FAIR VALUATION OF INSURANCE LIABILITIES

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Fair value of insurance liabilities

1. INTRODUCTION TO FAIR VALUE
2. RISK NEUTRAL PRICING AND DEFLATORS
3. EXAMPLES : THE BINOMIAL and THE BLACK/ SCHOLES CASES
4. FAIR VALUE OF LIFE INSURANCE PARTICIPATING CONTRACTS
5. FAIR VALUE OF VARIABLE ANNUITIES
6. CONCLUSION

1. Introduction to FAIR VALUE

✓International norms IAS / IFRS for all financial institutions in Europe soon (…)

✓…A lot of discussions linked to accounting principles and artificial volatility

✓General principle of Fair valuation of elements for assets as well as for liabilities: market values instead of historical values

1. Introduction to FAIR VALUE

- Basic principle : from an historical or statutory accounting point of view to fair value bases

- **<u>Fair value</u>** : price at which an instrument would be traded if a liquid market existed for this instrument

- ASSETS : market values

- LIABILITIES : ???

  If no market value : principle of estimation of future cash flows properly discounted and taking into account the different kinds of risk

1. Introduction to FAIR VALUE

- Need to develop good models of valuation especially for actuarial liabilities where there is no market price
- Consistency between modern financial pricing theory and classical actuarial models

- Important example : **guarantees in life insurance and pension**:one of the most challenging risk nowaday

- Even if for competition reasons methods of pricing could remain very classical , fair valuation will require new insights taking into account modern finance

2. Risk neutral pricing and Deflators

<u>Purpose :</u>

- introduction to the modern financial paradigm of *risk neutral pricing*

- link with *deflator* methodology

- link with *classical actuarial principle* of discounting

- development in a simple discrete market model

2. Risk neutral pricing and Deflators

Paradigm of ***risk neutral pricing***:

Purpose: to compute the present price of a future stochastic cash flow correlated with financial market in an uncertain environment

What we could expect : *price = discounted expected value of the future cash flow:*

$$L(0) = \frac{1}{(1+i)^T} E(L(T)) \qquad \text{NO!!!}$$

2. Risk neutral pricing and Deflators

You have to change either the probability measure, either the discounting factor

<u>Change of probability measure</u> : *risk neutral method*:

You can stay with a classical discounting factor but the expectation of the cash flows must be done using another probability measure than the real one

<u>Change of discounting factor</u> : *deflators method:*

You can use the real probability measure but the discounting factor has to be changed and becomes stochastic

2. Risk neutral pricing and Deflators

Risk neutral pricing :

$$L(0) = \frac{1}{(1+r)^T} \; E_Q(L(T))$$

Q= risk neutral measure

Deflator method :

$$L(0) = E(D(T)\, L(T))$$

D(T) =deflator = stochastic discounting

2. Risk neutral pricing and Deflators

Single period market model :

✓ one riskless asset :

$$S_0(1) = S_0(0)(1+r) \;\; with \;\; r = riskfree \;\; rate$$

✓ d risky assets defined on a probability space :

$$\Omega = \{\omega_1, \omega_2, ..., \omega_N\} \;\; with \;\; p_j = P(\{\omega_j\}) \;\; j = 1,...,N$$

$$S_i(1) = (S_i(1,\omega_1), S_i(1,\omega_2), ..., S_i(1,\omega_N)) \;\; i = 1,...,d$$

✓classical assumption of absence of arbitrage opportunities

2. Risk neutral pricing and Deflators

✓STATE PRICE : Basic property :

if the market is without arbitrage there exists a random variable ψ such that for any asset :

$$\boxed{S_i(0) = \sum_{j=1}^{N} \Psi_j \, S_i(1, \omega_j)} \quad with \ \Psi_j = \Psi(\omega_j) \rangle 0$$

If the market is complete, the state price is unique.

Consequence : for asset i =0 (risk free asset)

$$\overline{\Psi} = \sum_{j=1}^{N} \Psi_j = \frac{1}{1+r}$$

2. Risk neutral pricing and Deflators

RISK NEUTRAL MEASURE

Definition : artificial probability measure given by:

$$q_j = Q(\omega = \omega_j) = \frac{\Psi_j}{\overline{\overline{\Psi}}} = (1+r)\,\Psi_j$$

Properties :

1°) $0 \leq q_j \leq 1$ and $\sum_{j=1}^{N} q_j = 1$

2°) for each asset: mean return= risk free rate

$$S_i(0) = \frac{1}{(1+r)} \sum_{j=1}^{N} q_j \, S_i(1, \omega_j)$$

2. Risk neutral pricing and Deflators

✓DEFLATOR : random variable D defined by :

$$D(\omega_j) = D_j = \frac{\Psi_j}{p_j}$$

<u>Properties</u> :

i. $$\sum_{j=1}^{N} p_j D_j = E(D) = \frac{1}{1+r}$$

(expected value of the deflator = classical discount)

ii. $$S_i(0) = \sum_{j=1}^{N} p_j D_j S_i(1, \omega_j)$$

2. Risk neutral pricing and Deflators

<u>Generalization</u> : if X is a financial instrument on this market (replicable by the underlying assets) and giving for scenario j a cash flow X(1,j) then the initial value of this instrument can be written as :

$$\text{i. State price vision: } X(0) = \sum_{j=1}^{N} \Psi_j \, X(1,j)$$

$$\text{ii. Risk neutral vision: } X(0) = \frac{1}{1+r} \sum_{j=1}^{N} q_j \, X(1,j) = \frac{1}{1+r} E_Q(X(1))$$

$$\text{iii. Deflator vision: } X(0) = \sum_{j=1}^{N} p_j D_j \, X(1,j) = E(D\,X(1))$$

2. Risk neutral pricing and Deflators

Multiple periods model : discrete time model (t=0,1,…, T)

✓Riskfree asset:

$$S_0(t) = S_0(0)(1+r)^t \quad with \;\; r = riskfree \;\; rate$$

✓Risky assets :

$$S_i(t) = (S_i(t,\omega_1), S_i(t,\omega_2), \ldots, S_i(t,\omega_N)) \quad i = 1,\ldots,d$$

✓STATE PRICE :

$$S_i(0) = \sum_{j=1}^{N} \Psi_j(t) S_i(t,\omega_j) \quad with \; \Psi_j(t) = \Psi(\omega_j,t) \rangle 0$$

2. Risk neutral pricing and Deflators

✓ DEFLATOR :

$$D_j(t) = \frac{\Psi_j(t)}{p_j} = discount\ \ factor\ from\ \ t\ \ to\ 0\ if\ scenario\ j$$

✓Pricing : if X is a financial replicable instrument on this market generating successive stochastic cash flows :

$$\{C(t,\omega); t = 1,...,T; \omega \in \Omega\}$$

Then the initial price of X can be written alternatively :

$$X(0) = \sum_{t=1}^{T} \sum_{j=1}^{N} C(t,\omega_j)\Psi_j(t)$$

2. Risk neutral pricing and Deflators

Or with risk <u>*neutral measure*</u>:

$$X(0) = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \sum_{j=1}^{N} q_j \, C(t, \omega_j)$$

Or with <u>*deflators*</u> :

$$X(0) = \sum_{t=1}^{T} \sum_{j=1}^{N} p_j \, C(t, \omega_j) \, D_j(t) = \sum_{t=1}^{T} E(D(t) \, C(t))$$

3.1. THE BINOMIAL CASE

Single period model:

✓Risky asset :

$$S_1(1) = S_1(0) \cdot u \qquad \text{with probability p}$$

$$= S_1(0) \cdot d \qquad \text{with probability 1-p}$$

Absence of arbitrage opportunities if:

$$0 < d < 1 + r < u$$

Other form of the risky asset :

$$u = 1 + r + \lambda + \mu \qquad d = 1 + r + \lambda - \mu$$

3.1. THE BINOMIAL CASE

With condition : $0 < \lambda < \mu$

$\lambda = risk\ \ premium \qquad \mu = volatility$

<u>Equations of the STATE PRICE :</u>

For i=0: $(1+r)\Psi_1 + (1+r)\Psi_2 = 1$

For i=1: $u\Psi_1 + d\Psi_2 = 1$

3.1. THE BINOMIAL CASE

Solution for the STATE PRICE:

up $$\Psi_1 = \frac{1+r-d}{(1+r)(u-d)} = \frac{\mu-\lambda}{2\mu(1+r)}$$

down $$\Psi_2 = \frac{u-(1+r)}{(1+r)(u-d)} = \frac{\mu+\lambda}{2\mu(1+r)}$$

Safety principle :

$$\Psi_2 = \Psi_1 \quad if \quad \lambda = 0$$

$$\Psi_2 \rangle \Psi_1 \quad if \quad \lambda > 0 (normal\ case)$$

3.1. THE BINOMIAL CASE

Fair value in a binomial environment – single period :

If X is a financial instrument on this market with future stochastic cash flows given respectively by :

$$X(1,\omega_1) = X_1 \quad and \quad X(1,\omega_2) = X_2$$

Then the initial fair value of X is given by :

$$X(0) = X_1 \Psi_1 + X_2 \Psi_2$$

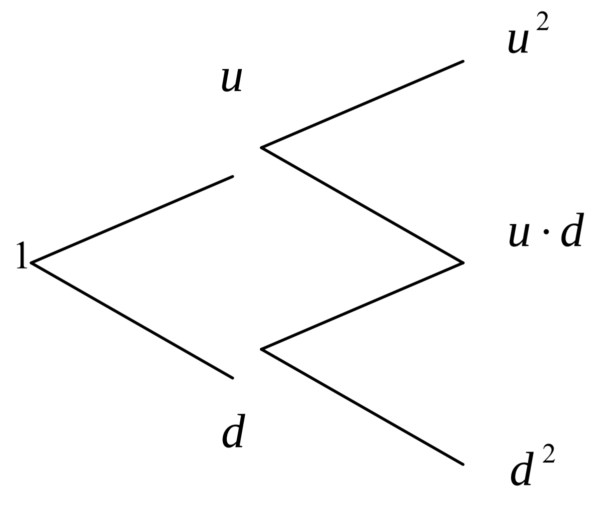
Or :

$$X(0) = \frac{1}{2}(X_1 + X_2)\frac{1}{(1+r)} + \frac{1}{2}(X_2 - X_1)\frac{\lambda}{\mu(1+r)}$$

3.1. THE BINOMIAL CASE

Multiple periods model:

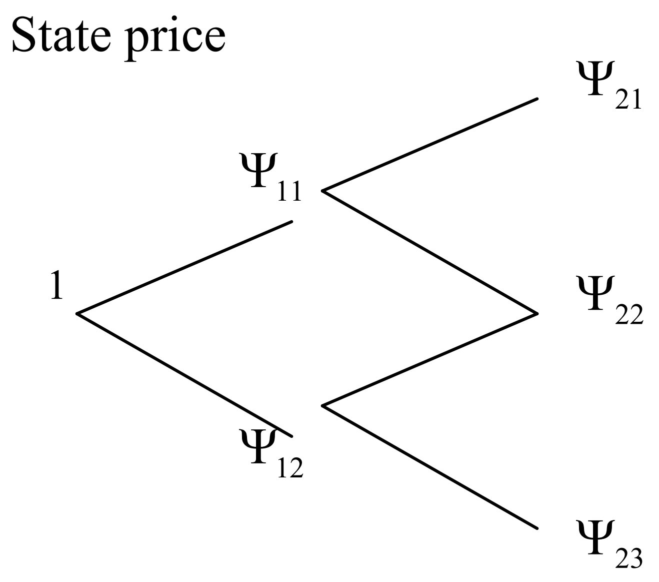
Risky asset



3.1. THE BINOMIAL CASE

Structure of STATE PRICES in multiple periods:

Assumption: financial product having successive cash flows depending only on the current situation of the market (no path dependant).



$$\Psi_{t\,j} = \textit{state price at time } t \;\; \textit{if scenario } j$$

3.1. THE BINOMIAL CASE

Value of the STATE PRICES:

$$\Psi_{t\,j} = C_t^{\,j-1}\,\Psi_1^{\,j-1}\,\Psi_2^{\,t-j-1}$$

Where j-1 = number of up (j=1,..,t+1)
t-j-1 = number of down

And: $C_t^{\,j-1}$ is the number of paths in the tree with j-1 up in t periods

3.1. THE BINOMIAL CASE

Fair valuation in multiple periods – binomial :

If X is a financial instrument having successive cash flows in the tree given by :

$$X_{t\,j} = cash\ flow\ at\ time\ t\ if\ scenario\ j$$

Then the initial fair value of X is given by :

$$X(0) = \sum_{t=1}^{T} \sum_{j=1}^{t+1} X_{t\,j}\, \Psi_{t\,j}$$

3.2. THE BLACK / SCHOLES CASE

Continuous extension : **Black and Scholes model**

Riskless asset :

$$dS_0(t) = r\,S_0(t)\,dt$$

Risky asset :

$$dS_1(t) = \delta S_1(t)\,dt + \sigma S_1(t)\,dw(t)$$

Where:- w is a standard brownian motion
- δ is the mean return of the asset ($\delta > r$)
- σ is the volatility of this return

3.2. THE BLACK / SCHOLES CASE

Risk neutral measure :

$$dS_1(t) = r\,S_1(t)\,dt + S_1(t)((\delta - r)\,dt + \sigma\,dw(t))$$
$$= r\,S_1(t)\,dt + \sigma\,S_1(t)\,dw^*(t)$$

$$\text{with : } w^*(t) = w(t) + \frac{\delta - r}{\sigma}t$$

Q = risk neutral measure (Girsanov Theorem)
Under Q the process w*is a standard brownian motion

3.2. THE BLACK / SCHOLES CASE

Deflators: stochastic process D such that for i=1 (risk free asset) and for i=2 (risky asset):

$$S_i(0) = E(D(t) S_i(t))$$

Solution:

$$D(t) = e^{-rt} e^{-\left(\frac{\delta - r}{\sigma}\right) w(t) - \frac{1}{2}\left(\frac{\delta - r}{\sigma}\right)^2 t}$$

Safety principle :

Lower values of « w » give higher values for deflator

4. Fair value of participating contracts

4.1. Liability side:

Life insurance contract with profit :

guaranteed interest rate + participation

4.2. Asset side :

Strategic asset allocation:

Cash + Bonds + Stocks

4.3. Valuation of the contract :

Fair value and equilibrium condition

4. Fair value of participating contracts

<u>Need for a consistent ALM approach:</u>

Double link between asset and liability in this kind of product :

Liability ⟶ Asset :

Investment strategy must take into account the specificities of the underlying liability

Asset ⟶ Liability :

Participation liability linked with investment results

4.1. Liability side

Pure Endowment contract :

- initial age at t=0 : x
- maturity : N
- Benefit if alive at time t=N : 1
- Benefit in case of death before N : 0
- Contract with single or periodical premiums (pure premium)
- Technical parameters :
  - mortality table: $\{l_x\}$
  - guaranteed technical rate : i
  - participation rate on surplus: η

4.1. Liability side

<u>Bonus systems :</u>

general definition: percentage of the surplus (Assets – Liabilities)

Three possible schemes :

- Terminal bonus : bonus computed at maturity
- Reversionary bonus : bonus computed each year and fully integrated in the contract as additional premium
- Cash bonus : bonus computed each year and paid directly to the client

4.2. Asset side

Cash – Bonds- Stocks model (CBS model):

The underlying portfolio of the insurer is supposed to be invested in three big classes of asset:

- Cash : short term position (money account)
- Bonds : zero coupon bonds with a maturity not necessarily matched with the duration of the contract
- Stocks : stock index

4.2. Asset side

Cash model :

Money market account :

$$d\beta(t) = r(t)\,\beta(t)\,dt$$

with $r(t)$ = risk free rate

Risk free rate :Ornstein-Uhlenbeck process

$$dr(t) = a(b - r(t))\,dt + \sigma_r\,dw_1(t)$$

with w_1 = standard brownian motion

4.2. Asset side

Bond model

P(t,T)= price at time t of a zero coupon with maturity T

General evolution equation :

$$dP(t,T) = P(t,T)\,\mu(t,T)\,dt - P(t,T)\,\sigma(t,T)\,dw_1(t)$$

Particular case : VASICEK Model

$$\mu(t,T) = r(t) + \lambda\sigma_r\, B(T-t)$$

$$\sigma(t,T) = \sigma_r\, B(T-t)$$

$$\text{with : } B(s) = \frac{1}{a}(1 - e^{-as})$$

4.2. Asset side

Stocks model :

S(t)= value at time t of a stock index

$$dS(t) = S(t)(\mu\, dt + \sigma_S(\rho\, dw_1(t) + \sqrt{1-\rho^2}\, dw_2(t)))$$

with ρ = correlation stocks / interest rates

w_2 = standard brownian motion independant of w_1

4.2. Asset side

Portfolio :

$$\alpha_C = \text{proportion in cash}$$

$$\alpha_B = \text{proportion in bonds}$$

$$\alpha_S = \text{proportion in stocks}$$

$$\alpha_C + \alpha_B + \alpha_S = 1$$

2 main assumptions :

- proportions remain constant
  (continuous rebalancing)
- self financed strategy

4.2. Asset side

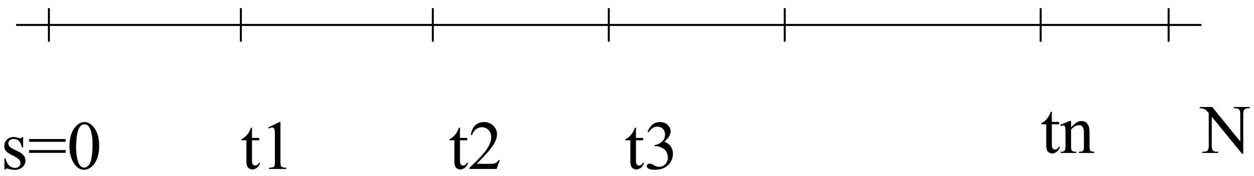
Bond strategy :

Assumption : at each time t zero coupon bonds of only one maturity can be held but the maturity has not necessarily to match the duration of the liability
(*price of mismatching-*
long duration of life insurance contract)

Strategy 1 : matched strategy : T=N
Strategy 2 : shorter maturity of the bond
and successive reinvestments till end of the contract

4.2. Asset side

Bond strategy



$$s = 0,1,\ldots,n : \text{number of reinvestments}$$

$$t_0 = 0, t_1, t_2, \ldots, t_n : \text{instants of reinvestment}$$

Particular case : matched strategy : n=0

4.2. Asset side

Evolution of the portfolio :

$V(t) =$ market value of the underlying portfolio

Evolution equation :

$$\frac{dV(t)}{V(t)} = \alpha_S \frac{dS(t)}{S(t)} + \alpha_B \frac{dP(t,t_i)}{P(t,t_i)} + \alpha_C \frac{d\beta(t)}{\beta(t)}$$

$$(\text{for } t_{i-1} < t < t_i)$$

4.2. Asset side

Value at maturity of the assets :

$$\ln V(N) - \ln V(0) = \sum_{s=0}^{n} (\ln V(t_{s+1}) - \ln V(t_s))$$

with each term $(\ln V(t_{s+1}) - \ln V(t_s)) = \text{normally distributed}$

$$\mu(N) = E(\ln V(N) - \ln V(0))$$

$$\sigma^2(N) = \text{var}(\ln V(N) - \ln V(0))$$

4.2. Asset side

Explicit form :

$$\ln V(t_{s+1}) - \ln V(t_s) = \int_{t_s}^{t_{s+1}} (\alpha_S \mu + (\alpha_B + \alpha_C) r(u) + \alpha_B \lambda \sigma_r B(t_{s+1} - u))\,du$$

$$+ \int_{t_s}^{t_{s+1}} \alpha_S \sigma_S \sqrt{1-\rho^2}\, dw_2(u) + \int_{t_s}^{t_{s+1}} (\alpha_S \sigma_S \rho - \alpha_B \sigma_r B(t_{s+1} - u))\,dw_1(u)$$

$$-\frac{1}{2} \int_{t_s}^{t_{s+1}} (\alpha_S{}^2 \sigma_S{}^2 + \alpha_B{}^2 \sigma_B{}^2 B^2(t_{s+1} - u) - 2\alpha_S \alpha_B \sigma_S \sigma_r \rho B(t_{s+1} - u))\,du$$

4.3. Valuation of the contract

Example of a contract with single premium and terminal bonus :

Fair value at maturity = pay off of the contract

$$FV(N) = (1+i)^N + \eta \max(V(N) - (1+i)^N ;\, 0)$$

Initial fair value : (? In line with the single premium)

$$FV(0) = {}_N p_x \left((1+i)^N P(0,N) + \eta \, Call(V; N; (1+i)^N) \right)$$

4.3. Valuation of the contract

Equilibrium condition :

$$(1+i)^N P(0,N) + \eta \ \text{call}(V;N;(1+i)^N) = 1$$

Consequences :

1°) $i < R(0,N)$

$$(\text{with } P(0,N) = \frac{1}{(1+R(0,N))^N})$$

2°) depends on the investment strategy through the value of $V(N)$

4.3. Valuation of the contract

3°) implicit relation for the technical rate i ;
4°) explicit relation for the participation rate η:

$$\eta = \frac{1-(1+i)^N P(0,N)}{\text{call}(V;N;(1+i)^N)}$$

An explicit formula of the call can be obtained in the CBS model presented before.

4.3. Valuation of the contract

Risk forward neutral measure method :

$$\text{call} = P(0,N)\, E_{Q_N}(\max((V(N)-(1+i)^T);0))$$

$$\text{where } Q_N = \text{forward risk neutral measure}$$

In the CBS model we have :

$$\text{call} = \Phi(D_+(i)) - (1+i)^N P(0,N)\, \Phi(D_-(i))$$

with :

$$D_\pm(i) = \frac{-N\ln(1+i) - \ln P(0,T) \pm 1/2\ \upsilon^2}{\upsilon}$$

4.3. Valuation of the contract

With for instance for the matched strategy :

$$\upsilon^2 = \alpha_S{}^2 \sigma_S{}^2 N + (1-\alpha_B)^2 \sigma_r{}^2 B_2(N) + 2\alpha_S (1-\alpha_B) \sigma_S \sigma_r \rho B_1(N)$$

with :

$$B_1(N) = \frac{N}{a} - \frac{1}{a^2}(1 - e^{-aN})$$

$$B_2(N) = \frac{N}{a^2} + \frac{1}{2a^3}(1 - e^{-2aN}) - \frac{2}{a^3}(1 - e^{-aN})$$

5. FAIR VALUE OF VARIABLE ANNUITIES

Purposes :

- How to valuate pension annuities not in terms of technical basis but in terms of market fair values;
- Influence of reversionary bonus (variable annuities) on the level of provision;
- Sensitivity of the provision with respect to financial parameters;
- How to fix the technical interest rate.

5.1. Liability side

- Immediate lifetime annuity for an affiliate to a pension fund
- x : initial age at time t=0
- Liability to pay: 2 cases :

1) *fixed annuity : L*

2) *variable annuity :*

$L_{t,j}$ = amount to pay at time t for scenario j

(possibility to increase yearly the pension depending on the financial performances – asset side)

- Payment at the end of the year till death or during a fixed period of n years

5.1. Liability side

<u>Actuarial first order bases :</u>

$$i = technical\ discount\ rate$$

$$_t p_x = survival\ \ probability\ at\ time\ t$$

<u>Technical provision for a constant pension</u> (case 1):

$$L_{t,j} = L$$

$$_n V_x = \ L\, a_{x\overline{n|}} = L \sum_{t=1}^{n} {}_t p_x \frac{1}{(1+i)^t}$$

5.2. Asset side

Binomial model :

mixed financial strategy of the pension fund between riskless asset (r= riskfree rate) and risky asset (binomial model u / d)

γ : part invested in the risky asset

$1-\gamma$: part invested in the riskless asset

$$(0 \leq \gamma \leq 1)$$

5.3. Bonus scheme

Definition of the reversionary bonus for variable < annuities

Used rule of bonus : comparison each year between the effective return of the assets and the riskfree rate; a part of this surplus is given back to the affiliate:

$0 \leq \beta \leq 1$: participation rate

5.3. Bonus scheme

Yearly rate of increase of the pension :

➢ ***If the risky asset is up :***

$$1+k = 1+\beta\left(\frac{\gamma u+(1-\gamma)(1+r)}{(1+r)}-1\right)^{+}$$

or $$1+k = 1+\beta\gamma\left(\frac{\lambda+\mu}{1+r}\right)$$

➢ ***If the risky asset is down*** :

$$1+l = 1+\beta\left(\frac{\gamma d+(1-\gamma)(1+r)}{(1+r)}-1\right)^{+} = 1$$

5.3. Bonus scheme

Final form of the liabilities of the variable annuity:

$$L_{t,j} = L \cdot (1+k)^{t-j+1}$$

Where t-j+1 is the number of times of up permitting to give a bonus.

As expected
THE LIABILITY DEPENDS ON TIME AND IS STOCHASTIC

5.4. Valuation of the contract

Computation of the fair value of the liabilities :
(fixed annuity)

$$FV(L)_{x,n} = \sum_{t=1}^{n} {}_t p_x \left[\sum_{j=1}^{t+1} L \, \Psi_{tj} \right]$$

$$= L \sum_{t=1}^{n} {}_t p_x \left[\sum_{j=1}^{t+1} \Psi_{tj} \right]$$

$$= L \sum_{t=1}^{n} {}_t p_x \left(\frac{1}{1+r} \right)^t \quad = L \, a^r_{x\overline{n|}}$$

5.4. Valuation of the contract

Computation of the fair value of the liabilities
(variable annuity)

- Actuarial valuation : not so simple: liabilities not deterministic
- Fair valuation : general formula of valuation :

$$FV(L_k)_{x,n} = \sum_{t=1}^{n} {}_t p_x \left[\sum_{j=1}^{t+1} L_{t,j} \Psi_{tj} \right]$$

$$= L \sum_{t=1}^{n} {}_t p_x \left[\sum_{j=1}^{t+1} C_t^{j-1} \Psi_2^{j-1} \left(\Psi_1 (1+k) \right)^{t-j+1} \right]$$

$$= L \sum_{t=1}^{n} {}_t p_x \left[\left(\Psi_2 + \Psi_1 (1+k) \right)^t \right]$$

5.4. Valuation of the contract

Computation of the fair value of the liabilities
(variable annuity)

$$FV(L_k)_{x,n} = L \sum_{t=1}^{n} {}_t p_x \left[\frac{1}{1+r} \right]^t \left[1 + \beta\gamma \frac{\mu^2 - \lambda^2}{2\mu(1+r)} \right]^t$$

$$= L \sum_{t=1}^{n} {}_t p_x \left(\frac{1}{1+i^*} \right)^t = L\, a^{i^*}{}_{x\overline{n|}}$$

5.4. Valuation of the contract

<u>Equilibrium relation</u> :

i* = equilibrium constant discount rate given by :

$$i^* = \left(r - \beta\gamma \frac{\mu^2 - \lambda^2}{2\mu(1+r)}\right) \Big/ \left(1 + \beta\gamma \frac{\mu^2 - \lambda^2}{2\mu(1+r)}\right)$$

$$\rightarrow \textit{if} \;\; \beta = 0 \;\textit{or}\; \gamma = 0 : i^* = r$$

$$\rightarrow \textit{if} \;\; \beta > 0 \;\textit{and}\; \gamma > 0 : i^* < r$$

5.5. Numerical illustration

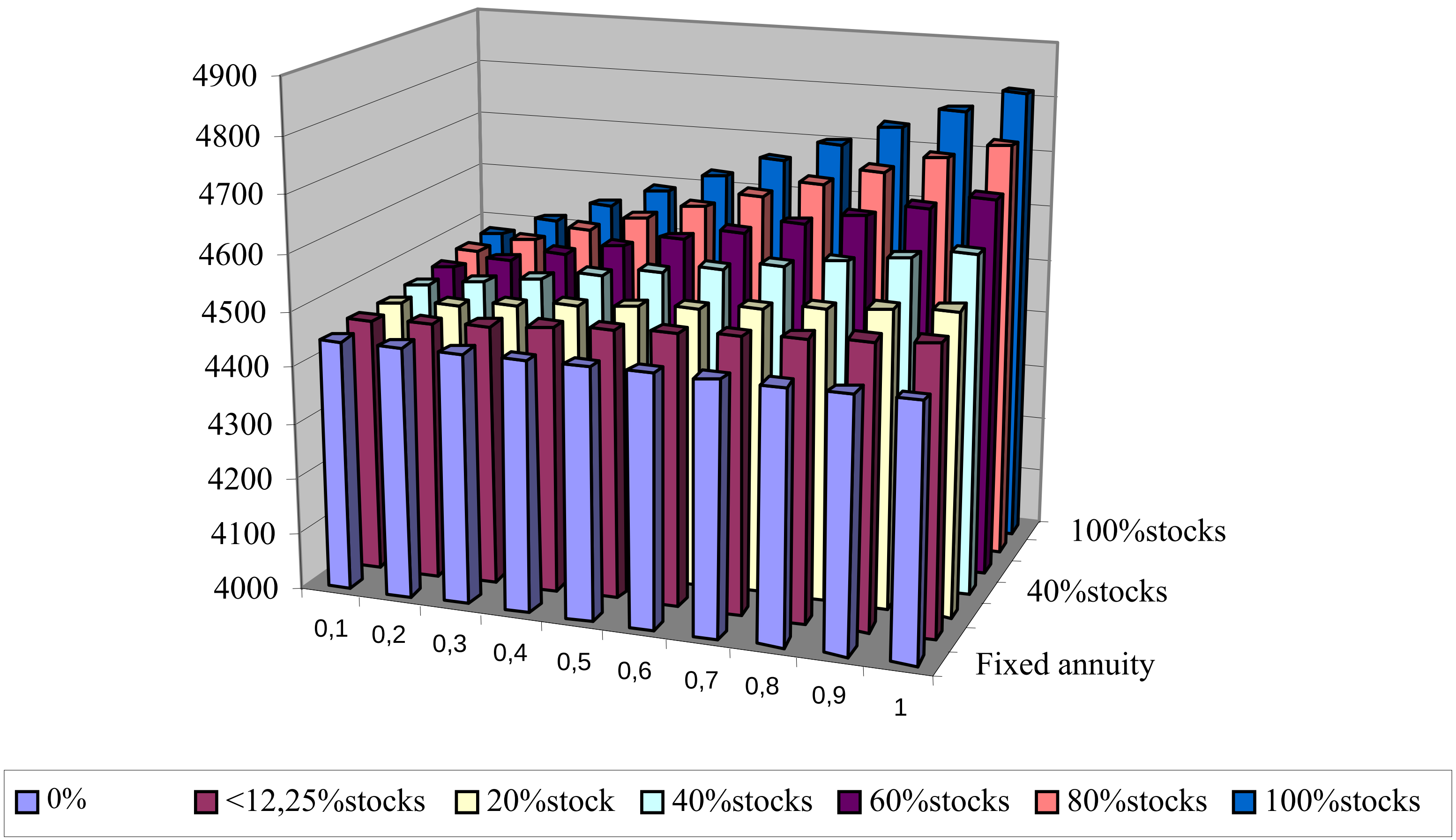
Central scenario:
$u=1.1$ $d=0.99$
$r=0.03$

$i=0.025$
Mortality: GRM 95

$n=5$

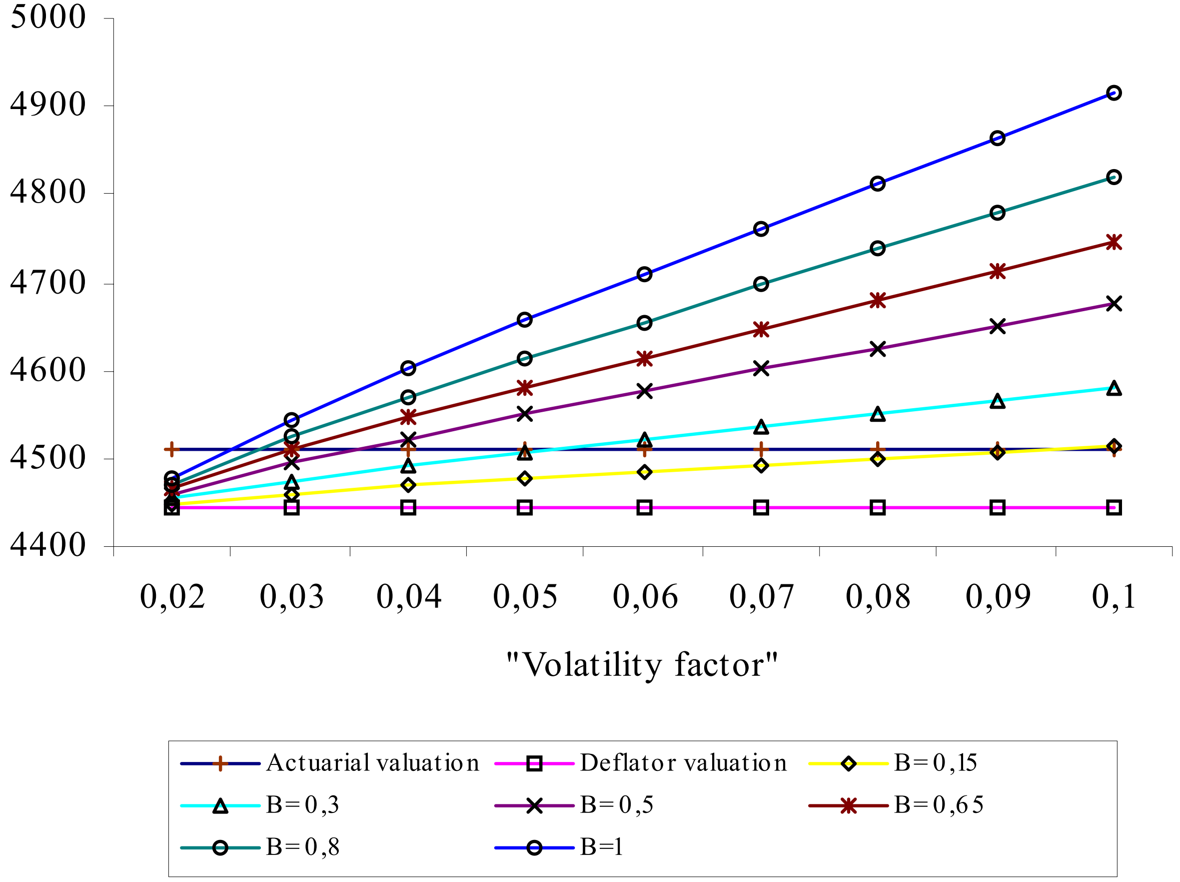
Risk premium : $\lambda = 0.02$
Volatility : $\mu = 0.06$

5.5. Numerical illustration



5.5. Numerical illustration

<u>Volatility sensitivity analysis</u> : (60% in risky asset)



5.5. Numerical illustration

Value of the equilibrium discount rate : central scenario

1° for $\beta = 0.5$ and $\gamma = 0.6$:

i*= 2.21%

2° for $\beta = 1$ and $\gamma = 0.6$:

i*= 1.42%

3° for $\beta = 0.9$ and $\gamma = 0.4$:

i*= 2.05%

4° for $\beta = 1$ and $\gamma = 1$:

i*= 0.4%

$\lambda = 2\%$

$\mu = 6\%$

r=3%

i=2.5%

5.5. Numerical illustration

Value of the equilibrium discount rate : other scenario
(less volatile)

1° for $\beta = 0.5$ and $\gamma = 0.6$:
i*= 2.60% versus 2.21%

2° for $\beta = 1$ and $\gamma = 0.6$:
i*= 2.21% versus 1.42%

3° for $\beta = 0.9$ and $\gamma = 0.4$:
i*= 2.52% versus 2.05%

4° for $\beta = 1$ and $\gamma = 1$:
i*= 1.68% versus 0.4%

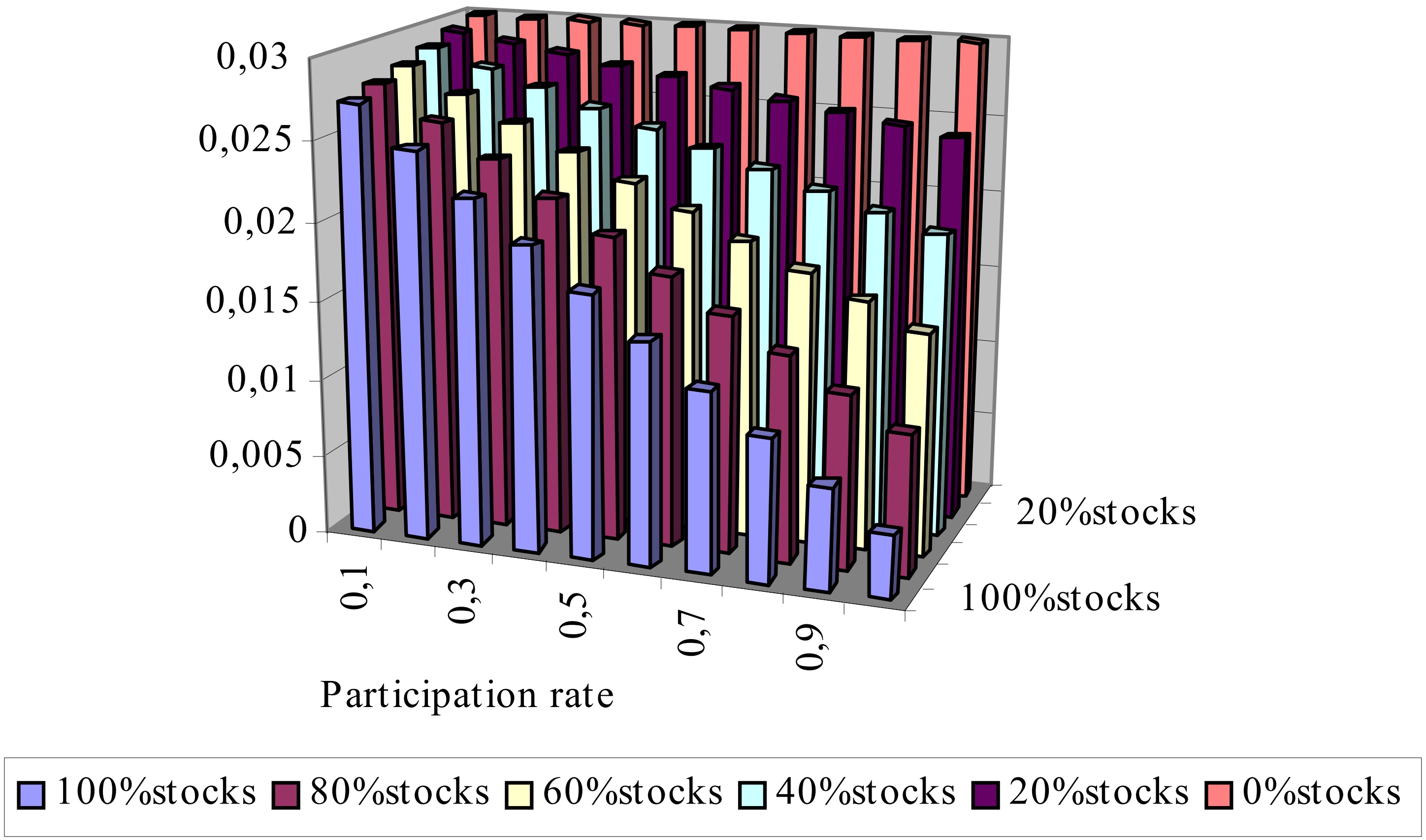
$\lambda = 1\%$

$\mu = 3\%$

r=3%
i=2.5%

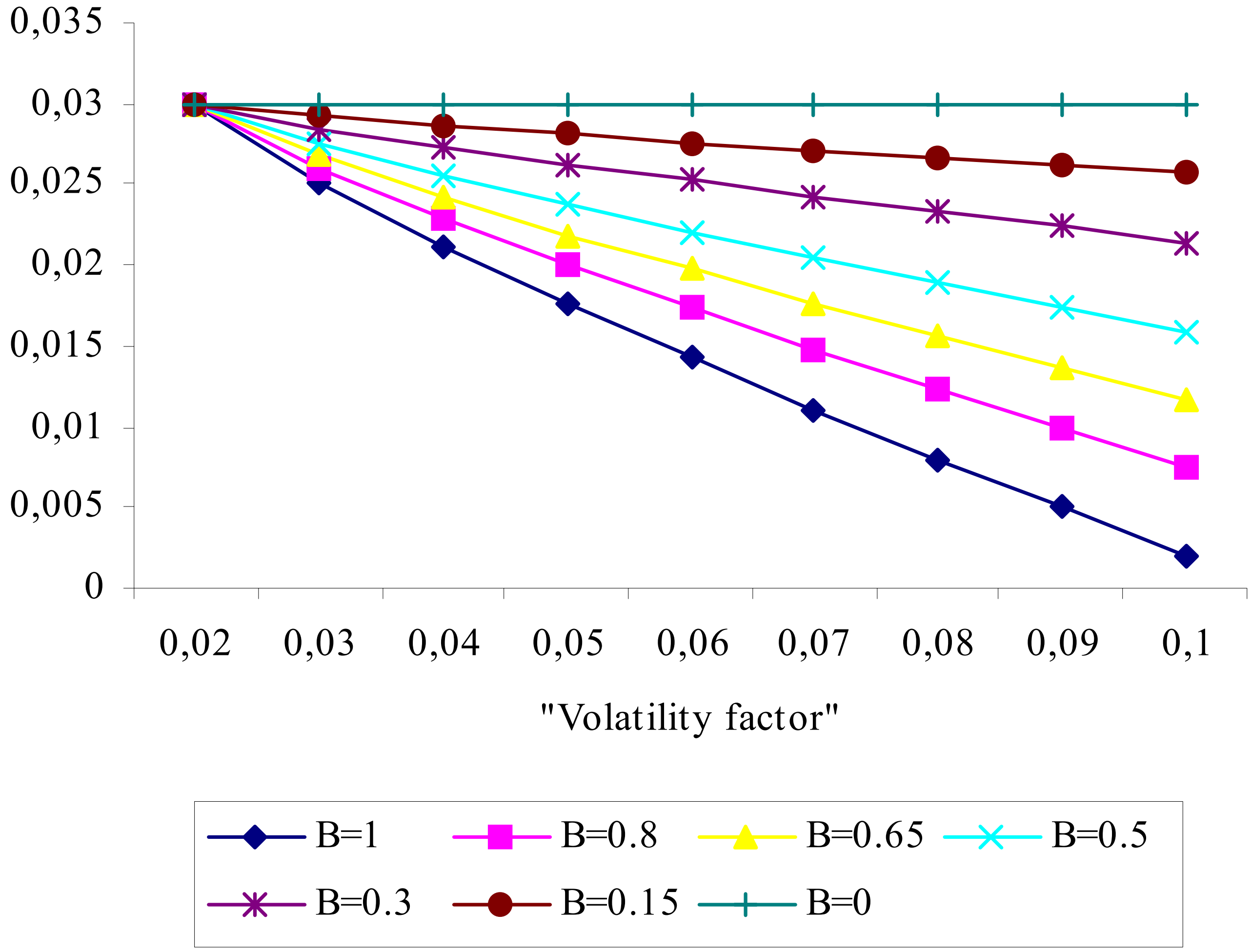
5.5. Numerical illustration

Equilibrium technical rate



5.5. Numerical illustration

EQUILIBRIUM TECHNICAL RATE(II)



6.CONCLUSION

- State prices , risk neutral measures and deflators are easy tools in order to valuate stochastic future cash flows correlated to future financial markets.

- This is exactly the situation of life insurance products

- One of the most important result is the way to valuate bonus and to define properly a technical interest rate in a complete ALM framework

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