Measuring Counterparty Credit Risk for Trading Products under Basel II

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1. Introduction

Counterparty credit risk is the risk that the counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract. Only the contracts privately negotiated between counterparties -- over-the-counter (OTC) derivatives and security financing transactions (SFT) – are subject to counterparty risk. Exchange-traded derivatives are not affected by counterparty risk because the exchange guarantees the cash flows promised by the derivative to the counterparties. Counterparty risk is similar to other forms of credit risk in that the cause of economic loss is the obligor's default. There are, however, two features that set counterparty risk apart from more traditional forms of credit risk: the uncertainty of exposure and the bilateral nature of credit risk.

If a counterparty in a derivative contract defaults, the surviving counterparty must close out its position with the defaulting counterparty and enter into a similar contract with another counterparty to maintain its market position. Therefore, counterparty exposure is determined by the contract's replacement cost at the time of default. Assuming no recoveries, let us determine our replacement cost if our counterparty defaults under these two scenarios:

- <u>Contract value is *negative* for us at the time of default</u>:
  - We close out the position by *paying* the defaulting counterparty the market value of the contract.
  - We enter into a similar contract with another counterparty and *receive* the market value of the contract.
  - Our net loss is zero.
- <u>Contract value is *positive* for us at the time of default</u>:
  - We close out the position, but *receive* nothing from the defaulting counterparty.
  - We enter into a similar contract with another counterparty and *pay* the market value of the contract.
  - Our net loss equals the contract's market value to us.

Thus, credit exposure of one counterparty to the other is the maximum of the contract's market value and zero. Since the contract value changes unpredictably over time as the market moves, only the current exposure is known with certainty, while future exposure is uncertain. Moreover, since the contract value can change sign and either counterparty can default, counterparty risk is bilateral.

Counterparty credit exposure can be dramatically reduced by means of netting agreements. In general, if there is more than one trade with the defaulted counterparty, the maximum loss for the surviving counterparty equals the sum of the contract-level credit exposures. A netting agreement is a legally binding contract between two counterparties that, in the event of default, allows aggregation of transactions between these counterparties. Derivatives with positive value at the time of default offset the ones with negative value and only the net value needs to be paid. Thus, the total credit exposure created by all transactions in a netting set (i.e., those under the jurisdiction of the netting agreement) is reduced to the maximum of the net portfolio value or zero.

Counterparty credit exposure can be further reduced by margin agreements. A margin agreement limits the potential exposure of one counterparty to the other by means of collateral requirements should the unsecured exposure exceed a pre-specified threshold. Under a bilateral collateral agreement, both counterparties periodically (typically, daily)

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mark their positions to market and check their net portfolio value against the other counterparty's threshold. If the net portfolio value exceeds the threshold, the other counterparty must provide collateral sufficient to cover this excess. Thus, collateral agreements essentially limit the exposure to the unsecured portion below the threshold. The threshold value depends primarily on the credit quality of the counterparty.

In this chapter, we describe treatment of counterparty credit risk under the Basel II Framework. Section 2 is devoted to minimum capital requirements for counterparty credit risk. Sections 3 and 4 describe non-internal and internal model methods, respectively, for calculating exposure at default. In Section 5, we suggest an exposure modeling framework that can be used for calculating exposure distribution at future dates (such a framework is required under Internal Model Method). Section 6 is devoted to treatment of margin agreements and modeling collateral.

2. Minimum Capital Requirements for Counterparty Risk

According to the Basel II framework, minimum capital requirements for counterparty credit risk in OTC derivatives and SFT are calculated by applying Basel II rules for corporate exposures. These rules are described in the Basel Committee's document entitled "International Convergence of Capital Measurement and Capital Standards" (BCBS, 2005b), known as the Revised Framework. Here is a brief summary of the minimum capital requirement calculation for corporate exposures under the Advanced Internal Ratings Based (AIRB) approach.

Regulatory Capital, C, is calculated according to

$$C = \mathrm{EAD} \times K(\mathrm{PD}, \mathrm{LGD}) \times \mathrm{MA}(\mathrm{PD}, M) \tag{2.1}$$

where EAD is exposure at default, PD is obligor's probability of default (subject to a floor of 0.03%), LGD is exposure-level expected loss given default, conditional on economic downturn, and M is the exposure's effective remaining maturity (subject to a floor of one year and a cap of five years).

$K(\mathrm{PD}, \mathrm{LGD})$ is default-only capital factor that is calculated from PD and LGD according to

$$K(\mathrm{PD}, \mathrm{LGD}) = \mathrm{LGD} \times \mathrm{N}\left(\frac{\mathrm{N}^{-1}(PD) + \sqrt{\rho}\,\mathrm{N}^{-1}(0.999)}{\sqrt{1-\rho}} \right) - \mathrm{LGD} \times \mathrm{PD} \tag{2.2}$$

where $\mathrm{N}(\cdot)$ is the standard normal cumulative distribution function and its inverse, respectively, and ρ is the asset correlation which depends on PD:

$$\rho = 0.12 \times \frac{1-\exp(-50 \times PD)}{1-\exp(-50)} + 0.24 \times \left(1 - \frac{1-\exp(-50 \times PD)}{1-\exp(-50)} \right) \tag{2.3}$$

$\mathrm{MA}(\mathrm{PD}, M)$ is maturity adjustment. It describes migration risk and is calculated from PD and M according to

$$\mathrm{MA}(\mathrm{PD}, M) = \frac{1 + (M - 2.5) \times b(\mathrm{PD})}{1 - 1.5 \times b(\mathrm{PD})} \tag{2.4}$$

where $b(\mathrm{PD})$ is a function of PD defined as

$$b(\mathrm{PD}) = \left[0.11852 - 0.05478 \times \ln(\mathrm{PD}) \right]^2 \tag{2.5}$$

The major difficulty in applying these rules to counterparty risk in SFT and OTC derivatives is the uncertainty of future exposure and complexity associated with calculation of future exposure distribution. Basel Committee's document entitled "The Application of Basel II to Trading Activities and the Treatment of Double-Default" (BCBS, 2005a) describes methods of calculating EAD.[3] For OTC derivatives, these methods include Current Exposure Method (CEM), Standardized Method (SM) and Internal Model Method (IMM) in the order of increasing sophistication. SFTs are treated either under the non-internal method for repo-style transactions or under the IMM.

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[3] This document appeared as a supplement to an earlier version of the Revised Framework. The current version of the Revised Framework, BCBS (2005b), incorporates the methods described in BCBS (2005a). Fleck and Schmidt (2005) provide an excellent review of these methods as well as the old Basel I rules.

Under the IMM, banks are allowed to compute distribution of exposure at future dates using their own models. Assuming that this distribution is available, the IMM prescribes a way of calculating EAD and effective maturity *M* from the expected exposure profile.

EAD is calculated at the *netting set* level under any of these methods. Netting set is a group of transactions with a single counterparty that are subjects to a legally enforceable bilateral netting agreement that satisfies certain legal and operational criteria described in Annex 4 of BCBS (2005b). Cross-product netting between SFTs and OTC derivatives is allowed only under the IMM.[4] Netting other than on a bilateral basis is not recognized for the purpose of calculating regulatory capital. Each transaction that is not subject to a legally enforceable bilateral netting agreement is interpreted as its own netting set.

In the remainder of this chapter, we will cover all three Basel methods for EAD calculation, describe the basic principles of exposure modeling, and discuss the modeling of margin agreements.

3. Non-Internal Model Methods

The banks can choose to apply either the CEM or the SM in estimating EAD for their OTC derivative positions. For SFTs, there is a separate non-internal method. These non-internal methods are designed to provide a simple and workable supervisory algorithm, which results in a number of simplifying assumptions.

3.1. *Current Exposure Method*

Under the CEM approach, EAD is computed according to

$$\text{EAD} = \text{RC} + \text{Add-on} \tag{3.1}$$

where RC is the current replacement cost and Add-on is the estimated amount of the potential future exposure (PFE). For a single transaction, Add-on is calculated as the product of the transaction notional and the *Add-on Factor*, which is determined from the regulatory tables on the basis of the remaining maturity and the type of underlying instrument (e.g., interest rates, foreign exchange, etc.).

For a portfolio of transactions covered under a legally enforceable bilateral netting agreement, the RC is simply the net replacement cost across derivative contracts in the netting set, given by the larger of net portfolio value or zero. However, the Add-on on the portfolio of transactions within a netting set is calculated under the Basel I formula with only limited netting, that is

$$\text{Add-on(P)} = \left(0.4+0.6\cdot\text{NGR}\right)\cdot\sum_i \text{Add-on}_i \tag{3.2}$$

where Add-on_i is the Add-on for transaction i and NGR is the ratio of the current net replacement cost (RC under full netting) to the current gross replacement cost (RC under no netting) for all transactions within the netting set.

Finally, for collateralized counterparty, the credit exposure for transactions with a netting set is calculated as

$$\text{EAD} = \max\left[0, \text{MtM(P)-C}_A\right] + \text{Add-on(P)} \tag{3.3}$$

where MtM(P) is mark-to-market of the portfolio, C_A is the volatility-adjusted collateral amount (i.e., the value of collateral minus haircut) and Add-on(P) is Add-on on the portfolio of transactions under the netting set.

3.2. *Standardized Method*

The standardized method (SM) in Basel II was designed for those banks that do not qualify to model counterparty exposure internally but would like to adopt a more risk-sensitive approach than the CEM.

Under the SM, one computes the EAD for derivative transactions within a netting set as follows:

[4] Cross-product netting is not allowed under non-internal methods because EAD calculation rules are product-specific.

(3.4) $$EAD = \beta \times \max\left[NCV, \sum_j NRP_j \times CCF_j \right]$$

where NCV is the current market value of transactions in the portfolio net the current market value of collateral assigned to the netting set; NRP_j is the absolute value of net risk position in the hedging set j; and CCF_j is the credit conversion factor with respect to the hedging set j, that converts the net risk position in the hedging set into a PFE measure. The long positions arising from transactions with linear risk profiles shall carry a positive sign, while short positions carry a negative sign. The positions with nonlinear risk profiles are to be represented by their delta-equivalent notional values.[5]

The use of delta-equivalent notional values for options implies that sold options enter the calculation of the risk positions under the SM approach. This is a notable difference in comparison to the CEM, which includes only purchased options, since the CEM adopts a transaction-by-transaction approach instead of considering the netting set as a portfolio. At the level of individual transactions, sold options will not generate exposure to counterparty credit risk as they imply only potential net liabilities to the bank if any outstanding option premiums are disregarded. However, from a portfolio perspective, a reduction in the price of a sold option, all else being equal, will increase the current market value of the netting set and thus, may lead to counterparty credit risk.

The hedging set is defined as the portfolio risk positions of the same category (such as currencies, remaining maturities, and market risk factors) that arise from transactions within the same netting set. Within each hedging set, offsets are fully recognized, that is, only the net amount of all risk positions within a hedging set is relevant for the exposure amount or EAD. The exposure amount for a counterparty is then the sum of the exposure amounts or EADs calculated across the netting sets with the counterparty. The calibration of CCFs is assumed for a one-year horizon on ATM forwards and swaps because the impact of volatility on market risk drivers are more significant for at-the-money trades. Thus, this calibration of CCFs results in a conservative estimate of PFE.

In the course of the Trading Book Review (ISDA-LIBA-TBMA, 2004), the regulators have received assistance from ISDA counterparty risk working group in calibrating the CCF in reference to the banks' internal PFE model outputs. The schedule below summarizes the CCFs for different risk drivers and compare the ISDA proposed measures with the matrix obtained by the Trading Book review.

Exhibit 3.1. CCFs proposed by ISDA and Trading Book Review (TBR)

FX	CCF	Equity	CCF	IRP 1yr	CCF	IRP 5yr	CCF
USD/EUR	3.22%	S&P	4.35%	USD	0.19%	USD	0.30%
USD/JPY	2.71%	FTSE	7.36%	EUR	0.14%	EUR	0.19%
USD/GBP	2.79%	DAX	6.62%	JPY	0.05%	JPY	0.13%
EUR/JPY	3.13%	NIKKEI	4.30%	GBP	0.12%	GBP	0.18%
FX (TBR)	2.50%	Equity (TBR)	7.00%	IRP (TBR)	0.20%	IRP (TBR)	0.20%

3.3. *Treatment of Repo-styled Transactions*

For repo-styled transactions, the exposures are calculated as the difference between market value of the securities and the collateral received, and given by

(3.5) $$EAD = \max\left[0; S \cdot (1 + H_S) - C \cdot (1 - H_C) \right]$$

where H_S is the haircut on the security and H_C is the haircut on the collateral. The haircuts must be applied to both the securities and collateral received in order to account for the risk arising from the decline in value of securities held or collateral received as result of future market movements. The levels of haircut are usually estimated according to market price volatility and the foreign exchange volatility in the case of securities denominated in

[5] A key simplification within the SM approach is that the non-linear positions are defined as "delta equivalent" positions. In other words, the method assumes that the positions with a forecasting horizon of one day shall remain open and unchanged throughout a one year horizon.

foreign currency. Furthermore, it takes into account the type of security, its credit rating and the duration of its maturity.

As an alternative method to the use of standard haircuts, the banks may take a VaR-based approach to reflect the price volatility of the exposure and collateral received. Under the VaR-based approach, the EAD or exposure is calculated for each master agreement as

(3.6) $$\text{EAD} = \max\left[0;\ \sum_i S_i - \sum_i C_i + \text{VaR}\right].$$

The advantage of the VaR model is to improve the rule-based aggregation under standard haircuts by taking into account correlation effects between security positions in the portfolio. The VaR-based approach is available to banks that have already received the approval for the use of internal models under the Market Risk Amendment (BCBS, 2005c). Other banks can separately apply for supervisory recognition to use their internal VaR models for the haircut calculation on repo-style transactions.

The quantitative and qualitative criteria for recognition of internal market risk models on repo-style transactions and other similar transactions are, in principle, the same as under the Market Risk Amendment. With regard to the holding period, the minimum will be 5-business days for repo-style transactions, rather than the 10-business days under the Market Risk Amendment. For other transactions eligible for the VaR models approach, the 10-business day holding period will be retained. The minimum holding period should be adjusted upwards for market instruments where such a holding period would be inappropriate given the liquidity of underlying instrument.

4. Internal Model Method

The Internal Model Method (IMM) is the most risk-sensitive approach for the exposure at default (EAD) calculation available under the Basel II framework. It is intended to provide incentives for banks to improve their measurement and management of counterparty credit risk by adopting more sophisticated practices. Under the IMM, both EAD and effective maturity *M* are computed from the output of bank's internal models of future exposure. These models must be approved by the bank's supervisors for the it to become eligible for the IMM.

Similar to the non-internal methods, EAD under the IMM is calculated at the netting set level. However, in contrast to non-internal methods, cross-product netting is permitted under the IMM, and the EAD calculated under IMM benefits full netting.

Under the IMM, EAD for a netting set is calculated according to

(4.1) $$EAD = \alpha \times Effective\_EPE$$

where Effective EPE is the Effective Expected Positive Exposure calculated for each netting set from the expected exposure (EE) profile as described below, and α is a multiplier.

4.1. *Calculating EPE and Effective EPE*

Based on the results from resent research (e.g., Canabarro, Picoult and Wilde, 2003) and the Trading Book Review document (ISDA-LIBA-TBMA, 2004), the expected positive exposure (EPE) is generally viewed as the appropriate EAD measure to determine the capital for counterparty credit risk in a trading book.

Typically, banks that model exposure internally compute exposure distributions at a set of future dates $\{t_1, t_2, \ldots, t_K\}$ using Monte Carlo simulations (we will describe a suitable exposure modeling framework in Section 5). For each simulation date t_k, the bank computes expectation of exposure EE_k as a simple average of all Monte Carlo realizations of exposure for that date.

EPE is defined as the average of the EE profile over the first year. Practically, it is computed as the weighted average of EE_k as follows:

(4.2) $$\text{EPE} = \sum_{k=1}^{\min(1yr, maturity)} \text{EE}_k \times \Delta t_k$$

where weights are defined as time intervals between simulation dates $\Delta t_k = t_k - t_{k-1}$.

Generally, as the number of trades with the counterparty and the number of unrealized cash flows in the portfolio decrease over time, the portfolio exposure decreases as a function of time. In particular, the EE profile can decrease because of the expiration of short-term trades. However, these short-term trades are likely to be replaced by new ones, but the EPE does not take this into account and can, therefore, understate the risk.

To account for this *roll-over risk*, the definition of EPE is modified as follows. First, *Effective EE* profile over the first year is obtained from the EE profile by adding the non-decreasing constraint. In our notations, this non-decreasing constraint can be represented via the simple recursive relation given by

(4.3) $$(\text{Effective EE})_k = \max\left[(\text{Effective EE})_{k-1}, \text{EE}_k\right],$$

with the initial condition of $(\text{Effective EE})_0$ being equal to the current exposure.

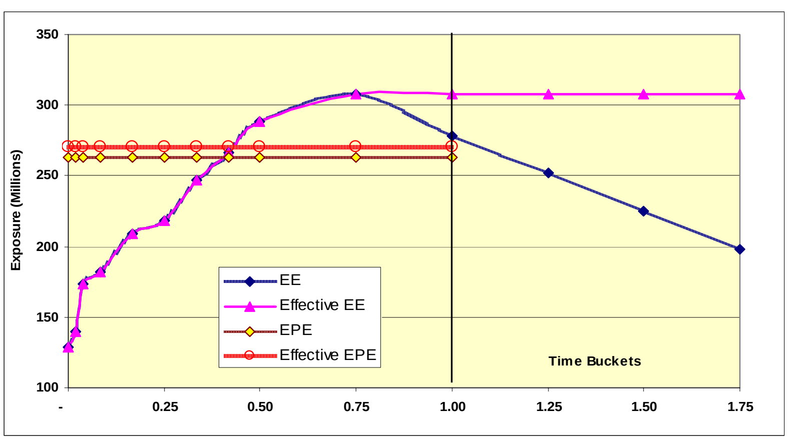
The *Effective EPE* is computed from Effective EE profile in exactly the same way as EPE is computed from EE profile:

(4.4) $$\text{Effective EPE} = \sum_{k=1}^{\min(1yr, maturity)} (\text{Effective EE})_k \times \Delta t_k$$

Similarly to EPE, the Effective EPE is calculated from the Effective EE profile up to one year. If all contracts in the netting set mature before one year, the Effective EPE is the average of Effective EE until all contracts in the netting set mature. The roll-over adjustment in (4.4) is effective only below one year and its impact becomes increasingly dominant on a portfolio of short-dated OTC derivative or repo-style trades.

Figure 1 shows a plot of EE, Effective EE, EPE and Effective EPE calculated on a typical exposure profile of a portfolio under a netting set.

Figure 1 Expected Exposure and Effective EPE



4.2. *Multiplier Alpha*

In contrast to loan exposures, derivative exposures are uncertain. This uncertainty makes modeling portfolio counterparty risk very complicated, as market risk factors that drive exposure should be modeled together with credit risk factors that drive counterparty credit quality. This complexity could be greatly reduced if we could make separate modeling of market and credit risk factors as follows: (i) model market risk factors to calculate exposure distribution; (ii) from exposure distribution determine deterministic loan equivalent exposure (LEE); (iii) use LEE with a loan portfolio model to calculate capital. The LEE should be defined as follows: we replace the derivative exposures by a single LEE for each counterparty in such a way that the portfolio default-only capital is not changed.

Canabarro, Picoult and Wilde (2003) showed that for an infinitely granular portfolio (i.e. portfolio with infinitely many counterparties with infinitely small exposures) with counterparty-level exposures independent both across themselves and from the counterparty credit quality, EPE becomes the true LEE.[6] Real portfolios, however, are not infinitely granular and counterparty exposures are not independent, as they are driven by the same set of market risk factors. Additionally, credit exposure may be correlated to the counterparty's credit quality, which results in right- or wrong-way risk. Thus, used without correction for real portfolios, EPE is bound to understate portfolio capital.

Picoult (2002) introduced the concept of a multiplier, α, defined as the ratio of default-only portfolio capital computed via the full model with uncertain exposures to the one computed via the reduced model obtained from the full model by replacing uncertain exposures by EPE. Since capital in the reduced model is a homogeneous function of exposures, scaling EPE by α would match the capital produced by the full model. Thus, $\alpha \times \text{EPE}$ is the true LEE in the sense that using this quantity as EAD removes the difference in capital treatment between economically equivalent loans and derivatives.

Canabarro, Picoult and Wilde (2003) used a one-factor credit risk portfolio model to study the sensitivity of α to various model inputs and parameters under the assumption of wrong-way risk. Their base case resulted in $\alpha = 1.09$. After adding wrong-way risk to the same model, Wilde (2005) obtained for the same base case $\alpha = 1.21$. ISDA-LIBA-TBMA (2004) reports estimates of α in the range 1.07-1.10 calculated by four banks using their internal models for their own portfolios.

Under the IMM, α is fixed at a rather conservative level of 1.4. However, banks using the IMM have an option to compute their own estimate of α, subject to supervisory approval and a floor of 1.2.

4.3. *Maturity Adjustment*

Under Basel II, default-only capital is scaled by some maturity adjustment given by Equation (2.4). The maturity adjustment accounts for the migration of risk exposures that extend beyond the one-year horizon. , Banks are required to compute the EE profile out to the expiration of the longest contract in the netting set. For exposures with remaining maturity greater than 1 year, the Effective Maturity *M* is given by

(4.5) $$M = \min\left[1 + \Delta M, 5 years\right]$$

where

(4.6) $$\Delta M = \frac{\sum_{t_k > 1 year}^{maturity} \left[\text{EE}_k \times \Delta t_k \times \text{DF}_k\right]}{\sum_{k=1}^{t_k \le 1 year} \left[\left(\text{Effective EE}\right)_k \times \Delta t_k \times \text{DF}_k\right]},$$

where DF_k is the discount factor from the simulation date t_k to today. When all the trades in a netting set mature in less than one year, $\Delta M = 0$ and there is no maturity adjustment. On other hand, if the trades in a netting set are out-of-money today, but there is non-zero credit exposure at some future date, the denominator in (4.6) may become very small and the effective maturity in this case can potentially extend beyond the contractual maturity of the netting set.

The one-year floor does not apply to certain short-term exposures that have remaining maturity of less than one-year and that are collateralized. The instruments included in this category are SFTs (eg., repo-style transactions). For transactions in this category and that are subject to a master netting agreement, Effective Maturity is calculated as

(4.7) $$M = \max\left[t_{\text{margin}}, M_{\text{wt avg}}\right],$$

[6] More details are given in Wilde (2005).

where t_{margin} is the minimum holding period for the transaction type, and $M_{\text{wt avg}}$, the weighted average maturity of the transactions, is defined as

$$(4.8) \qquad M_{\text{wt avg}} = \sum_{i=1}^{N} P_i \times T_i \Big/ \sum_{i=1}^{N} P_i$$

where P_i is the notional principal of transaction i, and T_i is the remaining time to maturity of ith- transaction within the same netting set. If there is more than one transaction type present in the netting set, the higher holding period[7] will be applied in determining the effective maturity.

5. Credit Exposure Framework

This section discusses a general framework for calculating the potential future exposure on OTC derivative products, particularly for path-dependent derivative instruments. Such a framework is necessary for banks to obtain supervisory approval for using the IMM. However, the value of credit exposure framework goes beyond calculating regulatory capital, as the same framework can be used for internal purposes such as the calculation of economic capital or measuring exposure against limits. Thus, it is extremely important for a bank to have robust and accurate exposure models, as well as systems infrastructure, to quantify the potential exposures of its derivatives positions.

While Basel II requires calculation of expected exposure profiles at the netting set level, internal applications may require calculations at the counterparty level. These calculations may involve characteristics of exposure distribution other than expectation (e.g., exposure profiles at some high confidence levels). The exposure framework we present here can cover both regulatory and internal applications because it allows one to calculate exposure distribution (and, therefore, any of its characteristics) at both netting set and counterparty levels.

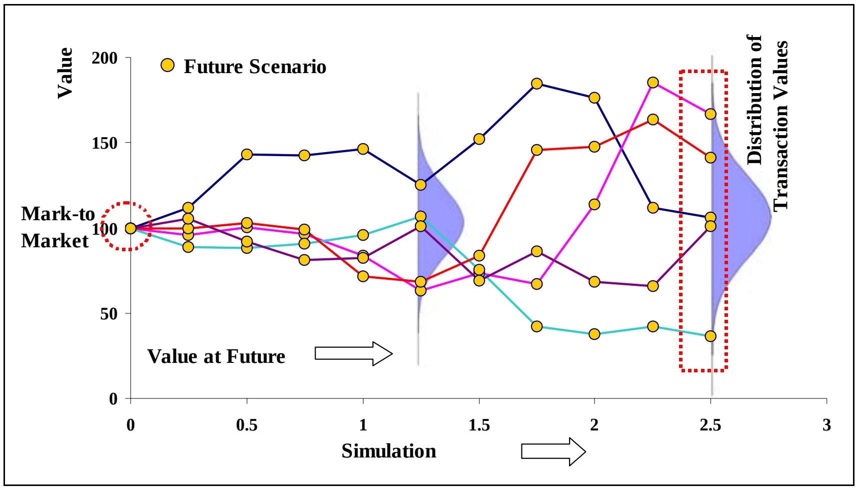
There are three main components in calculating the distribution of netting-set-level or counterparty-level credit exposure:

- Scenario generation: Future market scenarios are usually simulated for a fixed set of simulation dates using evolution models of the risk factors. The simulation is done under the "real measure" instead of the "risk-neutral" pricing measure used for pricing.
- Instrument valuation: For each simulation date and for each realization of the underlying market risk factors, instrument valuation is performed.
- Aggregation: For each simulation date and for each realization of the underlying market risk factors, instrument values are added within each netting set to obtain netting set portfolio value. Realization of netting-set-level exposure is obtained by taking the maximum of zero and the netting set portfolio value. Realization of counterparty-level exposure is calculated by summing the realizations of the netting-set-level exposures across all netting sets of the counterparty.

The outcome of this process is a set of realizations of netting-set-level and/or counterparty-level exposures (each realization corresponds to one market scenario) for each simulation date. The exposure simulation process is schematically illustrated in Figure 2.

[7] Minimum 5 business days for repo-style transactions and 10 business days for other OTC derivative transactions.

Figure 2   Simulation Framework for Credit Exposure



Because of computational intensity required to calculate counterparty exposures, especially for a bank with a large portfolio, compromises are usually made with regards to the number of simulation times and/or the number of market scenarios. For example, the simulation times (also called "time buckets") used by most banks to calculate credit exposure usually have daily or weekly intervals up to a month, then monthly up to a year and yearly up to five years, etc.

Computational intensity is also the main reason why most front office valuation models cannot be used in the credit exposure framework. While front office can afford to spend several minutes or even hours for a trade valuation, valuations in the credit exposure framework must be done much faster because each instrument in the portfolio must be valued at each simulation date for each market risk scenario. Therefore, such valuation models that involve Monte Carlo simulations or numerical solutions of partial differential equations do not satisfy the requirements on computation time. Typically, analytical approximations or simplified valuation models are used.

Path-dependent derivatives present additional difficulty for valuation that precludes direct application of front office models. The basic problem in valuing path-dependent instruments in the credit exposure framework is that we simulate the future scenarios only at discrete set of dates, while the value of instrument may depend on the full continuous path prior to the simulation date or on a discrete set of dates different from the given set of simulation dates. Most of the pricing functions used by the front office are inadequate to calculate value of a path-dependent instrument at a future date because the value at a given future date may be contingent on either some event at an earlier time (such as exercise of an option) or in some cases on the entire path leading to the future date (such as the case of knock-in and knock-out barriers).

5.1. *Scenario Generation*

The first step in calculating credit exposure is to generate potential market scenarios (e.g., FX rates, equity prices, interest rates, etc.) at different times in the future. One obvious choice is to use the same model for instrument pricing to generate the scenarios, but the evolution dynamics of this type of models are often constrained by arbitrage arguments. In contrast, the dynamics for risk measurement are usually built on a real measure based on historical data and not necessarily constrained to a risk neutral framework. For example, in a front-office pricing model, the interest rate scenarios are usually generated by construction[8] of zero rates or discount factors using the market prices of cash, euros and swap rates. However, such construction is often computationally expensive, as it requires the search algorithm for the business day count library. Furthermore, the forward rates implied from these scenarios can be nonsensical as a result of arbitrage-free constraints.

For some exposure types (e.g., equity derivatives), a simple lognormal model for underlying risk factors is sufficient

[8] The construction of zero curves is commonly referred as curve construction, which is a necessary step in pricing most interest rate instruments.

(5.1) $$X(t) = X_0 \exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)t + \sigma W(t)\right]$$

where $W(t)$ is the Brownian motion, μ is the drift and σ is the volatility. However, distinction must be made between scenario generation and instrument valuation for these parameters. When the model is used for pricing or instrument valuation, we know that the volatility $\sigma = \sigma_{iv}$, i.e., the implied volatility for the option on underlying price, and the drift is set under the risk-neutral measure to $\mu = r - d$ for stock or indices with r is interest rate and d is dividend yield. On the other hand, when the model is used for generating future scenarios for risk management, we normally use the drift $\mu = \mu_h$ and volatility $\sigma = \sigma_h$, which are usually estimated from the historical data as follows:

(5.2) $$\sigma_h = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\ln\left[\frac{X(t)}{X(t-1)}\right] - \mu_h\right)^2}, \qquad \mu_h = \frac{1}{T}\sum_{t=1}^{T}\ln\left[\frac{X(t)}{X(t-1)}\right]$$

and we then adjust the drift $\mu = \mu_h + \frac{1}{2}\sigma^2$ to compensate $-\frac{1}{2}\sigma^2$ term in the model (5.1).

There are two ways that we can generate the possible future values of the market factors. The first is to generate a "path" of the market factors through time, i.e., each simulation describes a possible trajectory from time $t = 0$ to the longest simulation time $t = T$. The other method is to simulate directly from time $t = 0$ to the relevant simulation date *t*. We will refer to the first method as "Path-Dependent Simulation (PDS)" and the second method as "Direct Jump to Simulation Date (DJS)".

Path-Dependent Simulation is a form of d*iscrete-event simulation* where a market factor is evolved through discrete time intervals. Therefore, the value of a market factor at any given simulation date is dependent on its values at previous simulation dates. A PDS scenario is a *path* that a market factor takes through time. With PDS, the market factor dynamics is expressed in term of the previous simulated values $X(t_i)$ and the difference in simulated times.

For example, the lognormal evolution function takes the form

(5.3) $$X(t_{i+1}) = X(t_i)\exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)(t_{i+1} - t_i) + z\sigma\sqrt{t_{i+1} - t_i}\right]$$

where *z* is a normal variant and $X(t_{i+1})$ represents the shocked market factor at time t_{i+1} that connects from the particular scenario $X(t_{i+1})$ at previous time t_i.

Figure 3A illustrates a sample path for $X(t_i)$ while

Figure 3B shows a 30-scenario random simulation of a stock price using PDS.

Direct-Jump to Simulation is the evolution process that depends on the initial value, $X(t_0)$, and the distance, t_{i+1}, from the original time point t_0.

For example, the lognormal evolution function takes the form:

(5.4) $$X(t) = X(0)\exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)t + z\sigma\sqrt{t}\right]$$

where *z* is a normal variate and $X(t)$ represents the shocked market factor at time *t*.

*Figure 3*B illustrates a direct-jump to a simulation date while *Figure 4*B shows a 30-scenario DJS random simulation of a stock price using a lognormal evolution model.

Figure 3 Two ways of generating market scenarios

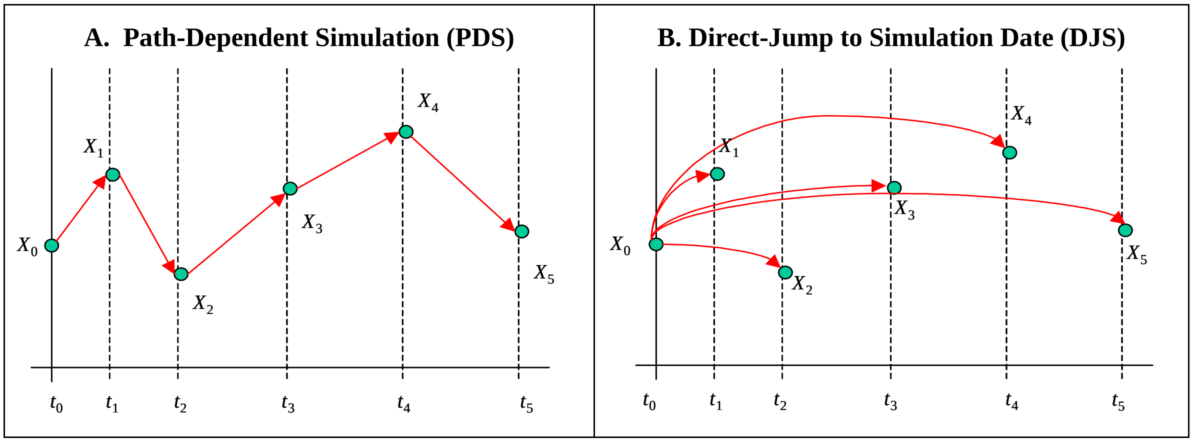
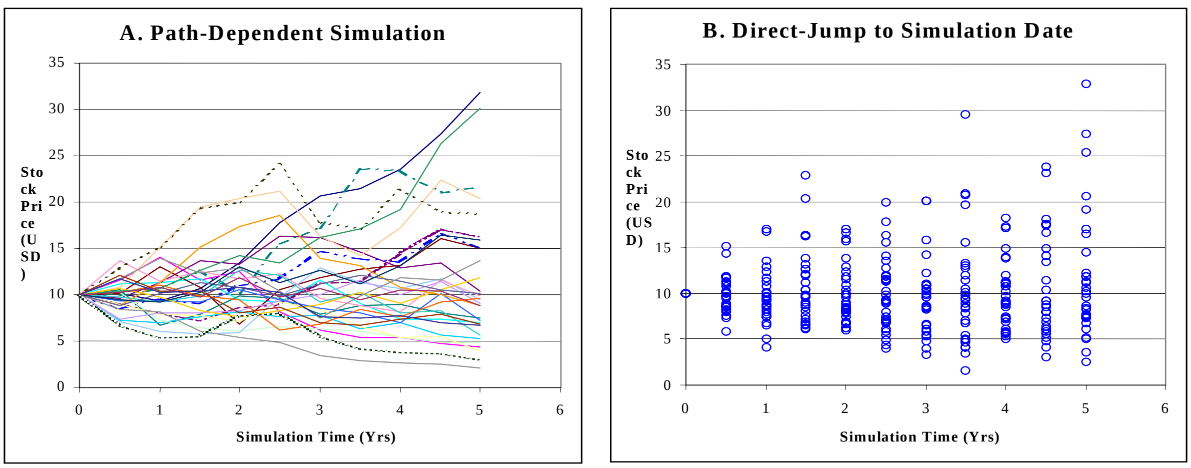
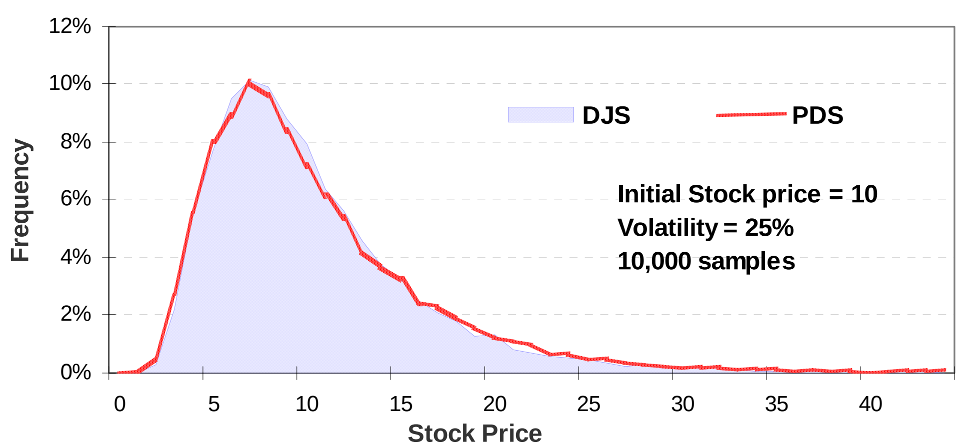


Figure 4 Path-Dependent Simulation (20 scenarios - One-Factor Lognormal Model)



Since continuous-time models are used for the market factor evolution (rather than simple discretization), the market factor distribution at a given simulation date using either PDS or DJS will be indistinguishable in the limit of large number of samples. Since we will be using the conditional valuation in this paper, what matters is the distribution of the market factor scenarios at a simulation date rather than the path it took to get there.

Figure 5 Distribution of Stock Price at the 5-Year Simulation Time Using PDS and DJS



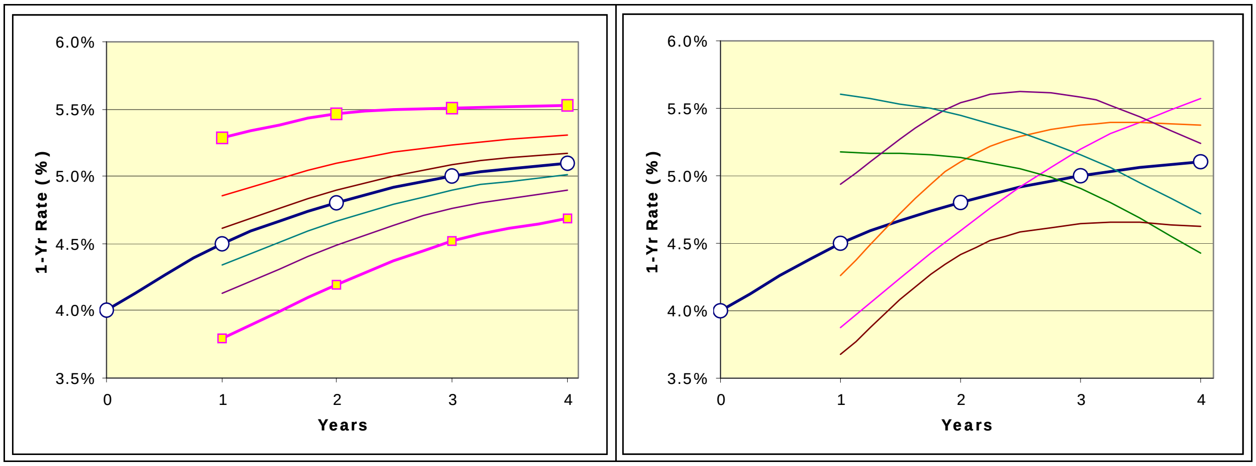
The results of simulation test in *Figure 5* on a lognormal evolution model show that the price factor distribution is almost indistinguishable (even for only 2,000 simulation scenarios) between PDS and DJS methods.

One criticism of the lognormal model in (5.1), when used to simulate the FX or interest rates, is the chance that the FX or interest rates will become unrealistically large or small, especially for long time horizons. To alleviate such the modeling deficiency, we add the effects of mean-reversion into the model as follows,

(5.5) $$d \ln X(t) = \lambda \left(\ln \bar{X} - \ln X(t) \right) dt + \sigma dW(t),$$

Where λ is the mean reversion speed, $\bar{X}$ is the long-term mean of the risk factor being modeled. The calibration of these parameters should be based on much longer period of historical data since the mean-reversion is considered as an equilibrium of the FX or interest rates observed for developed country (i.e., G10) over a very long time horizon. Furthermore, when modeling interest rates, the one-factor model in (5.5) accounts only for parallel shifts and fails to capture the non-parallel variability in yield curve movements that are observed historically. Thus, the common choice is a multi-factor simulation of principal factors in modeling the yield curve movements. Using principal component analysis (PCA) similar to that in Rebonato (2003), it can be shown that yield curve movements can be largely explained using two or three factors. These factors correspond to parallel shifts, twists and butterfly movements of yield curves as below.

Figure 6 Parallel Shifts and Non-parallel Shifts of Yield Curve Movements



5.2. *Instrument Valuation*

The second step in credit exposure calculation is to value the instrument at different future times using the simulated scenarios. The valuation models used to calculate exposure could be very different from the front-office pricing models. In credit exposure calculations, pricing is not an end in itself. What is important is the distribution of the instrument values (under the real measure) at different times in the future. The valuation models need to be optimized in order to perform sufficiently large number of calculations required to obtain such distribution. Term structure models[9] such as HW, HJM and BGM are not adequate for exposure calculation because these models require either Monte-Carlo or lattice-based modeling, which is computationally intensive. Analytical approximations or simplified models are typically used instead. Furthermore, the standard valuation models used to price the instruments for mark-to-market are not applicable for calculating exposures on many path-dependent products whose value at the future time may depend on past scenarios leading to the future date. For such path-dependent instruments, we recommend the conditional valuation approach as a probabilistic technique to "adjust" the mark-to-market valuation model to account for the events contingent on past scenarios.

We again emphasize that the need for conditional valuation stems from the fact that we can only simulate future scenarios at discrete time intervals because of limited computer resources. However, the value of a derivative product at any of these dates may depend on the full path over the continuum of dates prior to the simulation date. We provide the formulation of the conditional valuation approach for calculating credit exposures that are consistent

[9] Rebonato (2003) provides an excellent overview of interest rate term structure models.

across all derivative products, path-dependent or not. For simplicity of exposition, we will assume in this chapter that the direct-jump to simulation date (DJS) approach is used to generate market scenarios. We will use the following notations to describe the evolution of market risk factor and exposure calculation:

- Discrete simulation dates: $\{t_k = t_1, t_2, \ldots, t_N\}$
- Market risk factor scenarios: $\{X(t_k) = X(t_1), X(t_2), \ldots, X(t_N)\}$
- The future values of the transaction: $\{V(t_k) = V(t_1), V(t_2), \ldots, V(t_N)\}$

There are many situations where the future value of a given transaction is not uniquely determined by the state of underlying risk factors at the simulation date. For example, we consider a swap-settled swaption (or a Bermudan in a general case) where the future value of such transaction can be ambiguous on the simulation date past the expiry date of option, because we could either have a swap as the result of option exercised or nothing if the swaption expires worthless. Other examples include barrier (i.e., knock-in or knock-out) and average options, where the payoff is truly path-dependent in the sense that the future values of such options at the simulation date depend on the entire history of the underlying market factor. For path-dependent instruments, their values at the future simulation date may thus depend on either the event occurring at a time before the simulation date or the entire scenario path leading to the simulation date. Hence, the valuation at the future date can be formulated as the conditional expectation

(5.6) $$V(t_k, x) = \mathrm{E}\left[f\left(t_k, \{X(t)\}_{0\le t<t_k}\right) \middle| X(t_k) = x \right]$$

where $\{X(t): 0 \le t < t_k\}$ denotes the entire path of risk factor evolution, and the conditioning is on the state of market risk factor $X(t_k) = x$ for a particular scenario at t_k. When an instrument is not path-dependent, such as in the case of swap, forward and cash-settled option, the conditional expectation in (5.6) above simply degenerates to

(5.7) $$V(t_k, x) = f(t_k, X(t_k) = x)$$

which is just a simple MtM valuation at the simulation date t_k. In general, conditional valuation as defined in (5.6) is not the MtM valuation at the future simulation date. To illustrate the difference, let us consider two special cases in the formulation (5.6) where the valuation function is separable in the following sense:

(5.8) $$f\left(t_k, \{X(t)\}_{0\le t<t_k}\right) = g(t_k, X(t_k)) \cdot h\left(\{X(t)\}_{0\le t<t_k}\right)$$

(5.9) $$f\left(t_k, \{X(t)\}_{0\le t<t_k}\right) = g(t_k, X(t_k)) + h\left(\{X(t)\}_{0\le t<t_k}\right)$$

In each of two cases, we can respectively rewrite the conditional expectation explicitly

(5.10) $$V(t_k, x) = g(t_k, x) \cdot \mathrm{E}\left\{ h\left(\{X(t)\}_{0\le t<t_k}\right) \middle| X(t_k) = x \right\}$$

(5.11) $$V(t_k, x) = g(t_k, x) + \mathrm{E}\left\{ h\left(\{X(t)\}_{0\le t<t_k}\right) \middle| X(t_k) = x \right\}$$

where $g(t_k, x)$ is the mark-to-market valuation of such transaction at the simulation date. As shown later in this chapter, the barrier option is an example of the case (5.8) where

(5.12) $$h\left(\{X(t)\}_{0\le t<t_k}\right) = I_{\{X(t)<H:\ 0\le t<t_k\}}$$

is the indicator function of breaching the up barrier. The average option is an example of the case (5.9) where

(5.13) $$h\left(\{X(t)\}_{0\le t<t_k}\right) = \frac{1}{t_k - t} \sum_{t_0}^{t_k} X(t_i)$$

is the average of the scenario history leading to t_k . These options are commonly embedded in many structured products.

The conditional valuation in (5.6) provides the consistent framework within which the transactions of various types (i.e., path-dependent or path-independent) can be aggregated to recognize the benefits of netting rule across multiple risk factors. The conditional valuation approach is relatively easy to implement, because it is feasible in many cases as shown in Lomibao and Zhu (2005) to explicitly compute the conditional expectation (5.6) for many instruments such as the barrier option, average option and swap-settled swaptions.

5.3. *Exposure Profiles*

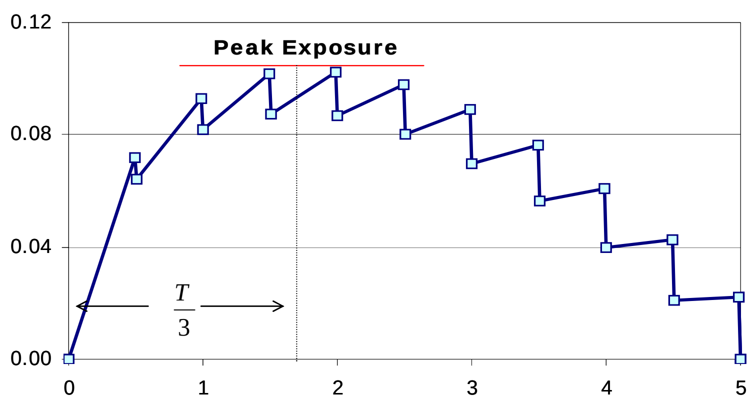
Uncertain future exposure can be visualized by means of exposure profiles. We have already seen one type of exposure profile when we discussed the calculation of EPE and Effective EPE. Expected exposure profile is obtained by computing expectation of exposure at each simulation date. One can use other parameters of the exposure distribution to arrive at different profiles. For example, a high-level (e.g., 95%) percentile of exposure can be computed at each simulation date to arrive at potential future exposure profile (such profiles are popular for measuring exposure against credit limits). While profiles obtained from different exposure measures have different magnitude, they have similar shapes. In this sub-section we will illustrate some of the common properties of exposure profiles without specifying exposure measure.

There are two main factors that determine the credit exposure over time for a single transaction or a portfolio of transactions with the same counterparty: diffusion and amortization

- As time passes, the "diffusion effect" tends to increase the exposure since there is greater variability and uncertainty of market risk variables, such as the FX or interest rates, to move away from current levels.
- As time goes by, the "amortization effect" tends to decrease the exposure as periodic payments are made reducing the remaining cash flows that are exposed to default.

These two effects act in opposite directions, the diffusion effect increasing the credit exposure and the amortization effect decreasing it over time. For single cash flow products, such as FX forwards, the potential exposure peaks at maturity of transaction because there are purely diffusion effect but no amortization effect until maturity date. On other hand, for product with multiple cash flows, such as interest rate swap, the potential exposure usually peaks at about one-third to one-half of the way into the life of the transaction as shown in the following exhibit

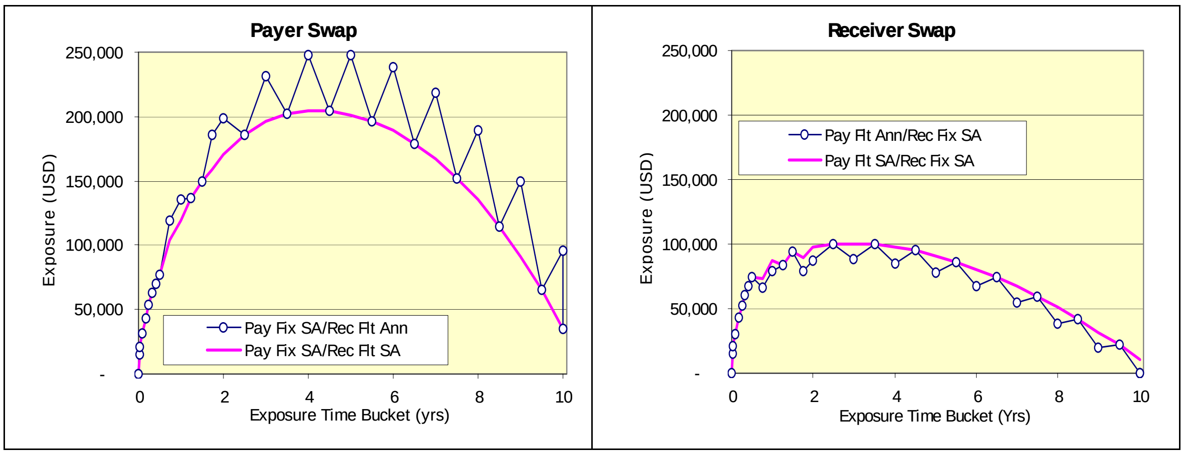
Figure 7 Exposure Profile of Interest Rate Swap



Furthermore, different type of instrument can generate very different credit exposure profiles and the exposure profile of same instrument may also vary under different market conditions. When yield curve is upward sloping, the exposure is greater for a payer swap than the same receiver swap because the fixed payments in early periods are greater than the floating payments, resulting in positive forward values on the payer swap. The opposite is true if the yield curve is downward sloping. However, for humped yield curve, it is not clear which swap carries more risk as the forward value on a payer swap is initially positive and then becomes negative (and vice versa for a receiver swap), and thus, the overall effect implies that both are almost “equally risky”, i.e. the exposure is roughly the same between a payer swap and a receiver swap.

The exposure profile on interest rate swap can be affected substantially when there are unequal payments, i.e. the payment frequency is different between the fixed leg and the floating leg. The graph below shows the change in the exposure profile of the payer swap when the fixed rate is paid semi-annually rather than annually, and this has the effect of increasing the exposure, since half on the fixed leg has been paid off by six month’s time, and thus, effectively increasing the exposure by annual payment date of the floating amount to be received. The opposite effect is observed for the receiver swap where the fixed rate is paid semi-annually rather the annual payment from the floating leg. In this case, half of the payment amount on the fixed leg has been received by six month’s time and ,thus, effectively decreasing the exposure by annual payment date of the floating amount to be paid, as we stand to lose this amount if default occurs in six month period.

Figure 8 Exposure Profile of Interest Rate Swap with Unequal Payments



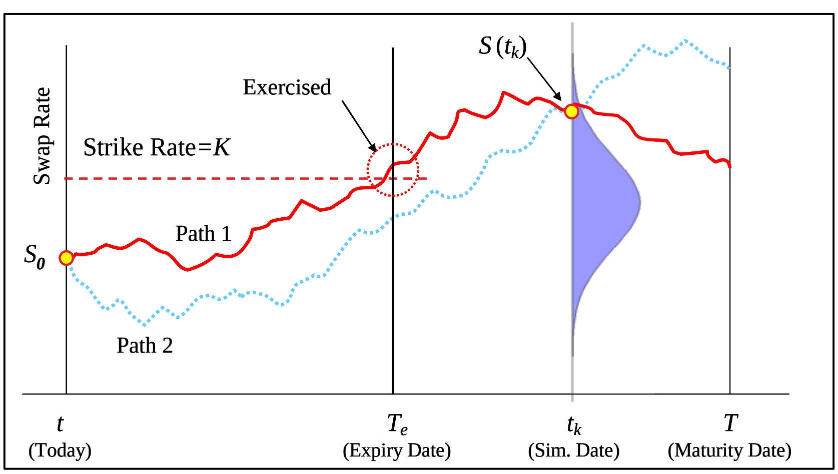
Finally, the exposure profile on swap-settled swaption will be simulated by applying conditional valuation (5.6) such that for any future time $t_k > t_E$ (= exercise date), we have

$$V_{Swaption}(t_k, x) = P\left[S(t_E) > K \middle| S(t_k) = x\right] \times V_{Swap}\left(S(t_k), T\right) \quad (5.14)$$

where K is the strike rate, $S(t_k)$ is the swap rate at time t_k , and $S(t_E)$ is the swap rate at exercise date t_E. Also, $P\left[S(t_E) > K \middle| S(t_k) = x\right]$ is the conditional probability that the swaption is exercised into the underlying swap at expiry date, and $V_{Swap}\left(S(t_k), T\right)$ is the value of remaining portion of underlying swap from time t_k to its maturity T given that the swap rate equals $S(t_k)$.

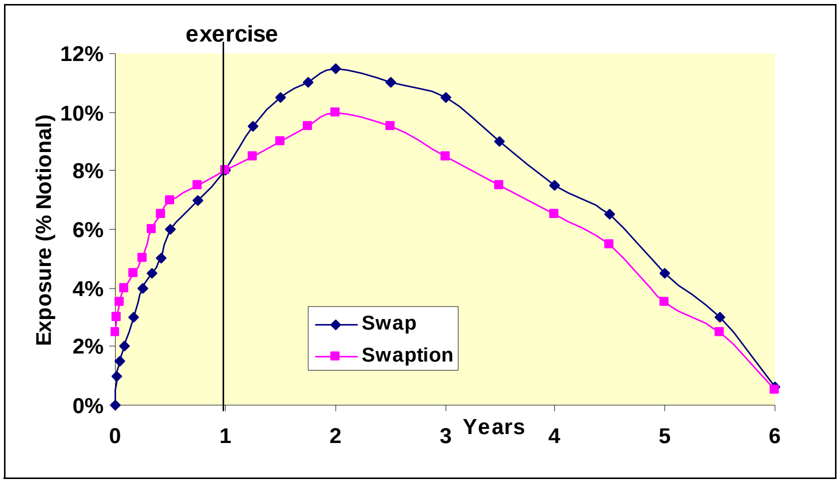
Both the exercise probability and the value of the underlying swap can be explicitly computed using the Brownian bridge technique as described in Lomibao and Zhu (2005).

Figure 9 Conditional Valuation of Swap-settled Swaption



The swap-settled swaption generates an exposure profile beyond the expiry of the option as shown in *Figure 10* because the exposure after the expiry date is contingent on the exercise of the option.

Figure 10 Exposure Profile of Swap and Swap-Settled Swaption



Compared with the underlying forward swap, it always have a higher exposure before the exercise date since the value of swaption must, by definition, be greater than its intrinsic value of its underlying forward swap. However, after the expiry date, the exposure of the forward swap will always be greater because there are some scenarios under which the swaption may not be exercised.

5.4. *Effective EPE for Margined Portfolios*

Banks that are active in the OTC derivative markets increasingly use margin agreements to reduce counterparty credit risk. A margin agreement is a legally binding contract that requires one or both counterparties to post collateral when the difference between the uncollateralized exposure and already posted collateral exceeds a threshold. If this difference becomes less than the threshold, part of the posted collateral (if there is any) is returned in order to bring the difference back to the threshold. To reduce the frequency of collateral exchange, a minimum transfer amount (MTA) is specified. No transfer of collateral occurs unless the required transfer amount exceeds the MTA.

The following time periods are essential for margin agreements:

- Call Period: defines the frequency at which the collateral is monitored and called for (typically, one day).
- Cure Period: time interval necessary to close out the counterparty and re-hedge the resulting market risk.
- Margin Period of Risk: time interval from the last exchange of collateral until the defaulting counterparty is closed out and the resulting market risk is re-hedged.

Margin period of risk is usually assumed to be the sum of the call and cure periods. Basel II provides a floor for the margin period of risk: five business days for repo-style transactions and ten days for OTC derivatives.

While margin agreements can reduce the counterparty exposure, they pose a challenge in modeling the effect of margin calls in the calculation of EPE measures because the future collateral amount and margin calls must also be modeled at same time. Gibson (2005) has provided a quasi-analytic approach for calculating EPE with margin under the similar assumption that the mark-to-market value of the portfolio follows a random walk with Gaussian increments.

Under the IMM, there are two ways of calculating the Effective EPE for counterparties with margin agreements:

- modeling collateralized exposure
- the Shortcut Method.

Under the former approach (it does not have an official name), collateral is modeled together with uncollateralized exposure. For each market scenario at each simulation date, collateralized exposure is calculated as the difference between the uncollateralized exposure and collateral values at that scenario and that simulation date. The distribution of collateralized exposure obtained this way is used for calculation of both Effective EPE and Effective Maturity. The Shortcut Method is a simple and conservative approximation that does not require modeling of collateral.

5.5. *Modeling Collateralized Exposure*

There is no single regulatory approved model for collateral when using simulation approaches. Here we will discuss general principles of collateral modeling and suggest an approximation that reduces computation time.

The most straightforward approach to account for the effect of delivery lags on the exposure profile is to run a Monte Carlo simulation with a daily time-step to determine the amount of transferred collateral for each day. However, this method is not practical because of required computation time. One would need to compute collateral only for the given set of simulation dates for which uncollateralized exposure is modeled. Thus, a more practical approach using the given simulation dates in the credit system should be considered.

General Procedure for Handling of Collateral

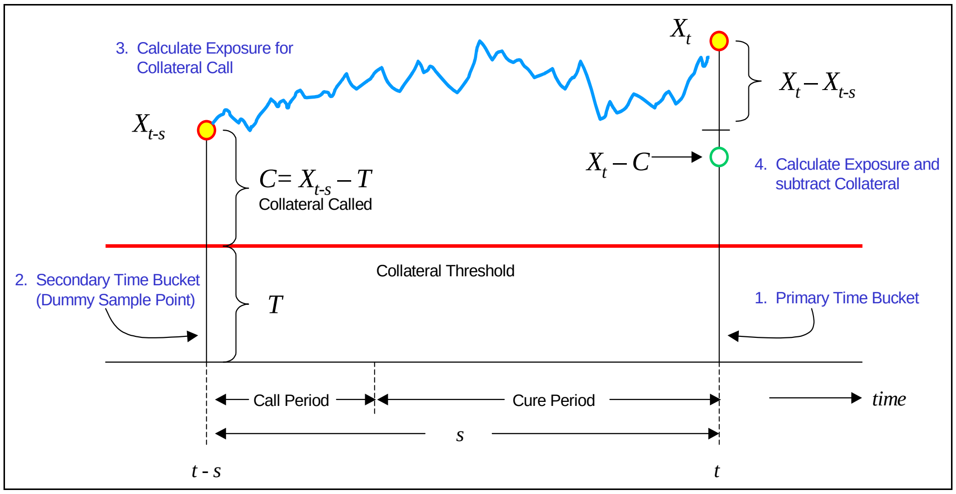
Collateral available at a given simulation date t is determined by the uncollateralized exposure at time $t-s$, where s is the margin period of risk. Therefore, we can place additional simulation dates (secondary time buckets) prior to the main simulation dates to capture the effect of the margin period of risk. Each primary simulation date $t > s$ will have one corresponding secondary date $t-s$. Since the margin period of risk can be different for different margin agreements, secondary time buckets are not fixed.

The general procedure for handling collateral at each simulation date t is given by the following:

1. The uncollateralized exposure X_t at the primary time bucket is calculated for all trades under the margin agreement.
2. The secondary time bucket (or synthetic margin call dates) is determined from each primary time bucket t (or simulation date) by subtracting the margin period of risk s (given by the sum of call lag, delivery lag, and the cure period, subject to the regulatory-specified floors) from the primary time bucket. For example, if the call period is daily, delivery lag is 1 day and the cure period is 12 days, then we subtract 14 days from the primary time bucket.
3. The uncollateralized exposure X_{t-s} at the secondary time bucket is calculated for all trades under the margin agreement.
4. Collateral C_t available at primary time bucket t is calculated by subtracting the threshold T from the exposure at the secondary time bucket X_{t-s}. If this amount is less than the MTA, the collateral is set to zero.
5. Collateralized exposure at the primary time bucket t is calculated by subtracting the collateral C_t from the uncollateralized exposure X_t.

Exposure at the secondary time buckets is used only for intermediate calculation of collateral, as illustrated in Figure 11..

Figure 11 Treatment of Collateral at Secondary Time Bucket



Figures 12 and 13 describe the effect of collateral calls on exposure reduction. The collateral is called as soon as the exposure rises above the threshold, which would result in reduction of exposure by the collateral held. In practice, there is a lag (i.e., margin call lag) between the time collateral is called and the time collateral is posted. This lag exposes the bank with additional exposure above the threshold, which is normally referred to as collateralized exposure. Clearly, the longer the margin call lag the larger the collateralized exposure.

Figure 12 Effect of Collateral on Exposure (7-Day Lag)

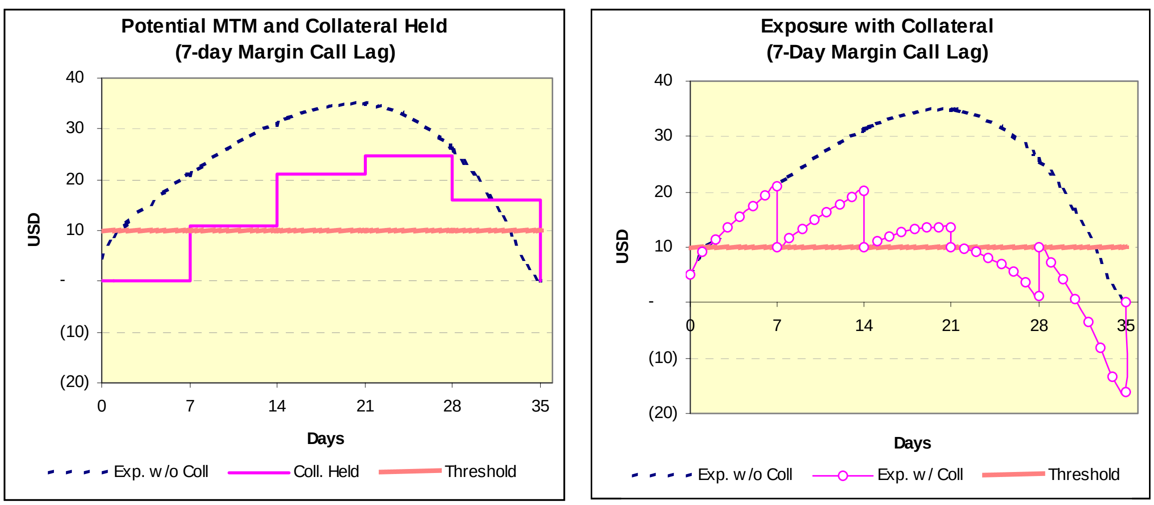
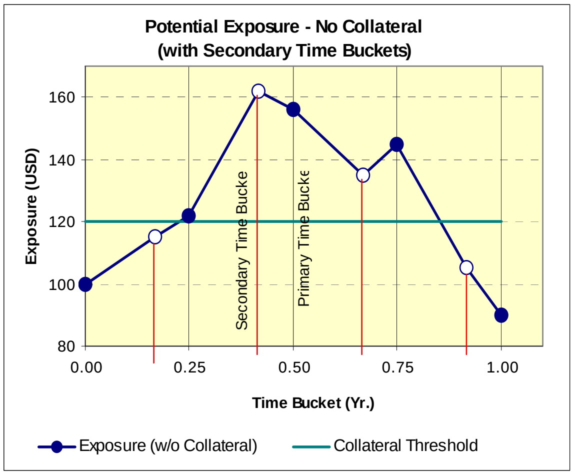
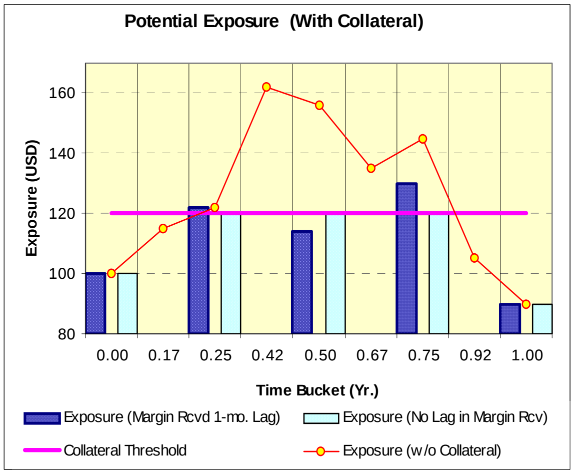


Figure 13 Future Exposure with Secondary Time Buckets (With and Without Collateral)





Simplified Calculation of Collateral

As we have seen above, the collateral C_t available at primary time bucket *t* is calculated as the greater of zero and the difference between the uncollateralized exposure X_{t-s} at the secondary time bucket $t-s$ and the threshold *T*,

(5.15) $$C_t = \max\{X_{t-s} - T, 0\} = [X_{t-s} - T]^+.$$

Thus, collateral calculation requires the knowledge of exposure at the secondary time bucket. The obvious approach is calculating this exposure by simulation. This, however, requires major changes in the system (e.g., adding variable time buckets) and results in doubling the computation time. Instead, we present a simplified approach to calculating the collateral.

For each primary time bucket *t*, the exposure simulation as described in Section 5 produces many realizations of exposure $x_t^{(i)}$, where *i* is the number of realization (scenario). Let us utilize, for simplicity, the "direct jump to simulation date" approach, which means that exposure realizations at any primary time bucket are related only to exposure at the present date, x_0, and are not related to exposure values at any prior primary time bucket. Our task is to calculate exposure value $x_{t-s}^{(i)}$ at the secondary time bucket $t-s$ that corresponds to the primary time bucket exposure realization $x_t^{(i)}$. Then, for each scenario *i*, collateral at the primary time bucket will be determined according to equation (5.15).

We will use the conditional valuation approach of Lomibao and Zhu (2005) described above. Under this approach, we will calculate exposure value at the secondary time bucket $t-s$ for scenario *i* as the expectation conditional on our knowledge of initial exposure x_0 and exposure at the primary time bucket $x_t^{(i)}$:

(5.16) $$x_{t-s}^{(i)} = \mathrm{E}\left[X_{t-s} \middle| \{X_0 = x_0\} \,\&\, \{X_t = x_t^{(i)}\} \right]$$

The evaluation of this expression under the actual exposure dynamics is not feasible because the dynamics can be very complicated. We can, however, make simplifying assumptions about the stochastic process describing the evolution of exposure. The result of the evaluation will depend on these assumptions. One of the simplest assumption one can make is that exposure follows a Brownian motion process. Under this assumption, equation (5.16) simplifies to a linear interpolation between x_0 and $x_t^{(i)}$:

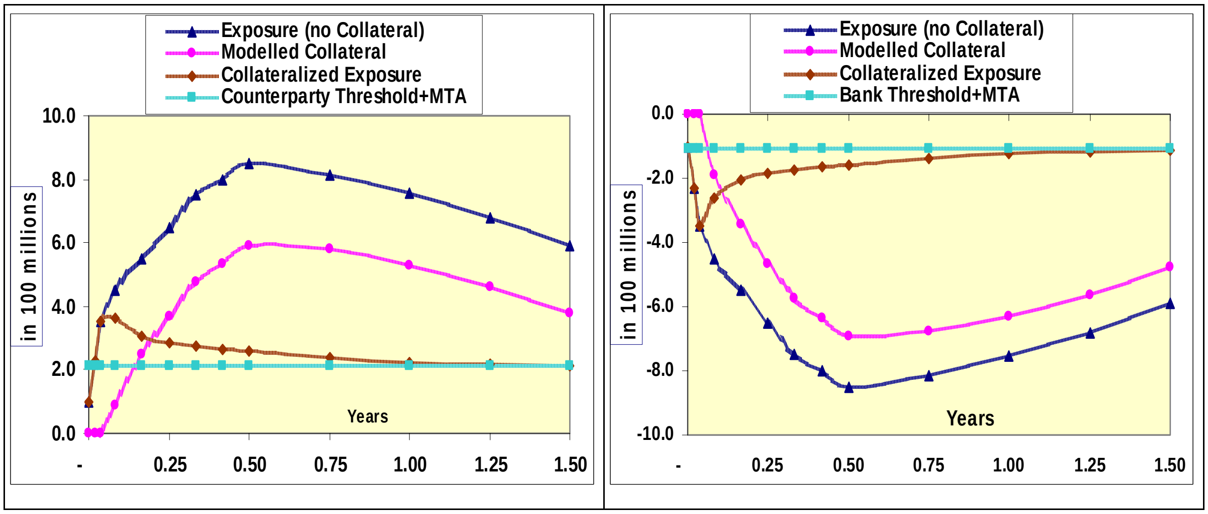
(5.17) $$x_{t-s}^{(i)} = \frac{s}{t} x_0 + \frac{t-s}{t} x_t^{(i)}$$

This approach can be easily extended to the path-dependent simulation (PDS) approach, where exposure at any simulation date is obtained by evolution from the previous simulation date. Under PDS, expectation in the right-hand side of Equation (6.2) should be conditioned on the realization of exposure at all simulation dates prior to the current simulation date. Equation (6.3) would then describe the linear interpolation between the previous simulation date t_{k-1} and the current simulation date t_k :

(5.18) $$x^{(i)}_{t_k - s} = \frac{s}{t_k - t_{k-1}} x_{t_{k-1}} + \frac{t_k - t_{k-1} - s}{t_k - t_{k-1}} x^{(i)}_{t_k}$$

Figure 14 shows how the modeled collateral and collateralized exposure evolve relative to a particular realization of exposure scenario over the future time points.

Figure 14 Exposure Profile with Collateral Modeling



In practice, the collateralized exposures (cPE) are calculated with respect to both the collateral threshold (T) and minimum transfer amount (MTA). The collateral threshold may be different between the bank and the counterparty, and the threshold levels are usually negotiated on the credit standing of the counterparty relative to the bank. At a future time *t*, if the exposure is positive to the bank, i.e. bank has exposure to counterparty such as the case depicted on the left panel of Figure 14, the exposure X_{t-s} at the secondary time bucket (*t-s*) will be compared to the counterparty threshold T_{Cpty} plus the MTA. A collateral call to the counterparty will be triggered only when

(5.19) $$X_{t-s} > T_{Cpty} + MTA.$$

Thus, the modeled collateral and collateralized exposure shall be computed according to (5.15) as follows:

(5.20) $$C_t = \max\{0, X_{t-s} - (T_{cpty} + MTA)\}, \quad \text{and} \quad \text{cPE}_t = X_t - C_t > 0.$$

On other hand, if the exposure is negative to the bank,[10] i.e. the counterparty has the exposure to the bank such as the case in the right panel of Figure 14, the exposure X_{t-s} at the secondary time bucket (*t-s*) will be compared to the counterparty threshold T_{Bank} plus the MTA, and a collateral call to the bank will be triggered only when

(5.21) $$X_{t-s} < -T_{Bank} - MTA.$$

Thus, the modelled collateral and collateralized exposure shall be computed according to (5.15) as follows:

(5.22) $$C_t = \min\{0, X_{t-s} + (T_{cpty} + MTA)\}, \quad \text{and} \quad \text{cPE}_t = X_t - C_t < 0$$

[10] In this context, negative exposure simply means that the counterparty is exposed to the bank.

Consequently, the collateralized exposure at some future time can be either positive or negative to the bank, which reflects the bilateral nature of the transactions between the bank and the counterparty.

5.6. *Shortcut Method*

As we have seen in modeling of collateralized exposure is more complicated than the modeling of uncollateralized exposure. For banks that can model the latter, but are not sophisticated enough to model the former, the Shortcut Method is available under the IMM. The Shortcut Method is a simple and conservative approach for calculating the Effective EPE for a netting set with an accompanying margin agreement. The $(\text{Effective EPE})_{\text{margin}}$ at a netting set level with a margin agreement can be computed as the sum of the threshold, MTA, and the change of Effective EE over the margin period of risk *s* beginning from today, subject to a cap equal to the Effective EPE at the netting set level without the collateral. Mathematically, this can be written as

(5.23) $$(\text{Effective EPE})_{\text{margin}} = \min\left\{(\text{Threshold} + \text{MTA} + \Delta\text{EE}_s), (\text{Effective EPE})_{\text{no margin}}\right\},$$

where

(5.24) $$\Delta\text{EE}_s = (\text{Effective EE})_{t=s} - \text{EE}_{t=0}$$

A supervisory floor of 5 business days is required for netting sets consisting only of repo-style transactions subject to daily re-margining and daily mark-to-market. A floor of 10 business days for all other netting sets is imposed on the margin period of risk used for this purpose.

6. Summary

In this chapter we described the treatment of counterparty credit risk of OTC derivatives under Basel II. According to this framework, minimum capital requirements for counterparty credit risk are to be calculated according to the corporate loan rules applied to the appropriate exposure at default (EAD) calculated at the netting set level. We described both Non-Internal and Internal Model Methods (IMM) of calculating this EAD. To obtain supervisory approval for the IMM, banks must be able to calculate expected exposure at the netting set level for a set of future dates. We also discussed a modeling framework that can be used for calculating exposure distribution at a set of future dates and, in particular, for calculating expected exposure profiles. This framework can be used for both regulatory and internal purposes. Additionally, we explained the treatment of margin agreements under the IMM that allows one to calculate the collateralized EPE measures: modeling collateralized exposure and the Shortcut Method. We discussed a general approach to modeling collateralized exposure that enables one to compute the collateral at a future date as a function of uncollateralized exposure at another date that precedes the primary date by the margin period of risk. Finally, we suggested a simple and fast method under this approach for modeling collateral that avoids the simulation of exposure at the secondary dates.

7. References

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