Exposure Draft

A framework for estimating and extrapolating the term structure of interest rates

Barrie+Hibbert

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Contents

Executive summary

Is there any more fundamental valuation challenge than placing a value on a known cash flow at some time in the future? Risk-free yield curves are the basic building blocks for the valuation of future financial claims and long-term risk management work. Despite their fundamental importance, it turns out that measuring and estimating suitable risk-free interest rates presents some major challenges for analysts. In highly-developed fixed income markets we may be able to observe bonds or interest rate swap contracts with maturities of up to 50 years. In less developed markets liquid bond quotations might be limited to only a few years. In exceptional circumstances there may be no traded risk-free instruments at all. Of course, the liabilities of long-term financial institutions frequently extend beyond the term of available market instruments. In order to value these ultra long-term claims and assess risk, practitioners must extrapolate yield curves to generate a set of 'pseudo-prices' for the assumed, inferred prices of discount bonds beyond the term of the longest available traded cash flow. A good yield curve estimation method must deliver extrapolated curves that are credible at a single point in time and where *changes* over time in extrapolated rates can be justified.

This Exposure Draft summarises the methods to be applied in yield curve construction work by Barrie & Hibbert from 30th September 2008. Further details of our yield curve construction methods can be found in two documents which have been reviewed in an earlier draft form by the Technical Advisory Panel[1]:

1. '*Calibration Methods - Fitting the yield curve: Spline interpolation and Nelson-Siegel extrapolation*', September 2008.
2. '*Calibration Methods - Interest rate calibration: How to set long-term interest rates in the absence of market prices*', September 2008.

All of these documents will be reviewed at the next meeting of the Technical Advisory Panel (currently scheduled for November 18th 2008) alongside any comments received from clients and other interested parties before end-October 2008. Following this meeting, either this *Exposure Draft* will re re-issued with amendments or published in the form of a final *Statement of Practice*.

Note the following:

- Yield curves for all currencies for which we provide calibrations are generated to a standard term of 120 years.
- Yield curves can be represented in various ways – as a set of coupon yields, par yields, spot rates or forward interest rates. We could choose to work with any of these but for the purpose of this Exposure Draft we choose to work with continuously-compounded (log) forward interest rates. Note that any set of forward interest rates will uniquely define a set of spot rates, discount bond prices and par or coupon yields.

Yield curve construction work requires completing two fundamental tasks. First, collating market data and fitting a continuous curve to the term of the longest available market instrument. Second, we must extrapolate from the longest available market data towards some long-term assumption for forward interest rates. Since we aim to generate yield curves to a term of 120 years, this second task is required for *any* currency.

Regulators and policymakers have tackled these tasks in different ways. For example, the CFO Forum establishes a general principle by saying that yield curves "*should be extended using an appropriate methodology*". In the QIS4 technical guidance for Solvency II yield curve "*interpolations and extrapolations are based on the assumption that forwards are constant*". Bitter experience of dealing with worldwide yield curves suggests that, whilst simple rules will work fine at some times and in some places, their universal application produces results that we cannot justify. Note that our proposed method does not preclude the

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[1] *Further details of the Technical Advisory Panel's objectives and working practices can be found on the Barrie + Hibbert web site at http://www.barrhibb.com/index.php/technical_advisory_panel*

application of a simpler rule; an analyst can replace our extrapolated estimates with ruled-based values. Our method is aimed at simply giving our ***best*** answer to the questions posed above.

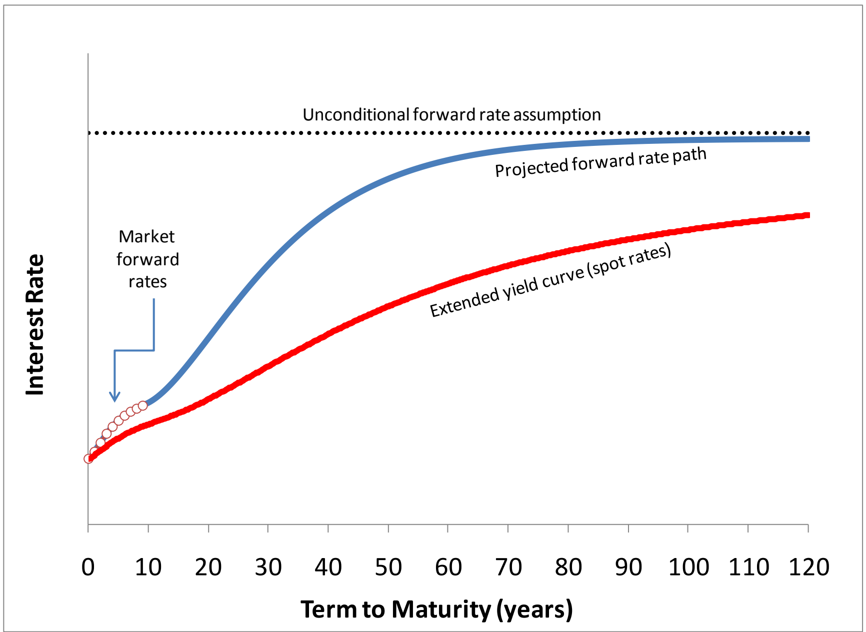
The first task requires a methodology for market data selection and for fitting forward interest rates which are both smooth and exhibit credible stability through time. Further, we believe that a rational method should deliver forward rate changes where variability declines with maturity. Our proposed method involves fitting a smooth curve (a 'regression spline') through a small number of 'knot' points in order to simultaneously fit the observed data closely and meet the smoothness criteria. Further we propose to restrict the gradient (and variability) of the longest observed market forward rates before extrapolating towards an unconditional forward rate target using the well-known ***Nelson-Siegel*** methodology[2].

The extrapolation requires us to answer two questions concerning how unobservable 'pseudo' forward rates might be expected to be determined beyond the term of available market data.

- First, we must set an unconditional (i.e. very long term assumption) for forward rates based on long-term expectations for the components of forward interest rates: the long-term real rate of interest, long-term inflation expectations, bond term premia and a technical 'convexity' adjustment. Based on long term empirical data across 16 economies, we set a global unconditional nominal forward interest rate of 5.3% and an unconditional nominal swap forward rate of 5.7% consistent with a forward swap spread assumption of 40bps at the end of September 2008.
- Second, we must define how quickly forward rates move toward this unconditional assumption. Here, a path is chosen so that variation in forward rates over time is matched (so far as this is possible) to a set of targets for forward rate volatility that we judge to be reasonable. This variation decreases over time which is consistent with the view that unobservable ultra long-term forward rates can be expected to be pretty stable.

The chart below provides a high-level illustration of our new methods in a situation where market instruments are available to a maximum term of 10 years. In practice the available term of market data as well as the shape of curves and extrapolations will vary considerably across currencies and over time.

Exhibit
Yield curve extrapolation – the basic idea



[2] *Nelson, C. and A. Siegel (1987). Parsimonious Modelling of Yield Curves. Journal of Business 60-4, p473–489*

1 Overview of the problem and our general approach

Risk-free yield curves are the basic building blocks for the valuation of future financial claims and long-term risk management work. In highly-developed fixed income markets we may be able to observe bonds or interest rate swap contracts with maturities of up to 50 years. In less developed markets liquid bond quotations might be limited to only a few years. In exceptional circumstances there may be no traded risk-free instruments at all. Of course, the liabilities of long-term financial institutions often extend beyond the term of available market instruments. In order to value these ultra long-term claims and assess risk, practitioners will extrapolate yield curves to generate a set of 'pseudo-prices' for the assumed, inferred prices of discount bonds beyond the term of the longest available traded cash flow. In Barrie & Hibbert's yield curve calibration work, extrapolations are performed to a standard term of 120 years.

The purpose of this *Exposure Draft (ED)* is to document the specific practices to be applied in B+H's work. Our aim is to codify practice in this area so far as possible. Prior to June 2008 our practice was to extrapolate yields from existing market data without explicit targets for the behaviour of very long-term forward rates. The procedures set out in this ED modify our working practices to use economic fundamentals to 'anchor' unobservable long-term yields. Changing our methodology will have an impact on the market-consistent and real-world interest rate calibrations.

Interest rates at the long (untraded) end of the curve and changes in those rates can have a material impact on value and risk capital calculations for long-term financial institutions. Furthermore, spurious variation in these yields can feed through to give material balance sheet and valuation variation. With this in mind it becomes extremely important to develop a robust yield curve construction method: one which reflects (to some degree) current market conditions and empirical views of long rate volatility, while simultaneously displaying adequate stability. We aim to answer the following questions:

- *How to interpolate a yield curve with discontinuous market data?* In theory, a forward rate curve should be a smooth. The interesting question is '*How smooth?*' In the real world, available government bond prices and swap rates from liquid markets are not quoted across a continuous term structure. Variations in issue size and liquidity and other bond-specific factors mean observed yield curves can look quite 'lumpy'. A method is required to specify how to join up sparse market rates and impose an appropriate degree of smoothness.
- *How to extrapolate the yield curve to terms where there are no liquid traded risk free instruments?*
- *What is a suitable target for the forward interest at a long-term horizon?* To extrapolate beyond traded maturities, we need to set an assumption for the unconditional forward interest rate. Should we simply use the average of empirical data or use financial economic theory to justify an assumption? A sound methodology to 'anchor' a relatively consistent and stable long term conditional target is desirable.
- *Should unconditional forward rate assumptions vary significantly across economies?* It is widely believed that short-term interest rates of certain currencies include a risk premium. Can we justify permanent differences between economies for any of the components that make up the unconditional forward rate: expectations for real interest rates, inflation, risk premia and convexity?

These are not simple questions. Regulators and policymakers have addressed them in different ways. For example, the CFO Forum establishes a general principle by saying that yield curves "***should be extended using an appropriate methodology***". They then offer some example methods which we believe will be appropriate in some circumstances but fail elsewhere[3]. In the QIS4 technical guidance for Solvency II yield curve "***interpolations and extrapolations are based on the assumption that forwards are constant***". Bitter experience of dealing with worldwide yield curves suggests that, while this simple rule will work fine at some times and in some places, its universal application would produce results that we cannot justify. It is also important to note that where a regulator or convention prescribes a simple extrapolation rule our proposed method does not preclude it; an analyst can replace our extrapolated estimates with ruled-based values. Our method is aimed at simply giving our best answer to the questions posed above.

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[3] *CFO Forum: "Market Consistent Embedded Value Principles", June 2008*

2 Yield curve interpolation and extrapolation

2.1 General principles

We define some general properties which we believe are desirable properties for the yield curves we will construct:

1. Where a liquid market exists the yield curve should accurately price that market.
2. The (forward rate) curve should be continuous.
3. The (forward rate) curve should be smooth. That is, its first derivative should be continuous.
4. On average, the variability of long-term (forward) interest rates should be lower than shorter term (forward) rates.

2.2 Interpolation

We fit the market bond data and swap data using a regression spline technique. We fit a natural cubic spline to the instantaneous forward interest rate curve and calculate the corresponding zero coupon bond prices by:

$$P(T) = exp\left[-\int_0^T F(t)\,dt\right] \tag{1}$$

Where $P(T)$ is the price of a zero coupon bond with maturity T and $F(t)$ are the set of instantaneous forward rates. With a set of zero coupon bonds we can evaluate swap rates and bond prices and compare with the market. We adjust the points of the spline in order to alter the forward rate curve and achieve better agreement with the market data. These points allow us to pivot the curve so as to obtain a close fit and are known as 'knot-points'. It is possible to fit a spline through many knot-points. However, in practice a more credible curve (conforming to our general principles) is produced by fitting through a smaller number of points (typically between 3 to 10) and accepting that a perfect fit may not be produced for individual points. Furthermore, a lower number of knot-points also helps to minimize oscillations in the fitted curve through time.

2.3 Fitting market swap rates and bond prices

In practice, we use two types of market data to specify the yield curve: bond price data or (swap) interest rate data. In the market, the swap rate is normally quoted in the form of a par yield for a given maturity. Government bond data can be quoted in the form of either bond yields or market bond prices. Therefore, instead of defining our approach for swap rates and bond prices directly, we define it for market yields and price data.

When fitting the market yields we adjust the forward rate at the spline knot-points to minimise

$$X_S = \sum_{i=1}^{N} [S_i - \pi_i]^2 \tag{2}$$

Where S_i is the market yield, π_i is the yield obtained from the fitted forward curve and N is the number of market data points. All market data points are weighted equally.

When fitting the market *prices* we minimise

$$X_G = \sum_{i=1}^{M} \left[\frac{B_i - \pi_i}{D_i}\right]^2 \tag{3}$$

Where B_i is the market price, π_i is the price obtained from the splined forward curve, D_i is the duration and M is the number of market data points. Here, we weight the objective function by the bond duration. Since the sensitivity of bond prices will vary with bond duration, we have weighted prices using their duration to give approximately the same weighting strategy as for yields. Without such weighting the fit to the market prices will tend to suffer for short-term maturities, especially when a smoothing constraint is applied.

When interpolating the swap and government rate curve, we apply a smoothing constraint to dampen the volatility of the forward rate curve at long maturities.

$$X'_{S,G} = X_{S,G} + \omega \int_{T_1}^{T_2} \left[\frac{dF(t)}{dt}\right]^2 dt, \tag{4}$$

Where ω is a constant parameter that specifies the strength of the smoothing constraint. We only employ this smoothing at the end of the forward curve and $(T_2 - T_1)$ which corresponds to the final 20% of the available market data. Using a smoothness condition inevitably affects the quality of fit. We minimise the gradient squared of the forward curve whilst remaining approximately 5 bps from the bond market data and 2 bps from the swap market data.

Note that, were we *not* to apply this smoothing constraint, any extrapolation methodology which is constrained to be smooth at the longest market forward rate, would produce increasing volatility in forward rate volatility for some real-world yield curves which exhibit strong curvature at the long end. We believe this is largely a consequence of the difficulty of measuring long-term forward rates and impose what we believe is an economically rational constraint on forward rate behaviour.

2.4 Extrapolation

Extrapolation requires answering two questions about the path of forward interest rates beyond the longest market data point. Firstly, ***what is an appropriate assumption for the infinite-maturity, unconditional forward rate of interest?*** Secondly, ***what path is chosen between the longest (smoothed) market forward rate and this ultra long-term rate?*** In particular, the analyst needs to determine the speed at which the extrapolated forward rate tends towards the long-term asymptote.

We extrapolate the forward curve from the end of the market data to an unconditional forward rate target using the *Nelson-Siegel* parametric form:

$$F(t) = \beta_1 + \left(\beta_2 + \beta_3(t - t_{max})\right) \exp\left(-\lambda(t - t_{max})\right) \qquad (5)$$

Here β_1 corresponds to the unconditional forward rate, λ is a free parameter which we can use to control the speed of convergence and β_2 and β_3 can be specified by matching the value and derivative of the curve at the last market data point. The terminal market maturity is denoted by t_{max}. We fix the parameter λ by constraining the standard deviation of extrapolated forward rates to meet a certain target at a particular horizon. Let us look at this in more detail.

The choice of λ parameter controls the variation through time in extrapolated forward rates. A high value for λ will lead to faster convergence of the forward curve to the unconditional forward rate target, so the forward rate volatility will decrease. A low value for λ will lead to slower convergence to the unconditional target so forward rate variation will be higher. Here, forward rate volatility at the longest market data point will be transmitted to adjacent maturities and only decay slowly. We can set the value of λ by estimating the forward rate volatility term structure using historical forward rate data and market data sources. This task turns out to be difficult. Our approach and conclusions on suitable assumptions for forward rate volatility at various horizons are set out in detail in our forthcoming report *"How to construct a volatility term-structure of interest rates in the absence of market prices"* (Z. Liu, 2008). A summary of forward rate volatility targets is shown below in exhibit 2.1.

Initially (i.e. at September 2008) we use a value of $\lambda = 0.06$ to be consistent with the target forward rate volatility decay shown below.

Exhibit 2.1

Volatility targets for the year-to-year proportional changes of instantaneous forward rates

Maturity/years	1	5	10	15	30	60	120	Infinity
vol: target	27.9	14.8	12.8	12.4	9.6	3.1	0. 2	0

The following section explains how the unconditional forward interest rate β_1 is chosen.

3 Assumptions for the long-term forward interest rate and extrapolation path

3.1 General principles

Similar to yield curve construction, we set out some general principles for estimation of the unconditional nominal forward rate.

Stability: The unconditional nominal forward rate *cannot be materially affected by short term economic changes*.

Consistency: If the long-term forward rate is approached at some very long-term horizon, we *would expect this value to be broadly the same around the world*.

Simplicity: Whilst it is inherently difficult to estimate the long-term forward interest rates, we *aim to use a simple approach that is easy to understand* that can be applied to a large number of economies.

3.2 Methodology for setting an unconditional forward interest rate

The long term (unconditional) nominal forward interest rate is defined as the forward interest rate for an infinite maturity. In practice, for the range of parameters we judge to be suitable, there is a high degree of convergence in forward rates over the typical 50-120 year horizon over which the methodology is used.

No arbitrage argument can be used to show that the nominal long-term (unconditional) forward interest rate (UFR) consists of four components:

Long term nominal forward interest rate = *Real expected short-term interest rate*

\+ *Long term expected inflation*

\+ *Long term nominal term premium*

\+ *Long term nominal convexity effect*

In general, we have based our estimates for each component of the UFR on empirical data for countries with available long historical time-series of annual data. We construct each of the components of the long term nominal forward rate using a weighting scheme of historical data that attributes a larger weight to more recent observations. Details of how we propose to estimate each of the components of the unconditional nominal forward rate is set out in detail in the report '*Calibration Methods - Interest rate calibration: How to set long-term interest rates in the absence of market prices*'.

3.3 Targets for a global unconditional nominal forward rate

Exhibit 3.1 summarises our assumptions for the unconditional nominal forward rate at end-September2008. The unconditional nominal forward rate is set at 5.3% comprised of an expected real cash rate of 1.8%, expected inflation of 2.4%, an unconditional term premium of 1.5% and a technical convexity adjustment of 0.4%. All of these estimates are based on slow-moving historical exponentially weighted averages with the exception of expected inflation. Here, our estimate is weighted 80% towards a forward-looking view derived from an average of central bank inflation targets.

Exhibit 3.1

Long-term nominal government and swap forward interest rate

Assumptions for 2008	Government bond calibration	Swap calibration
Expected real interest rate	**1.8%**	**2.0%**
Expected inflation	**2.4%**	**2.4%**
Expected short rate	4.2%	4.4%
Term premium	**1.5%**	**1.7%**
Convexity adjustment	**-0.4%**	**-0.4%**
Unconditional forward rate	5.3%	5.7%

Exhibit location: summary_tables.xls

In the swap calibration we add an additional spread of 40bps to the long-term level of the unconditional forward rate, giving a long-term unconditional forward rate in the swap calibration of 5.7%. We plan to revise these estimates in the first quarter each year. Our research provides more background on this assumption[4]. In summary we believe that there are three possible explanations for this spread:

i. Government bonds and the swap rare are referenced to different interest rates.

ii. The term (risk) premia embedded in the government and swap rates are different.

iii. If either explanation is correct, the relative variance of the prices of the instruments may differ and contribute to the spread.

Finally, note that, while we add all of this spread to our estimate of the unconditional forward rate in the swap calibration, we attribute only 20 bps to the long term real short rate.

These assumptions are used for all economies where we currently construct yield-curves.

[4] *See Barrie+Hibbert draft calibration note: 'Understanding the long-term swap spread for real-world calibration'. This note will be discussed with the TAP in Autumn 2008.*

4 Next Steps & Comments

This Exposure Draft summarises the methods to be applied in yield curve construction work by Barrie & Hibbert from 30th September 2008. Further details of our yield curve construction methods can be found in two documents which have been reviewed in an earlier draft form by the Technical Advisory Panel:

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Any comments should be sent using email to Ruosha.Li@barrhibb.com

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