5) FIFO Allocation	(6) Total Allocation = (3)+(5)
1 – LMM	150	94	56	56	150
2	5		5	5	5
3 – LMM	1000	94	906	290	384
4 -- LMM	500	94	406		94
5	1500		1500		
Total		282		351	633

Source: CME Group

Daily Settlement Prices

The exchange bears sole responsibility not only for establishing the final settlement price of an expiring contract, but also for setting contract daily settlement prices, by which marks-to-market are determined. The daily settlement price for any of the nearest 12 Quarterly contracts (any GE contract in the White, Red, or Green delivery years) is based on the levels of bid and offered prices on the CME Globex electronic trading platform between 1:59 and 2:00 pm Chicago time.

The daily settlement price for any Monthly Serial futures contract is likewise based on information drawn from the CME Globex electronic trading platform between 1:59 and 2:00pm CT – either as a transaction-volume-weighted average price or, in the absence of any transactions, as the midpoint of the prevailing bid and offered prices.

In all instances, these daily settlement prices may incorporate adjustments, within the prevailing bid/offered price spread, that reflect trading conditions in Packs, Bundles, and other GE futures spread combinations.

Daily settlement prices for Quarterly contracts for more distant delivery months – those in the Blue, Gold, Purple, Orange, Pink, Silver, or Copper delivery years – are determined on the basis of market quotes in the Eurodollar futures trading pit, as canvassed by exchange officials at 1:50, 1:55 pm, and 2:00 pm CT.²⁰

²⁰ Reminder: There is no daily settlement price for any Pack, Bundle, or other combination or spread.

Almost a Forward Rate, but Not Quite: Convexity Bias

A close cousin of the GE futures contract is the three-month forward rate agreement (FRA), which trades widely in both over-the-counter interbank markets and dealer-to-customer markets for treasury management products. Despite numerous similarities, GE futures and FRAs are by no means identical. Among the salient distinctions between them is that their market values respond differently to interest rate volatility.

The upshot of this difference is that GE futures contract rates systematically exceed the “true” forward interest rates at which the corresponding FRAs are priced. Market practitioners refer to this phenomenon as the convexity bias in GE futures prices. The following passages describe how convexity bias comes about, and how market participants quantify it in practice.

Convexity in FRAs vs Linearity in GE Futures

To begin, consider some heuristic examples in which the term structure of spot and forward interest rates is assumed to be flat at 4 percent per annum --

Example 18

A par FRA is struck at a forward-starting 90-day rate equal to 4 percent per annum, on a notional \$1 million interbank loan, for forward settlement two years hence and for maturity two years three months hence. Suppose all interest rates increase by one basis point. Two and a quarter years from now, this change in market conditions will cost \$25 for the seller of the FRA (notionally, the future lender) and will reward the FRA buyer (notionally, the future borrower) by the same amount:

$$\$25 = \$1 \text{ million} \times (0.01 \text{ percent per year} / 100) \times (90 \text{ days} / 360 \text{ days per year})$$

Market convention calls for the FRA’s profit/loss to be reckoned in terms of its present value, rather than its value 2.25 years from now. At an interest rate of 4.01 percent per annum, one dollar due in 2.25 years is worth approximately 91.53 cents today. Accordingly, the profit/loss is booked at \$22.88 instead of \$25:

$$\$22.88 = \$25 \times 0.915337 = \$25 \times (1 + (4.01 \text{ percent}/100))^{-2.25}$$

Example 19

The same applies to an FRA for any expiry date. Thus, consider another FRA with identical terms -- forward-starting 90-day rate of 4 percent per annum on a notional \$1 million interbank loan -- except that it is struck for forward settlement nine years hence instead of two years hence. A one-basis-point rise in interest rates to 4.01 percent will translate to a forward profit/loss of \$25, to be recognized 9.25 years in the future. At a rate of discount of 4.01 percent per annum, a dollar due in 9.25 years is worth 69.51 cents today. Thus, the profit/loss on the FRA to be booked today is:

$$\$17.38 = \$25 \times 0.695113 = \$25 \times (1 + (4.01 \text{ percent}/100))^{-9.25}$$

Example 20

Now compare the FRAs in Examples 18 and 19 to their corresponding GE futures. With all interest rates assumed to start out at 4 percent per annum, a naïve but seemingly reasonable assumption is that GE futures for delivery either in two years or in nine years both should be priced at 96.000, equal to 100 minus 4 percent.²¹ In both cases, moreover, a one-basis-point rise in all expected future interest rates presumably would reduce price to 95.990, equal to 100 minus 4.01 percent. Most important, both of the ensuing marks-to-market presumably would equal \$25 because, unlike their corresponding FRAs, GE futures are defined so that *an interest rate basis point is always worth \$25, irrespective of when the futures contract expires.*²²

²¹ Assuming this example takes place on the eve of the nearby Quarterly contract expiration, these two GE futures would be First Green and First Copper, respectively, in terms of the color-coding scheme for delivery months (see **Delivery Month and Last Trading Day** on pages 4-5).

²² See **Contract Size and Price Increments** on page 8.

Because the interest rate sensitivity of each GE contract is \$25 per basis point, always and everywhere, its response to interest rate changes is described as “linear.” Because the interest rate sensitivity of the FRA depends on both the prevailing level of interest rates and the FRA’s term to expiry, the relationship between changes in its value and changes in its reference interest rate is described as “convex.”

A key consequence of this difference is that the pricing of GE futures does not take place as conjectured in Example 20. How GE futures do get priced is revealed in the dynamics of their cash-futures basis.

Volatility and the GE cash-futures basis

The difference in price responsiveness to interest rate changes – linear for GE futures, convex for FRAs – causes cash-futures spread positions to be sensitive to interest rate volatility in a peculiar way: In the absence of any correction for the structural differential between linear pricing of futures and convex pricing of FRAs, any movement in interest rates almost always benefits the buyer of the basis position (the notional future lender in the FRA and seller of GE futures) to the detriment of the basis seller (the notional future borrower in the FRA and buyer of GE futures). The next two examples illustrate.

Example 21

Return to the FRA in Example 19, and assume the notional amount is \$100 million instead of \$1 million. The profit/loss to be realized on the FRA 9.25 years from now will be \$2,500 for every one basis point change in the FRA’s underlying reference interest rate. As above, for the purpose of reckoning the profit/loss to be booked today, the DV01 of the FRA is the present value of \$2,500 to be received 9.25 years in the future:

$$\$1,738 = \$2,500 \times 0.695113$$

Thus, to buy the corresponding cash-futures basis spread, one would sell \$100 mln notional of the FRA as the cash leg and sell 70 GE futures for the First Copper delivery month.²³ The spread ratio of 70 futures is chosen to bring the cash-futures spread as close as possible to (local) DV01-neutrality:

$$\begin{aligned} \text{GE leg DV01} &= \text{FRA leg DV01} \\ 70 \text{ contracts} \times (\$25 \text{ per bp per contract}) &= \$1,750 \text{ per bp} \approx \$1,738 \text{ per bp} \end{aligned}$$

Now assume all interest rates rise 10 bps, to 4.10 percent. The basis spread position earns a profit of \$261, with the FRA component losing \$17,239, while the short GE component earns \$17,500 --

$$\begin{aligned} \text{Loss on FRA position} &= -\$17,239 \\ &= 10 \text{ bps} \times (-\$2,500 \text{ per bp}) \times (1 + (4.10 \text{ percent}/100))^{-9.25} \end{aligned}$$

$$\begin{aligned} \text{Earnings on short position in GE futures} &= \$17,500 \\ &= (-10 \text{ GE price ticks}) \times (\$25 \text{ per tick per contract}) \times (-70 \text{ contracts}) \end{aligned}$$

Next, assume instead that all interest rates fall 10 bps, to 3.90 percent. Once again the spread earns a profit, this time \$49. Calculations similar to those above show that the FRA position has earned \$17,549, while the short GE position has lost \$17,500.

²³ Market practitioners familiar with the cash-futures basis in, eg, CBOT Treasury futures or Grain futures will be accustomed to thinking of a long basis position as “long cash plus short futures” – that is, as comprising a long position in the futures contract’s reference commodity plus a short position in futures. By comparison, the definition of the GE futures basis -- as a sale of an FRA combined with a sale of the corresponding GE futures -- may look odd. The explanation is that purchases and sales of FRAs are, by convention, defined in terms of interest rate exposure rather than asset price exposure. To be clear, the FRA is a contract for a cash payment at maturity based on the difference between a future spot interest rate and the FRA contract rate. At the FRA’s expiration, if the spot interest rate is higher than the FRA contract rate, then market custom holds that the FRA “seller” shall pay the “buyer” the difference between the FRA contract rate and the spot rate (multiplied by the FRA’s notional principal amount). Conversely, if the spot rate is lower than the FRA contract rate, market custom dictates that the “buyer” shall pay the “seller” the appropriate amount upon the FRA’s expiry. Thus, the roles of “buyer” and “seller” in the FRA are opposite to the respective roles performed by the buyer and seller of a GE futures contract. See, eg, **Before Futures Expiration – Contract Price and Contract Interest Rate** on page 7.

The “peculiar” feature of the trade is obvious. Without any correction for the difference between linear pricing of futures and convex pricing of FRAs, the buyer of the cash-futures basis position enjoys a profit, and the seller of the basis sustains a loss, regardless of whether interest rates rise or fall.

Example 22

To appreciate this peculiarity in a more realistic setting, let’s revisit market conditions on 14 December 2010 (shown in Exhibit 2 on page 5):

The 2:00pm CT daily settlement price is 94.355 for the First Copper GE futures contract, for delivery 9.25 years hence in March 2020. This signifies a contract rate of 5.645 percent per annum on a notional three-month interbank deposit, for forward settlement in mid-March 2020 and for maturity 92 days later in mid-June.

Because the term of the notional forward deposit is 92 days (not 90 days as in previous examples) a one basis point change in the pertinent three-month forward interest rate means that the forward profit/loss to be recognized in mid-June 2020 is equal to \$2,555.56 per \$100 million notional deposit size (not \$2,500).

Levels of London interbank loan rates and Eurodollar futures contract rates for intervening delivery months jointly suggest that any profit or loss to be realized in mid-June 2020 should be discounted to present value at a rate of 3.76 percent per annum. Thus, the DV01 for reckoning profit/loss to be booked today on the corresponding FRA is:

$$\$1,780 = \$2,555.56 \times 0.704231 = \$2,555.56 \times (1 + (3.76 \text{ percent}/100))^{-9.5}$$

A cash-futures basis position built on \$100 million notional of the corresponding par FRA would require a countervailing position of 72 First Copper GE futures contracts:

$$72 \text{ GE contracts} \approx (\$1,780 \text{ per bp}) / (\$25 \text{ per bp per GE contract})$$

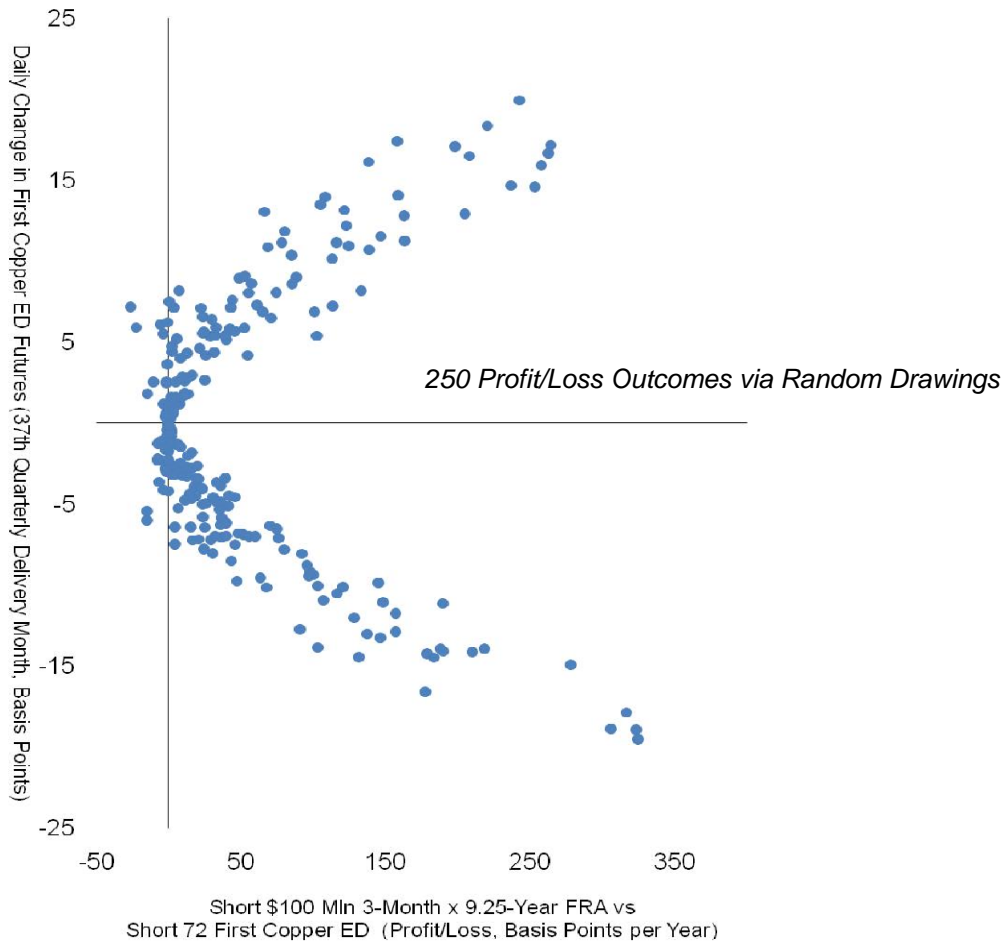
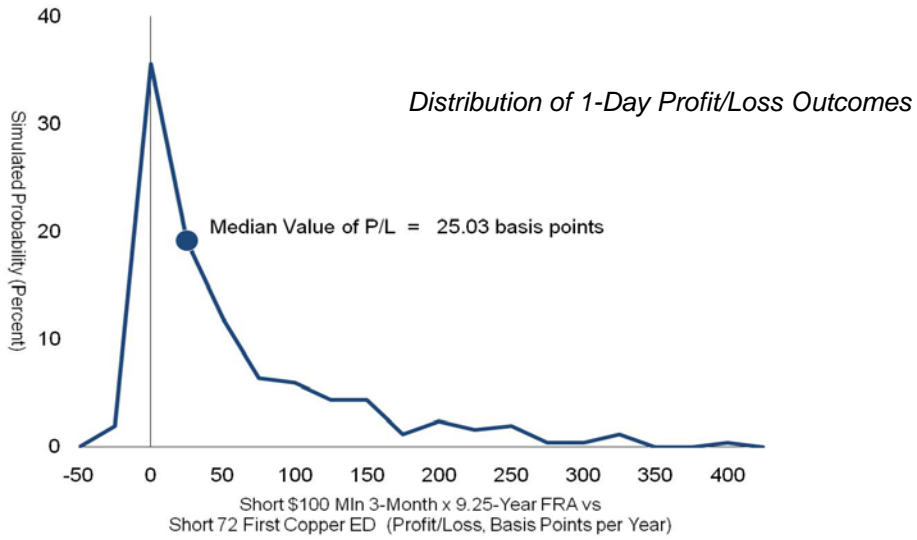
What sort of profit/loss might this cash-futures basis spread position generate? The upper panel of Exhibit 8 depicts an estimate of the distribution of one-day profit/loss outcomes, based on 250 random draws from a bivariate Gaussian distribution for which the mean is (0,0) and²⁴

standard deviation of daily changes in the First Copper GE contract rate = 8.13 bps;
standard deviation of daily changes in the 9.5-year discount rate = 7.32 bps;
correlation between daily changes in the two rates = 0.89 (on a scale of 0 to 1).

The lower panel of Exhibit 8 evidences the “peculiarity” illustrated in Example 21. A long position in the cash-futures basis (as before, a sale of the FRA combined with a short position in GE futures) seldom loses money -- in this particular simulation exercise, it produces a net loss on just 15 draws out of every 100 -- and it never loses large amounts of money.

²⁴ The standard deviations and correlation are computed with daily rate changes observed between mid-September and mid-December 2010. One can debate whether a bivariate normal distribution with this particular covariance matrix adequately reflects the expectations of the representative market participant on the afternoon of 14 December 2010. But it’s fair to assert that this distribution is a plausible reflection of prevailing market conditions.

Exhibit 8 – Estimating GE Futures Convexity Bias via Simulation: 14 December 2010
 (See Example 22 for details of simulation.)



Moreover, from the long basis holder's standpoint, the basis spread clearly rewards volatility. The higher the volatility, the more lavish the reward, irrespective of whether volatility produces a rise or a fall in the forward rate from which the FRA and the GE futures derive. The long-tailed skew in the marginal distribution of profit/loss, shown in the upper panel of Exhibit 8, dramatizes the point.

Examples 21 and 22 pose the same fundamental question. The cash-futures basis spread – short the FRA and short a DV01-equivalent number of GE futures – appears to slant sharply in favor of anyone who acquires interest rate volatility exposure by buying the basis. Given this, why would anyone sell volatility by selling the basis?

Convexity bias via leveling the playing field

The answer is that a market participant will sell the basis if she can buy the GE futures at a discount sufficient to give her at least an even shot at making a profit.

This phenomenon is well known to market practitioners. Indeed, for many years GE futures have traded at market prices that incorporate a systematic discount, precisely so that prospective returns get evened up between the long and short sides of the cash-futures basis.

The upper panel in Exhibit 8 suggests how the size of this discount might be determined. If there were no corrective discount to the GE futures price – as was presumed in Examples 20, 21, and 22 above – then the median point in the estimated distribution of profits accruing to the long holder of the cash-futures basis would be approximately 25 basis points per annum. That is, spot and forward interest rate volatility levels are such that a price reduction of 25 basis points (0.25 price points) applied to the First Copper GE futures contract would level the playing field so that the buyer and seller of the basis spread face equal odds of making a gain or loss on the trade.

If market participants make full and efficient use of the information available to them, including information about interest rate volatility, then the corrective price discount for convexity bias should already be built into observed GE futures prices:

$$\text{Observed GE contract rate} = \text{"True" forward rate} + \text{Correction for convexity bias}$$

Given that the First Copper GE contract rate at daily settlement on 14 December 2010 is 5.645 percent, and given that the estimated correction for convexity bias for that contract for that day is 0.25 percent, the "true" (FRA-equivalent) forward interest rate implied by the futures daily settlement price should equal 5.395:

$$5.645 \text{ percent} = 5.395 \text{ percent} + 0.25 \text{ percent}$$

Determinants and a rule of thumb

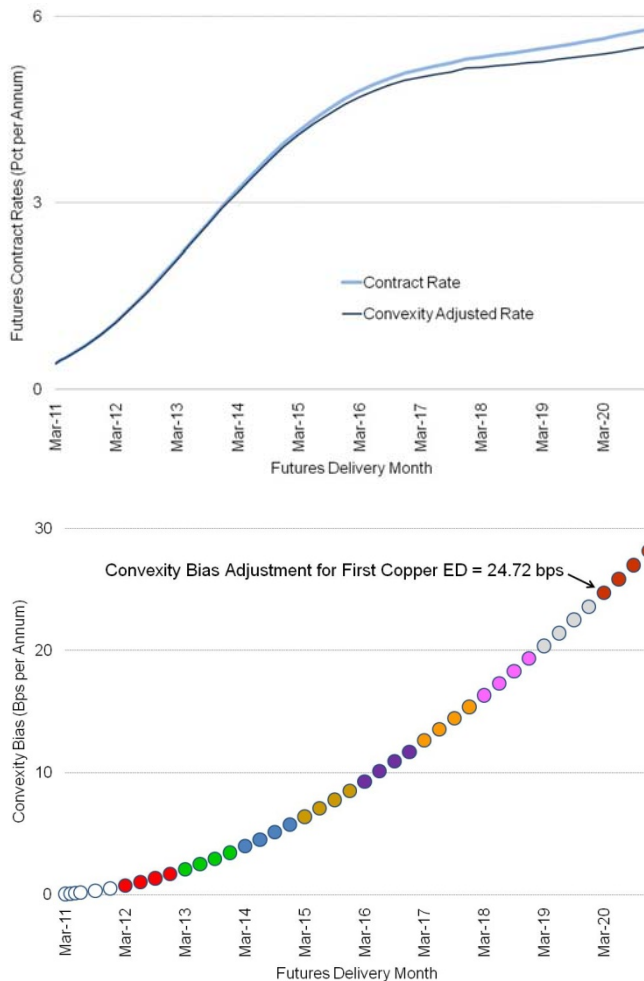
It would be cumbersome, but one could perform the same exercise as in Example 22 and Exhibit 8 for any trade date, for all GE futures delivery months. For instance, the upper panel of Exhibit 9 shows approximate results for all GE futures on 14 December 2010. The light blue line traces the term structure of raw GE futures contract rates (simply 100 minus the futures contract price, at each point). The dark blue line is the corresponding term structure of "true" FRA-equivalent forward interest rates (at each point, the raw GE futures contract rate minus the estimated correction for the futures contract's convexity bias). The lower panel shows the spread between the two term structures, ie, the magnitude of correction for convexity bias.

As a general proposition the magnitude of correction for any GE futures contract grows larger as:

- (1) the term between today and the GE futures expiry date lengthens (eg, as suggested by comparison of Examples 18 and 19); or
- (2) volatility increases in the forward-starting interest rate that the GE contract references; or
- (3) volatility increases in the spot rate used to determine the present value of changes in the GE contract's forward-starting interest rate; or
- (4) the degree of correlation increases between movements in the forward interest rate in (2) and movements in the spot rate used for discounting to present in (3).

Exhibit 9 -- GE Futures Convexity Bias: 14 December 2010

Upper Panel: GE Contract Rates with and without Adjustment for Convexity Bias
 Lower Panel: Size of Convexity Bias on 14 December 2010



Source: Bloomberg LP

Of these four factors, Exhibits 8 and 9 exemplify the two that are most salient: respectively, the level of interest rate volatility and the length of time to futures contract expiry. A popular rule of thumb elegantly and conveniently captures the interrelationship among them:

$$\text{Convexity bias} = (\sigma^2/2) \times t \times T$$

where

t = term from present to GE futures expiration (in years)

T = t + 3 months (ie, t + 0.25 years)

σ^2 = volatility of the forward interest rate for a notional interbank deposit for forward settlement on date t and for maturity three months later on date T

The volatility measure is typically implemented with the implied volatility of at-the-money standard options on the corresponding GE futures. Suppose that on 14 December 2010, σ for the First Copper GE contract rate equals 75

basis points per annum (approximately 4.75 basis points per day, equal to (75 bps per annum) / (252 business days per annum)^{1/2}). Then:

$$\begin{aligned}\sigma^2 &= 0.0000563 = 0.0075^2 \\ t &= 9.25 \text{ years} \\ T &= 9.5 \text{ years}\end{aligned}$$

Accordingly, the convexity bias is equal to:

$$0.002472 = (0.0000563 / 2) \times 9.25 \times 9.5$$

or 24.72 basis points per annum. Despite deep differences between the two methodologies, the 24.72 bps estimate obtained via the closed form approximation is strikingly close to the 25.02 bps estimate obtained earlier via numerical simulation.²⁵

In a nutshell

The GE futures contract is defined so that its response to interest rate changes is linear. The contract's DV01 with respect to its reference forward three-month interest rate is always \$25.

FRAs are customarily priced and marked so that their response to interest rate changes is convex. An FRA's DV01 with respect to its reference forward three-month interest rate is the present value of approximately²⁶ \$25 per basis point of change in the rate.

Because the GE futures buyer does not enjoy the benefit of being an owner of convexity, GE futures are systematically priced at a discount relative to the hypothetical price that would be determined as 100 minus the corresponding FRA forward rate.

In other words, GE futures contract rates, at fair value, are systematically higher than their corresponding FRA forward rates. The difference (GE futures contract rate minus FRA forward rate) is known as the convexity bias correction:

$$\text{GE contract rate} = \text{"true" FRA forward rate} + \text{convexity bias correction}$$

The convexity bias correction is a positive function of interest rate volatility and term to futures expiry.

²⁵ For an excellent discussion of the rule of thumb, see John Hull, **Options, Futures, and Other Derivatives, 7th Edition**, Pearson Prentice Hall, 2009, especially Chapters 6 and 30.

²⁶ The FRA's DV01 is the present value of exactly \$25 per basis point if the FRA's forward-starting three-month interval spans 90 days. Otherwise, the FRA's DV01 is the present value of some other slightly different amount per basis point. See **Notional Contract Size = \$1 Million ... More or Less** on pages 8-9.

One-Month Eurodollar Futures

One-Month Eurodollar (GLB) futures, introduced in April 1990, are just like their older GE siblings, except shorter and more concentrated. Exhibit 10 summarizes contract features.

Exhibit 10 -- One-Month Eurodollar Futures Contract Specifications

(All times of day are Chicago Time unless otherwise noted.)

Trading Unit	Interest on a Eurodollar interbank deposit, having approximately \$3 mln principal value for a term of one month, for spot settlement on 3 rd Wednesday of contract Delivery Month
Delivery Months	Nearest 24 calendar months
Price Basis	IMM price points: 100 points minus one-month London interbank offered rate for spot settlement on 3 rd Wednesday of Delivery Month (e.g., a 2.55 percent rate equals 97.45 points). One interest rate basis point (0.01 price points) equals \$25 per contract.
Price Increments	¼ of one interest rate bp (0.0025 price points) = \$6.25 per contract
Last Trading Day	Second London business day before 3 rd Wednesday of Delivery Month
Delivery Standard	One-month US dollar BBA LIBOR [®] as set on Last Trading Day for spot (T+2) settlement on 3 rd Wednesday of Delivery Month, rounded to nearest 1/100 th of one bp
Delivery Method	Cash settlement by mark-to-market to Final Settlement Price. Final Settlement Price = Price Basis evaluated at Delivery Standard. Final settlement occurs on Last Trading Day.
Trading Hours	CME Globex: 5pm to 4pm CT, Sun-Fri. Open outcry: 7:20am to 2pm CT, Mon-Fri. Trading of expiring contracts terminates at 11am London time on Last Trading Day.
Position Accountability	5,000+ contracts
Reportable Positions	100+ contracts
Block Trade Thresholds	<i>RTH (7am to 4pm CT):</i> 400+ contracts <i>ATH (4pm to 12am):</i> 100+ contracts <i>ETH (12am to 7am CT):</i> 200+ contracts
Designated Contract Market	CME Three-Month Eurodollar futures are listed with and subject to the rules and regulations of CME.

Source: CME Group

Trading Unit

The underlying cash-market reference for any GLB contract is interest on a Eurodollar interbank deposit having approximately \$3 million principal value, with a term to maturity of one month, for spot settlement on the 3rd Wednesday of the contract Delivery Month. The GLB contract trading unit is otherwise identical to the GE contract trading unit described on page 3.

Delivery Month and Last Trading Day

The exchange typically lists contracts for delivery in each of the nearest 24 calendar months, ie, 12 consecutive monthly contract expiries for each of the White and Red years. Identical to GE futures, each GLB contract ceases trading and expires on the second London business day (typically the Monday) before the 3rd Wednesday of its delivery month. When the nearby contract expires, a new twenty-fourth contract delivery month is listed for trading on the next following Exchange business day (typically the Tuesday before the 3rd Wednesday of the month).

Price

GLB futures prices are quoted in “100 minus rate” terms. The contract final settlement price is determined as 100 minus one-month US dollar BBA LIBOR[®] on the second London bank business day preceding the 3rd Wednesday of the contract delivery month. Before applying one-month BBA LIBOR[®] to set the final settlement price of an expiring contract, the Exchange rounds the value to four decimal places. Any tie value, ie, published one-month BBA LIBOR[®] ending in 0.00005, gets rounded up to four decimal places.

Contract Size and Price Increments

Contract notional scale is defined so that one basis point of contract interest rate exposure (0.01 contract price points or one “tick”) is always worth \$25. The minimum price movement for trading GLB futures is always one quarter of one interest rate basis point (0.0025 contract price points or ¼ tick), equal to \$6.25 per contract.

Similar to GE futures, the definition of GLB’s tick size determines the size of the notional 1-month bank placement that serves as the contract underlying reference, not the other way around. As Exhibit 11 demonstrates, the contract’s notional bank deposit is exactly \$3 million only if the length of its one-month term to maturity spans 30 days. Conversely, if the size of the interbank placement is fixed at \$3 million, then as the day count for the placement’s one-month term lengthens, the bank deposit’s DV01 rises (Column 3), and the number of GLB futures required to hedge it rises correspondingly (Column 4).

Exhibit 11 -- GLB Futures Notional Interbank Deposit Size, Interest Rate Sensitivity, and Day Count

(1) Length of Reference Bank Deposit’s 1-Month Interval (Days)	(2) Deposit Size Required for DV01 to Equal \$25 (Dollars)	(3) DV01 for Deposit = \$3 Million (Dollars per Basis Point)	(4) Number of GLB Futures Required to Hedge Deposit = \$3 Billion (1000 x (3) / \$25)
28	3,214,285.71	23.33	933
29	3,103,448.28	24.17	967
30	3,000,000.00	25.00	1,000
31	2,903,225.81	25.83	1,033
32	2,812,500.00	26.67	1,067

Source: CME Group

Spreads and Combinations

In addition to outright trades in GLB futures, CME Globex permits market participants to trade calendar spreads and butterflies (broadly similar to the GE calendar spreads and butterflies described on page 12) as well as GLB-GE inter-market spreads. In all cases, the minimum price increment for quoting and trading is ¼ tick, or 0.0025 contract price points, per spread.

Trade Matching for GLB Futures

For all GLB contracts, spreads, and combinations CME Globex matches buyers and sellers with the “Pro Rata Allocation with TOP Price” (or A) algorithm, described on pages 20-21.

Appendix: A Concise Guide to US Dollar LIBOR

LIBOR -- the London Interbank Offered Rate -- is the rate of interest at which banks borrow funds from each other, in marketable size, in the London interbank market. Since its introduction in 1986, LIBOR has become the most widely used benchmark for short-term interest rates.²⁷ It is set daily by the British Bankers' Association (BBATM), in conjunction with Reuters, for several terms to maturity, in 10 currencies, including euro, Japanese yen, Swiss franc, US dollar, UK pound, Canadian dollar, Australian dollar, New Zealand dollar, Swedish krona, and Danish kroner.

The following overview describes the BBA LIBOR[®] mechanism, as it applies to US dollar interbank deposits. Throughout, the information source is the British Bankers' Association (www.bbalibor.com).

Contributor Panel

For each of the 10 currency denominations, the banks that serve on the BBA LIBOR[®] Contributor Panel are selected by the BBATM Foreign Exchange and Money Markets (FX&MM) Committee. Three criteria form the basis for selection of Contributor Panelists:

- (1) scale of market activity
- (2) credit rating
- (3) perceived expertise in the currency at hand (here, US dollar).

In deciding whether to invite a bank to become a Contributor Panelist, the FX&MM Committee bases its determination upon confidential data regarding the candidate's trading volumes. For example, a candidate for participation on the US Dollar Contributor Panel must furnish quarterly trading volumes, dealt through its London offices, in both the US dollar interbank placement market and the forward US dollar FX swap market, in all terms to maturity up to one year.

At least once each year the FX & MM Committee evaluates each Contributor Panelist on the basis of a review by the BBATM. This review ranks Contributor Panelists according to the scale of their money market and swap market activity over the previous year -- criterion (1) above -- with due consideration given to the other two criteria. The annual review is not limited to extant Contributor Panelists. Any bank can submit itself to the evaluation process.

As of 1 February 2011, the US Dollar Contributor Panel expanded from 16 members to 20 members, pursuant to the FX & MM Committee's November 2010 review. As of 1 August 2011, the Panel stands at 19 members, reflecting the Committee's June 2011 review. Exhibit A1 details these changes in the Panel's composition.

²⁷ The antecedents of the BBA LIBOR[®] mechanism reach back to late 1984, with the introduction of the British Bankers' Association Interest Rates Settlement, or "BBAIRS."

Exhibit A1 -- BBA LIBOR® US Dollar Contributor Panelists

<i>May 2009 through January 2011</i>	<i>February through July 2011</i>	<i>From August 2011</i>
Bank of America	Bank of America	Bank of America
	Bank of Nova Scotia	Bank of Nova Scotia
Bank of Tokyo-Mitsubishi UFJ Ltd	Bank of Tokyo-Mitsubishi UFJ Ltd	Bank of Tokyo-Mitsubishi UFJ Ltd
Barclays Bank plc	Barclays Bank plc	Barclays Bank plc
	BNP Paribas	BNP Paribas
Citibank NA	Citibank NA	Citibank NA
	Credit Agricole Corp Investment Bank	Credit Agricole Corp Investment Bank
Credit Suisse	Credit Suisse	Credit Suisse
Deutsche Bank AG	Deutsche Bank AG	Deutsche Bank AG
HSBC	HSBC	HSBC
JP Morgan Chase	JP Morgan Chase	JP Morgan Chase
Lloyds Banking Group	Lloyds Banking Group	Lloyds Banking Group
The Norinchukin Bank	The Norinchukin Bank	The Norinchukin Bank
Rabobank	Rabobank	Rabobank
Royal Bank of Canada	Royal Bank of Canada	Royal Bank of Canada
Royal Bank of Scotland Group	Royal Bank of Scotland Group	Royal Bank of Scotland Group
Société Générale	Société Générale	Société Générale
	Sumitomo Mitsui Banking Corporation	Sumitomo Mitsui Banking Corporation
UBS AG	UBS AG	UBS AG
West LB AG	West LB AG	

Source: www.bbalibor.com

The BBA LIBOR® Standard

US dollar BBA LIBOR® is fixed daily for interbank placements at 15 terms to maturity: Overnight, 1-week, 2-week, and all monthly maturities from 1-month to 12-month, inclusive.

(1) Interest rate

For each term to maturity, each Contributor Panelist submits the interest rate...

(1a)...at which *it could borrow funds, were it to do so by asking for and then accepting interbank offers* in reasonable market size, just prior to 11am GMT, for an interbank placement that is:

made in the London market,
simple and uncollateralized,
governed by laws of England and Wales, and
subject to jurisdiction of the courts of England and Wales.

(1b)...that *conforms to prevailing quote conventions*. For instance, US dollar interbank deposit rates are always understood to be quoted on an actual/360 day count basis.

(1c)...for which the interest rate per annum is quoted to *at least two decimal places and no more than five decimal places*.

(2) Settlement dates

(2a) *For the overnight term to maturity*

The deposit rate must be quoted for same-day settlement. Moreover, “today” must be a business day in both London and New York.

(2b) *For all other terms to maturity*

The deposit rate must be quoted for spot (T+2) settlement. That is, the deposit is assumed to settle two London business days after “today,” as long as this is also a New York business day. If it is not, then the deposit is assumed to settle on the next following day that is both a London business day and a New York business day.

(3) Maturity date

(3a) *For overnight, 1-week, and 2-week terms to maturity*

The maturity date is always the first London-New York business day²⁸ that falls at least one, 7, or 14 days, respectively, from the settlement date.

(3b) *For monthly terms to maturity (1-month to 12-month, inclusive)*

The maturity date is the business day on or after the date, within the month of maturity, that corresponds to the settlement date.

Standard Procedure

If the corresponding date in the month of maturity is not a business day, then the Maturity Date gets pushed back to the next following business day.

Exception 1: Modified Next Business Day

If, and only if, the Standard Procedure pushes the Maturity Date into the next calendar month, then the Maturity Date gets pulled forward to the first preceding business day.

Exception 2: End-End

If the deposit's settlement date happens to occur on the final business day of the calendar month, then the Maturity Date is always set to the last business day of the month of maturity, rather than the corresponding date within the month of maturity. For example, in a non-leap year, a one-month deposit that settles on 28 February is understood to mature on 31 March, not on 28 March (assuming 28 February, 28 March, and 31 March are all business days).

Exhibit A2 illustrates how the Modified Next Business Day and End-End provisions would apply in the determination of maturity dates for one-month interbank placements made in late January 2009.

Exhibit A2 –

Settlement Date and Maturity Date Examples for 1-Month Interbank Placements, Jan 2009

Quote Date and BBA LIBOR [®] Fixing Date	Settlement Date	Maturity Date	Applicable Term to Maturity Rule
Mon, 26 Jan	Wed, 28 Jan	Fri, 27 Feb	Modified Next Business Day
Tue, 27 Jan	Thu, 29 Jan	Fri, 27 Feb	Modified Next Business Day
Wed, 28 Jan	Fri, 30 Jan	Fri, 27 Feb	End-End
Thu, 29 Jan	Mon, 2 Feb	Mon, 2 Mar	Standard Procedure
Fri, 30 Jan	Tue, 3 Feb	Wed, 3 Mar	Standard Procedure

Source: CME Group

(4) Every bank speaks for itself

Each Contributor Panel Bank shall submit its rate *without reference to the rates submitted by other Contributor Panel Banks*.

²⁸ In what follows, "business day" means a business day in both London and New York.

The BBA LIBOR[®] Fix

Consider the process by which BBA LIBOR[®] for US dollar three-month interbank deposits is established on Wednesday, 20 April 2011. Each Contributor Panelist submits its 3-month rate to the designated surveyor/distributor (Reuters) between 11am and 11:10am GMT. These appear in the “Rate Submissions” column of Exhibit A3 below, ranked from highest to lowest.

Exhibit A3 -- Three-Month US Dollar BBA LIBOR[®] on Wednesday, 20 April 2011

Contributor Panel Banks	Rate Submissions	Rate Submissions after Trimming
West LB AG	0.3800	
The Norinchukin Bank	0.3300	
Sumitomo Mitsui Banking Corporation	0.3200	
Société Générale	0.3125	
Bank of Tokyo-Mitsubishi UFJ Ltd	0.3100	
Credit Agricole Corporate Investment Bank	0.2975	0.2975
Barclays Bank plc	0.2800	0.2800
Lloyds Banking Group	0.2800	0.2800
Citibank NA	0.2750	0.2750
Bank of Nova Scotia	0.2700	0.2700
Rabobank	0.2700	0.2700
Royal Bank of Canada	0.2700	0.2700
The Royal Bank of Scotland Group	0.2700	0.2700
BNP Paribas	0.2650	0.2650
JP Morgan Chase	0.2600	0.2600
Credit Suisse	0.2550	
Bank of America	0.2500	
Deutsche Bank AG	0.2500	
HSBC	0.2500	
UBS AG	0.2450	
3-Month BBA LIBOR [®] , US Dollar		0.27375

Sources: Bloomberg LP, BBA[™]

Exactly what rate does each Contributor Panelist submit? As detailed above, it will be that individual bank’s assessment of the rate at which other banks offer to lend three-month money to it for spot settlement two business days hence, on Tuesday, 26 April 2011, and for maturity 91 days thereafter, on Tuesday, 26 July 2011.

Of the 20 interest rates submitted, the five occupying the lowest quartile are Credit Suisse and UBS, which face offered rates of 0.255 and 0.245 percent per annum, respectively, and Bank of America, Deutsche Bank, and HSBC, all of which face an offered rate of 0.25 percent. The highest quartile comprises West LB, Norinchukin Bank, Sumitomo Mitsui Banking Corp, Société Générale, and Bank of Tokyo-Mitsubishi-UFJ, which face offered rates ranging from 0.31 to 0.38 percent per annum. The data points submitted by these ten institutions – those in the highest and lowest quartiles – get trimmed from the sample.

3-Month US Dollar BBA LIBOR[®] is then computed as the interquartile mean -- the simple arithmetic average of the remaining ten data points -- as shown in the right-hand column of Exhibit A3.²⁹ Shortly after 11am GMT, Reuters publishes this result, with all 20 of the individual Contributor Panelist data points that entered into its computation.

²⁹ As of 1 August 2011, the US Dollar Contributor Panel has 19 members. (See **Exhibit A1 -- BBA LIBOR[®] US Dollar Contributor Panelists.**) Accordingly, US dollar BBA LIBOR[®] gets calculated by trimming out the highest five and the lowest five interest rate submissions, then computing the arithmetic average of the remaining nine rate submissions.

Resources

Galen Burghardt, **The Eurodollar Futures and Options Handbook**
McGraw-Hill and Chicago Mercantile Exchange, 2003

John Hull, **Options, Futures, and Other Derivatives, 7th Edition**
Pearson Prentice Hall, 2009

CME Rulebook Chapter 452 -- Three-Month Eurodollar Futures Terms and Conditions
www.cmegroup.com/rulebook/CME/V/450/452/

CME Rulebook Chapter 453 -- One-Month Eurodollar Futures Terms and Conditions
www.cmegroup.com/rulebook/CME/V/450/453/

CME Globex
www.cmegroup.com/globex/files/ElectronicTradingConcepts.pdf

British Bankers' Association -- BBA LIBOR[®]
www.bbalibor.com

UK Bank Holidays
www.direct.gov.uk/en/Governmentcitizensandrights/LivingintheUK/DG_073741

US Federal Holidays
www.newyorkfed.org/aboutthefed/holiday_schedule.html

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