

**Bond Relative Value Models and Term Structure of Credit Spreads:
A Simplified Approach**

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Abstract

Bond relative value models to detect mispriced bonds are widely used in the investment community. These range from simple yield to maturity comparisons to sophisticated stochastic models. The first step for many of these models is the determination of reference yield curves. There are numerous publications on these yield curve fitting approaches with related empirical research yet few actually document practical implementations for operational purposes. Accordingly, the first part of this article describes and then illustrates implementation of a number of these benchmarking models. Within such a fitting framework, bonds subject to credit risk can often not be handled since the number of bonds of equivalent credit quality is simply too small to derive reliable reference curves. Here the article proposes a novel approach to parameterize the term structure of credit spread. Its main benefit are intuitive model parameters that relate to the concept of how market practitioners like traders and asset manager tend to measure credit risk of fixed income securities.

Many of the models described herein have been implemented in EXCEL/VBA, some of which are generalized versions of models that have been developed for practical bond relative value research.

The files containing the models can be downloaded from the following website:

<http://www.mngt.waikato.ac.nz/kurt/>

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1 Introduction

Relative value model to detect indications of potential excess within a universe of fixed income securities are widely used in the investment community. In the bond markets, relative value usually refers to the process of comparing returns among fixed income securities but in a wider sense this definition can be extended to include comparisons with say related equity instruments. There are two fundamental factors that primarily affect the pricing of fixed income securities. These are firstly, the prevailing market interest rates and secondly, the specific credit risk of the bond¹. This paper will deal with models that address these two pricing aspects.

With regard to the interest rate factor, there are a number of theoretical models, many of them versions of seminal work by Vasicek (1977) and Cox, Ingersoll, & Ross (1985) that postulate an interest rate process as the driving state variable which in turn determines the shape of the yield curve and thus the pricing of bonds². Unfortunately, their practical application for bond pricing is limited. The yield curve shape and dynamics observed can often not be explained with these approaches as the true stochastic nature of the interest rate process remains elusive. The usual method is to calibrate such models with observed yield curves as for example in the models of Ho & Lee (1986) or Heath, Jarrow, & Morton (1992). This in turn requires methods to derive the term

¹ Other pricing factors such as taxation and liquidity premia have also been subject to research but these are typically not considered in models used by market participants. See Bliss (1997, p.6) or Ioannides (2003, footnote p. 5) for references to some of these studies.

² Rebonato (1998) describes most of these popular interest rate models in great detail.

structure from traded instruments or contracts such as bonds or interest rate swaps. The first part of this article will focus on this class of curve fitting models which — though their parameters do not have an actual economic meaning — have a much greater significance for the market practitioner. Section 2 characterizes them along the lines proposed by Bliss (1997) and then details how some of them have been implemented in EXCEL/VBA. Examples include versions of McCulloch (1971; 1975) and Nelson & Siegel (1987) as well as an unnamed simple approach that is a generalized version of a model tested in a trading environment. While it can be assumed that similar implementations are applied in industry, they have not been formally documented in the academic literature. Moreover, Excel/VBA is widely used for financial analysis, i.e. it provides a very popular platform to present these models.

In the context of pricing fixed income securities, credit risk is usually measured by the so-called credit spread which measures the difference in yield offered for a risky bond compared to an equivalent riskless government bond. Ever since the seminal work of Merton (1974) that pioneered the structural paradigm in credit risk modeling, the nature and dynamics of this term structure of credit spreads has been subject to substantial research. One puzzling result is that — even after accounting for possible taxation effects — one finds that expected default and recovery rates on bonds can explain only a part of the yield premium actually observed³. There is not only an academic debate as to how to explain the balance but Collin-Dufresne et al. (2001) illustrate that causes of spread changes are hard to pinpoint, too. Although they use a large number of proxies affecting credit risk, they fail to explain most of the observed dynamics. They conclude that the dominant

³ See for example Fons (1994). Elton et al.(2001) estimate spread components due to default risk, taxation with the balance explained as “risk premium for systemic risk”. Duffie & Lando (2001) model the imperfect, discrete nature of information flowing to investors to account for higher than expected credit spread observations.

component of credit spread changes are “local supply/demand shocks” not picked up by any of their proxies.

In the absence of any conclusive models, investor thus again have to rely on appropriate credit spread fitting models, similar to the ones discussed earlier in this section⁴. These give market practitioners like traders, which will generally be aware of such “local supply/demand factors”, a first indication of potential mispricing. Unfortunately, the basket of comparable bonds of the same credit quality is often limited and this prevents fitting reliable term structures of credit spreads to the data. This article thus presents a heuristic fitting method that can be applied in such situations. Starting point is a certain target credit spread, i.e. the spread which the investor deems appropriate for the particular bond or group of bonds. This is then complemented by a number of shape parameters. While the particular target spread is reviewed regularly, e.g. by means of statistical analysis, the characteristic of the shape parameters is more static and can also be commonly set for a larger segment of the bond market, for instance for all lower investment grade bonds⁵. It is not the ambition of this approach to compete with any of the more advanced fitting methods such as the ones derived from equilibrium term structure models⁶ but to simply provide the practitioner a tool to uncover apparently mispriced bonds in a first instance. The decision process is helped by the intuitive nature of target credit spread as the main model parameter. The illustrative Excel/VBA

⁴ Recent developments are new joint estimation techniques as presented in Houweling, Hoek, & Kleibergen (2001).

⁵ An investor might decide to define “lower investment grade” bonds as bonds with rating BBB- up to BBB+

⁶ e.g. see Anderson et al. (1996, chapter 4, p. 67)

model implementation presented in section 3 uses data for a sample of Swiss domestic industrial bonds.

2 Bond Relative Value with Yield Curve Fitting Models

As indicated in the introduction, static yield curve fitting models as a basis to detect mispriced bonds are very much applied in the markets. Examples are Merrill Lynch (2004) daily Rich/Cheap Reports for countless bond markets and segments. Krippner (2003) p. 2 also lists JP Morgan, HSBC Bank and UBS Bank as institutions producing bond relative value research based on yield curve fitting models. Evidence from various studies such as Sercu & Wu (1997) or more recently Ioannides (2003) indeed suggests that there is justification for applying such models. For both the Belgian, respectively UK government bond markets, these studies found significant excess returns for trading strategies based on buying (shortselling) bonds that are classified as undervalued (overvalued) relative to a particular estimated term structure model.

The following reviews and classifies these models in general which is then followed by subsections documenting the implementation of three of them.

2.1 Review of Static Term Structure of Interest Models

The term structure of interest, a concept central to economic and financial theory, plays a key role not just for the pricing of bonds but also any interest rate contingent claim. This section will focus on the work that has been done in the area of non-probabilistic yield curve modeling. It follows a framework proposed by Bliss (1997, p. 4) who sees three dimensions to such models or rather, there are three decisions required to estimate a term structure of interest as the basis for a bond relative value model: the pricing function, the approximation function and, finally, the estimation method.

2.1.1 Pricing Function

The most straightforward pricing function is certainly the present value (P) of the bond's promised cash flows (C):

$$P = \sum_{m=1}^M C_m e^{-r(m)t(m)} \quad (1)$$

where M are the number of remaining cash flows numbered 1 to m; $r(m)$ and $t(m)$ are the spot rates, respectively times at which these cash flows will occur. This is, however, not the only pricing relation used in the markets. The much simpler yield to maturity based bond valuation is still very much in use by investors. This because the simple yield measure akin to the well-known internal rate of return is typically the first piece of bond analytics listed by financial data providers and the media. Some markets like Australia and New Zealand even refrain from quoting fixed income securities by price but rather by yield to maturity which in turn is used to calculate the actual settlement price by means of a standardized formula⁷. A simplified version of such a formula, using continuously compounded rates and not considering the complexities of time measurement conventions⁸, would look like the pricing formula (1) above with constant rate r , no longer dependent on the time t of the m^{th} cash flow.

Whatever function is chosen, none will in practice exactly price all bonds in a particular reference basket. An inexact relation for the price P_i of a particular bond needs to be formulated:

⁷ See appendix 1 for the example of the New Zealand bond market formula as shown in RBNZ (1997, p. 12)

⁸ Christie (2003) provides some detailed description of how time measurement conventions including factors such as national holiday calendars affect bond yield calculations.

$$P_j = f[C_m, r(m)] + \varepsilon_j \quad (2)$$

where the function $f[\cdot]$ captures all that we assume what determines the price of the bond and $r(m)$ is fitted to minimize some function of the random residual term ε_j . In formulating $f[\cdot]$, researchers will often add terms (e.g. dummy variables) to the straight present value formula that attempt to capture effects of frictions in the markets such as tax effects or liquidity premia⁹. It is, however, the experience of this author that this is hardly done in operational models because such factors tend to have less tangible impact on the price of the instrument.

2.1.2 *Approximation Function*

As a next step, one must decide on the functional form to approximate either the discount rate function $r(m)$, or the discount function $d(m)$. This is necessary as there are limited numbers of bonds which requires a way of interpolating the rates, respectively discount function for arbitrary time horizons. The usual approach is to select an approximating function and then to estimate the parameters. We mention here just two mainstream methods¹⁰, a parsimonious representation defined by an exponential decay term pioneered by Nelson & Siegel (1987) and Svensson (1994) and cubic splines introduced to finance by McCulloch (1971; 1975). The most comprehensive comparative studies of these and other functional forms have been undertaken by Bliss (1997) and Ioannides (2003). While there are differences between the very many methods, none is disqualified by these

⁹ See footnote in introduction for references to alternative pricing factors.

¹⁰ Bliss (1997, p.6) and Ioannides (2003, p.3) list references to the major classes of such models.

researchers¹¹ for their goodness of fit. Weaknesses appear more in other aspects, e.g. difficulty to estimate parameters (see below) or unstable, respectively fluctuating forward rates implied. With regard to residual based bond relative models there is thus no cause to discount any of them.

2.1.3 Estimation Method

Lastly, the decision on the appropriate estimation technique is a more technical but nevertheless important issue. The aspect of estimation not only includes the choice of numerical algorithms for parameter estimation. These are often determined by the type of functions chosen before. One must also make decisions on error weighting functions and how to handle bid/ask spreads. Related are data integrity issues. One might define data filters to remove apparently erroneous data from the set. The tricky aspect remains that in contrast to empirical research with historical data, the data constellation in an operational trading application is not known beforehand so output plausibility checks are essential.

2.2 Implementations of Reference Curve Models

Three reference yield curve models are presented in this paper. Firstly, it shows a simple yield to maturity based benchmarking tool which is illustrated for a basket of Swiss government bonds. Next there is the JP Morgan Discount Factor Model (JPM), a version of McCulloch's (1971; 1975) cubic spline method and, finally, a bond relative value model using an extended Nelson & Siegel (1987) approach. The latter two models are illustrated with data of the small universe of New Zealand

¹¹ An exception is the Fisher, Nychka, & Zervos (1995) cubic spline which was found to be “performing poorly” by Bliss (1997, p. 26).

government bonds. The files containing the models can be downloaded from the website <http://www.mngt.waikato.ac.nz/kurt/>.

In following Table 1, the three models are characterized in line with the framework presented in the previous section. All the models are set up so they can easily be linked to a real time price data source. Note that some technical complications may arise from the treatment of accrued interest which depending on market conventions has to be paid upfront by the bond buyer. The models assume that prices quoted are so-called clean prices, excluding accrued interest. These and other issues related to bond analytics are discussed in specialized fixed income resources such as Fabozzi (1999).

Table 1: Model Classification

	I	II	III
Model	Yield to Maturity Benchmarking	JP Morgan Model Discount Factor Model	Extended Nelson & Siegel
Pricing function	Simplified price as function of yield to maturity, i.e. as in model II/III but constant $r(j, m) = r(j)$	$P_j = \sum_{m=1}^M C_{j,m} e^{-r(j,m)t(j,m)} + \varepsilon_j$	see formulas 1,2 for explanation of parameters
Approximation function	Model yield to maturity term structure as polynomial	Model discount function as a polynomial. Version of (McCulloch, 1971; 1975)	Spot rate modeled with exponential form as described in Bliss (1997, p. 11)
Estimation method	OLS of yield to maturity errors (equal weighting). Corresponding system of linear equations solved with LU Decomposition (Press et al. (1992, p. 43)	OLS of price errors (equal weighting) Corresponding system of linear equations solved Excel built-in LINEST function.	Duration weighted least square, minimized with Generalized Reduced Gradient (GRG2) nonlinear optimization code as implemented in Excel Solver

2.2.1 Simple yield to maturity based benchmarking model

Yield to maturity, also called redemption yield based measures to find relative value in a universe of bonds is the traditional method still used by many practitioners. As is commonly known, yield to maturity assumes that an investor holds the bond to maturity and all the bond's cash flows are reinvested at the computed yield to maturity. It is found by solving for the interest rate that will equate the current price to all cash flows from the bond to maturity. In this sense, it is the same as the internal rate of return (IRR) defined in many finance textbooks in e.g. in Reilly & Brown (1997, p. 529).

Needless to say that redemption yield based bond analytics has major shortcomings as was documented many years ago by Schaefer (1977) and also discussed in Anderson et al. (1996, p. 22). Reinvesting each coupon at the same rate is tantamount to assuming a flat term structure with identical spot rates for each maturity. If spot rates increase with maturity, yield to maturity will underestimate the spot rate. Conversely, it overestimates a downward sloping spot-rate curve. Having noted this, purely for bond relative value purposes, it remains a useful measure with the necessary caveats. Coupons should firstly be uniform, particularly within a particular maturity range. Similarly, errors are smaller if market yield levels, including coupon rates are low. There are many bond market segments that have comparably low liquidity with wide bid/ask spread. Applying an easier yield to maturity model in these cases is surely more honest because mispricing will also be detected with this more crude approach.

The implementation of this redemption yield based relative model is stored in the file named "ConfBenchmark (Feb04).xls". It is a generalization of a model that has been used for practical relative value research in the Swiss bond market for a number of years. It was applied to a number of homogeneous market segments providing information regarding the relative pricing of these

issues. The following briefly explains the mathematics of the fitting procedure and then elaborates on selected implementation issues.

Figure 1: Generic Time / Yield Chart with Bond Yield Curve

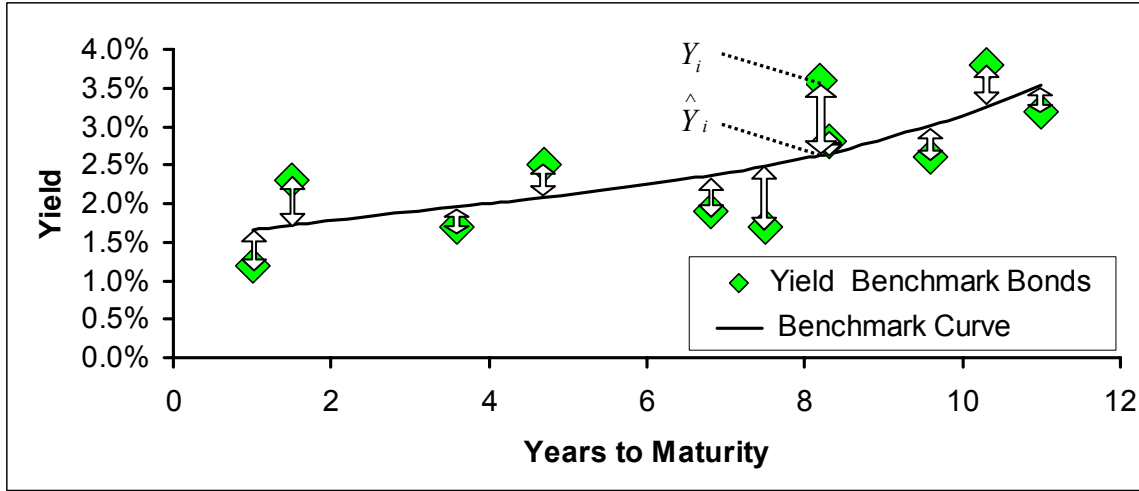


Figure 1 illustrates the principal method of fitting a polynomial into the time / yield to maturity plot of benchmark bonds. The yield Y_i of bond i ($i= 1 \dots n$ bonds in reference basket) is approximated by $\hat{Y}_i = a_m t_i^m + a_{m-1} t_i^{m-1} + \dots + a_1 t_i + a_0$ where t_i is time to maturity of bond i , and a_0, a_1, \dots, a_m are the constant coefficients of order $m+1$ polynomial.

In ordinary least square regression (OLS), minimizing the sum of squared yield errors means we have to set partial derivative to zero:

$$\text{Min} \sum_{i=1}^n \left(\hat{Y}_i - Y_i \right)^2 \Rightarrow \frac{\partial \sum_{i=1}^n \left(\hat{Y}_i - Y_i \right)^2}{\partial a_k} = 0 \text{ for } k = 0, 1, 2, \dots, m$$

This then yields $m+1$ equations for the unknown coefficients a_0, a_1, \dots, a_m .

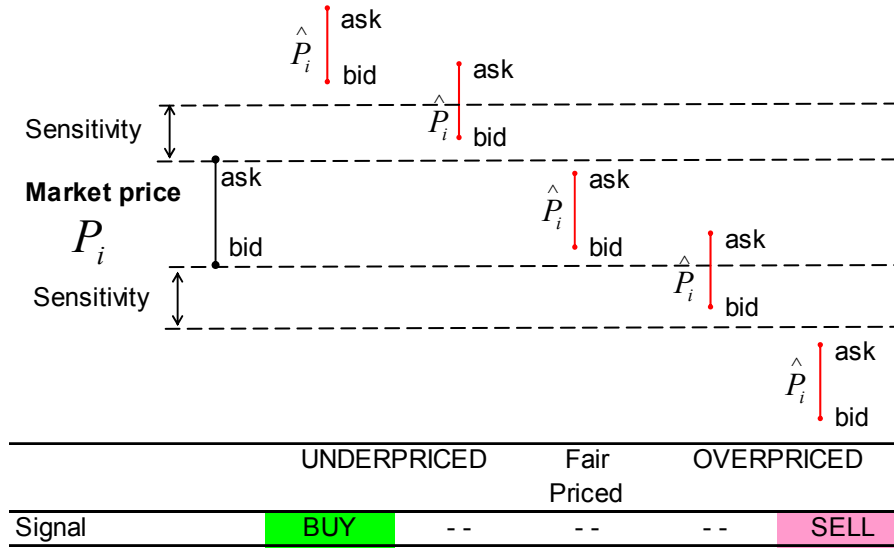
Some algebra shows that a_0, a_1, \dots, a_m must be a solution of the following system of linear equations using matrix notation:

$$\begin{bmatrix} n & \sum t_i & \sum t_i^2 & \dots & \dots & \sum t_i^m \\ \sum t_i & \sum t_i^2 & \sum t_i^3 & \dots & \dots & \sum t_i^{m+1} \\ \sum t_i^2 & \sum t_i^3 & \sum t_i^4 & \dots & \dots & \sum t_i^{m+2} \\ \vdots & \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \dots & \dots & \vdots \\ \sum t_i^m & \sum t_i^{m+1} & \sum t_i^{m+2} & \dots & \dots & \sum t_i^{2m} \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i t_i \\ \sum Y_i t_i^2 \\ \vdots \\ \vdots \\ \sum Y_i t_i^m \end{bmatrix}$$

In the illustration model, this system is solved numerically by the well known LU decomposition algorithm as described in Press et al. (1992, p. 43). As coefficients like $\sum_n t_i^{2m}$ become an extremely large number for higher dimensions of m , the algorithm will lose accuracy. However, this is not an issue in general because meaningful interpolations will not exceed 3rd to 4th order polynomials.

Once the approximated yields \hat{Y}_i have been calculated, one can then determine the corresponding model prices \hat{P}_i using an market convention yield formula (e.g. RBNZ, 1997, p. 12). As the model will derive benchmark polynomials for each the bid and ask yield, there will be both a bid and ask model prices. A buy (sell) signal is generated, if the bid (ask) price in the market exceeds (is below) the ask (bid) model price found by the model. There is a feature to decrease the sensitivity of the model by introducing a filter rule so recommendations are only generated if these prices are a set absolute amount apart. These simple rules for generating recommendations are illustrated in Figure 2.

Figure 2: Buy /Sell Signal Rules Simple Yield to Maturity Benchmarking Model



Another reality of operational models is that some data might suddenly be missing and this has to be handled. In the solution presented here, missing data are replaced by last known historical (e.g previous day) prices. It makes sense to suppress corresponding buy/sell signals in these instances.

The model is also set up to handle callable bonds in a simplified way. For each bond, it determines the so-called yield to worst which is the lower of either bond to maturity or the yield to the next call. If the call yield is lower, the bond's maturity will be set to the next call date for benchmarking purposes. It does thus not employ more advanced call feature analytics such as the often used option adjusted spread analysis¹².

¹² This methodology is described in Windas (1993) of Bloomberg based on the Black, Derman, & Toy (1990) interest model. Under this approach, a callable bond is viewed as a long position of an option-free bond plus a short call on the bond (sold to the issuer).

To mention a final feature, the model contains some utility macros for illustrative purposes. For operational use, it does not suffice to simply link the model to real time trading prices. The basket will be subject to continuous change and so one needs utilities to deal with additions to and deletions from the reference basket. Such macros would be data source specific but the scripts shown in the implementation give a flavor of what would have to be automated.

2.2.2 JP Morgan Discount Factor Model (JPM)

The JPM model illustrates how to overcome the weakness pure yield to maturity based analysis. The model is, respectively was known in the market as the JP Morgan discount factor model (JPM) but original documentation could not be uncovered for the purposes of this article. A review of the literature revealed that this model is in actual fact a simple version of the McCulloch (1971; 1975) spline approach without node points (as discussed in Anderson et al., 1996, p.25). In line with McCulloch, the model works with the discount function which means the minimizing function can be found using least squares as illustrated below. An advantage of refraining from modeling the spot rate curve is that the discount curve “much better behaved” to use a colloquial terms. This function has a clear boundary at time zero and is monotonically declining over time.

The implementation of this spline model is stored in the file named “Term structure JP Morgan Model (Feb04).xls”. It is more generalized than the earlier yield to maturity model in that it just fits one benchmark to the mid price which is calculated as the mean of bid and ask price. Accordingly, there is no selection of bonds as shown in figure 2 but simply a calculation of the residuals. Similarly, no examples of utilities are included. The following explains the mathematics of the discount factor fitting in this case and then the main model parameters on a screen shot.

In line with present value pricing function (1) the bonds in the reference basket with market prices $P = [p_1, p_2, \dots, p_n]^T$ should all be equal to the present value of future cash flows:

$$\begin{array}{rcccccc}
 c_1 d_{t_{1,1}} + & c_1 d_{t_{1,2}} + & (1+c_1) d_{t_{1,3}} & & & = p_1 \\
 c_2 d_{t_{2,1}} + & c_2 d_{t_{2,2}} + & c_2 d_{t_{2,3}} + & (1+c_2) d_{t_{2,4}} & & = p_2 \\
 & \vdots & \vdots & \vdots & \vdots & \vdots \\
 c_n d_{t_{n,1}} + & c_n d_{t_{n,2}} + \dots & \dots + c_n d_{t_{n,j}} + \dots & & \dots + (1+c_n) d_{t_{n,n}} & = p_n
 \end{array}$$

where c_i is the fixed coupon rate of bond $i = 1 \dots n$; $d_{t_{i,j}}$ is the discount factor at $t_{i,j}$, which is the time of the j^{th} coupon of bond i .

The approximation of $d_{t_{i,j}}$ is chosen as the polynomial $a_m t_{i,j}^m + a_{m-1} t_{i,j}^{m-1} + \dots + a_1 t_{i,j} + a_0$.

To simplify, the further solution is developed just for the three dimensional case of a cubic spline. Note, however, that the model implementation can cope with higher order polynomials.

Rewriting above equations for $m=3$ in matrix notation, one finds:

$$\begin{bmatrix}
 c_1 & c_1 \sum_j t_{1,j} & c_1 \sum_j t_{1,j}^2 & c_1 \sum_j t_{1,j}^3 \\
 c_2 & c_2 \sum_j t_{2,j} & c_2 \sum_j t_{2,j}^2 & c_2 \sum_j t_{2,j}^3 \\
 c_3 & c_3 \sum_j t_{3,j} & c_3 \sum_j t_{3,j}^2 & c_3 \sum_j t_{3,j}^3 \\
 \vdots & \vdots & \vdots & \vdots \\
 c_n & c_n \sum_j t_{n,j} & c_n \sum_j t_{n,j}^2 & c_n \sum_j t_{n,j}^3
 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{bmatrix}$$

We are thus looking for the vector of coefficients $A = [a_0, a_1, a_2, a_3]^T$ that minimizes the sum of the squared difference between the market price vector $P = [p_1, p_2 \dots p_n]^T$ and model price vector

$$\hat{P} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \dots \\ \hat{p}_n \end{bmatrix}^T, \text{ i.e. Min } \sum_{i=1}^n \left(\hat{P}_i - P_i \right)^2$$

Setting the partial derivatives to zero: $\frac{\partial \sum_{i=1}^n (\hat{p}_i - p_i)^2}{\partial a_k} = 0$ for $k = 0, 1, 2, 3$

..., yields four equations for the four unknown a_0, a_1, a_2, a_3 .

$$\text{Defining } C = \begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} \\ C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} \\ \vdots & \vdots & \vdots & \vdots \\ C_{n,1} & C_{n,2} & C_{n,3} & C_{n,4} \end{bmatrix} = \begin{bmatrix} c_1 & c_1 \sum_j t_{1,j} & c_1 \sum_j t_{1,j}^2 & c_1 \sum_j t_{1,j}^3 \\ c_2 & c_2 \sum_j t_{2,j} & c_2 \sum_j t_{2,j}^2 & c_2 \sum_j t_{2,j}^3 \\ \vdots & \vdots & \vdots & \vdots \\ c_n & c_n \sum_j t_{n,j} & c_n \sum_j t_{n,j}^2 & c_n \sum_j t_{n,j}^3 \end{bmatrix},$$

..., one must thus solve the system of following system of linear equations to find the coefficient vector A::

$$C^T C \times A = C^T \times P \Rightarrow A = (C^T C)^{-1} \times (C^T \times P)$$

The model prices are then found by multiplying matrix C with the coefficient vector A:

$$C \times A = \hat{P}$$

Bonds priced below [above] their corresponding model price are “cheap” [“rich”].

The residual vector of (under pricing)/over pricing $R = [r_0, r_1 \dots r_n]^T$ is then:

$$R = \hat{P} - P = C \times A - P$$

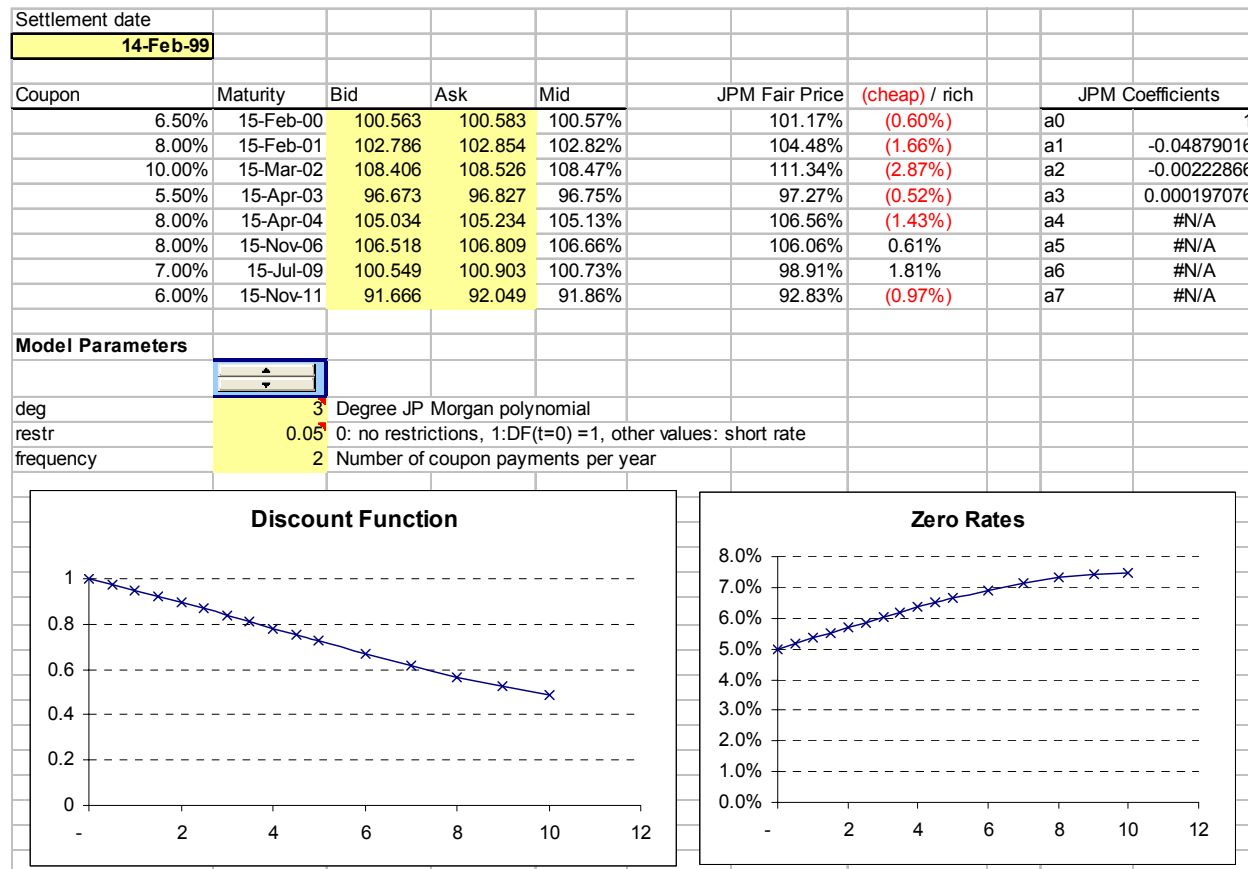
2.2.3 Boundary conditions

Typically constraints are introduced to force the discount factor at time zero to one which is obviously a most reasonable assumption. This means coefficient a_0 is set to 1. Moreover, one often chooses the first derivative of the discount function at time zero to fit a short-term rate, e.g. overnight bank rate observed in the market. For an assumed short-term rate r_0 , a_1 becomes minus the continuously compounded equivalent of the short rate, respectively in terms of the annual

equivalent rate $a_1 = -r_c = -\ln(1 + r_{ann})$ where r_{ann} is the short-term rate expressed as an annual equivalent yield and r_c is the short-term rate expressed as an continuously compounded yield.

This result is derived in footnote ¹³.

Figure 3: JPM model screenshot



¹³ The discount factor at time $(t_0 + \Delta t)$ for r_c continuously compounded yield with Taylor expansion of exponential function

$$a_0 + a_1(t_0 + \Delta t) + a_2(t_0 + \Delta t)^2 + \dots = e^{-(t_0 + \Delta t)r_c} = e^{-t_0 r_c} - r_c e^{-t_0 r_c} \Delta t + \frac{1}{2} r_c^2 e^{-t_0 r_c} \Delta t^2 + \dots$$

The parameters as well prices are set in yellow shaded areas. The major parameters are ...

deg: Allowing a choice of the degree of the discount function polynomial.

It is not meaningful to choose a much larger value for the small bond universe in the example.

restr: **0** means no restrictions on discount function, **1** means discount function is forced to one for $t=0$ which means coefficient a_0 is set to one. **Any other value** is assumed to be the short-term rate (annual effective yield, i.e. non-continuous yield). a_1 is calculated with the formula given before (a_0 is forced to one).

frequency: to set number of coupon payments per year.

for $t_0 = 0$, $a_0 = 1$ and omitting second and higher order terms of Δt

$$1 + a_1 \Delta t + a_2 \Delta t^2 + \dots = 1 - r_c \Delta t + \frac{1}{2} r_c \Delta t^2 + \dots \Rightarrow a_1 = -r_c = -\ln(1 + r_{ann})$$

2.2.4 Extended Nelson & Siegel Model

Nelson and Siegel (1987) proposed a parsimonious model of yield curves that are continuous and smooth. Unlike the spline models like JPM, Nelson and Siegel (N&S) model the forward or spot interest rate directly without modeling the discount function first. The model presented here is structurally similar to the JPM model just shown in that it relies on the same pricing function and also uses the same small data sample of New Zealand government bonds. It does however not employ its own estimation procedure, using Solver, Excel's built-in multidimensional optimization tool instead. The model is stored in file "NelsonSiegelYieldCurveModel.xls"

The following first explains the yield curve fitting according to the extended N&S model as described in Bliss (1997) and then talks about particular implementation issues.

Under the extended N&S model, the spot rates r as a function of time m are approximated by this exponential function:

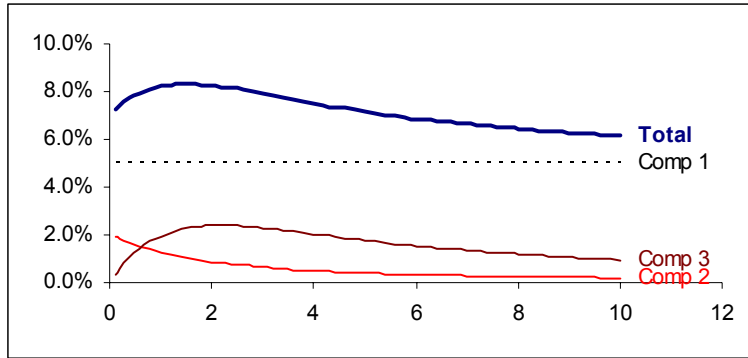
$$r_{t,j}(m, \Theta) = \beta_0 + \beta_1 \left(\frac{1 - e^{-m/\tau_1}}{m/\tau_1} \right) + \beta_2 \left(\frac{1 - e^{-m/\tau_2}}{m/\tau_2} - e^{-m/\tau_2} \right) + \varepsilon_{t,j}$$

with $\varepsilon_{t,j} \sim N(0, \sigma^2)$
 $\Theta = (\beta_0, \beta_1, \beta_2, \tau_1, \tau_2)$

There are five parameters in parameter vector Θ which can be interpreted as follows. β_0 represents the long-run level of interest rates as $m \rightarrow \infty$ and for very short times, r will converge to $\beta_0 + \beta_1$. β_2 could be interpreted as the medium term component as it will tend to zero both at the short and long end of the time scale. Finally, the two decay parameters τ_1 and τ_2 determine how quickly the effect of the short-term, respectively the medium-term component will

tend to zero. τ_1 and τ_2 should both be positive to ensure convergence. In the basic version of N&S (1987) they are set to equal values. Figure 4 visualizes the effect of these three components.

Figure 4: Nelson & Siegel (1987) Spot Rate Components



Contributions of the three N&S equation terms to the total spot rate r .

Example parameter values:

$$\beta_0 = 5\%, \beta_1 = 2\%, \beta_2 = 8\%$$

$$\tau_1 = 1, \tau_2 = 1.2$$

The parameter vector Θ must now be chosen to minimize the sum of squared price errors

$$\hat{P}_i - P_i, \text{ i.e. } \min \left(\sum_{i=1}^N (w_i \varepsilon_i)^2 \right) \text{ where } w_i = \frac{1/D_i}{\sum_{j=1}^N 1/D_j} \text{ and } \varepsilon_i = \hat{P}_i - P_i$$

Note that in this case the squared errors are actually weighted with the inverse of the bond's Macaulay duration D_i . This means the prices of short-term bonds are fitted much tighter to account for a greater variability of short-term bond yields.

The estimation procedure needs to be constrained to ensure rate r remain positive and the implied discount function non-increasing (non-negative forward rates):

$$0 \leq r(m_{\min}) \text{ and } 0 \leq r(m = \infty)$$

where m_{\min} is a small value just slightly higher than zero.

(Note that the N&S function is not defined for $t=0$.)

$$\exp(-r(m_k)m_k) \geq \exp(-r(m_{k+1})m_{k+1}) \forall m_k < m_{\max}$$

Finally, similarly to the JPM model, it is often meaningful to prescribe not just positive a sort term interest rate, i.e. an overnight lending rate, to fix the curve at the short end. This is tantamount to prescribing a constraint for the sum of β_0 and β_1 : $\beta_0 + \beta_1 = r(m_{\min})$.

Figure 5: Screenshot of Nelson & Siegel (1987) Bond Relative Value Model

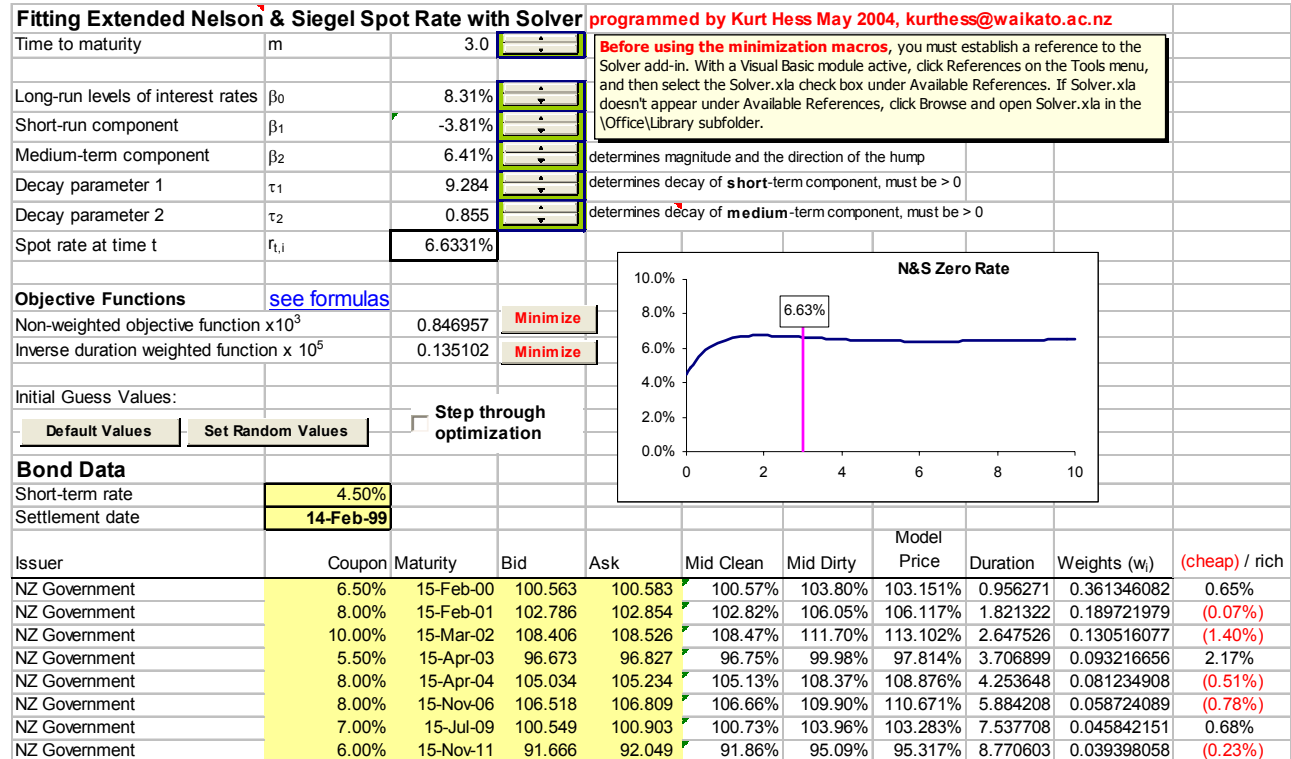


Figure 5 provides a screenshot of the N&S model implementation. There are some prerequisites for the Excel setup in order to use the Solver software that minimizes the objective function. One should not be confused by the markedly different shape of the zero yield curve compared to the curve found through JPM for the same sample universe of New Zealand government bonds. Fitting a model with five parameters to a purely illustrative basket of only eight bonds is bound to lead to over fitting problems. Accordingly, the buy/sell signals generated will be very different.

3 A Heuristic Fitting Method for the Term Structure of Credit Spread

With this section we tackle the second major topic in this paper. There is also the need for suitable bond relative valuation when credit risk, the second major bond pricing factor, becomes an important determinant of market price. This section will first expand on the discussion in the introduction on the nature and dynamics of credit spreads, elaborating first on aspects that affect credit spreads beyond factors generally assumed in standard academic models. This is followed by a review of the shape of the term structure of credit spreads, both predicted and observed in the market. Purpose of this discussion is to provide the rationale and motivation for a heuristic term structure of fitting method of detecting mispriced which is presented in the final subsection.

3.1 The nature of credit spreads

The credit spread is the most popular yardstick for practitioners to assess bonds subject to default risk. Measured in basis points, it is typically just derived from redemption yield differentials to a reference benchmark, e.g. a government bond curve. Once they have done so, they must make a judgment whether this yield premium compensates them adequately for the risk they assume. This in turn is much harder and they do not get much help from academic research where there is a healthy debate on how to explain credit spreads observed in the market in the first place. We mentioned articles of Fons (1994) and Elton et al. (2001) in the introduction who pointed out that pure default risk cannot possibly account for absolute yield spreads alone.

Whatever the components of the credit spread, practitioners are more concerned about the relative pricing of the bonds compared to instruments of equivalent credit quality. Quite naturally, they turn to credit curves for the same credit rating category but this is by no means the only

decision criteria. Research of Collin-Dufresne et al. (2001) drew attention to the fact that only about a quarter of spread changes can actually be attributed to factors one would theoretically expect to influence them. Failing to identify the true driver of spread changes, they coin the expression “local supply/demand shock” that are independent of both changes in credit risk and typical measures of liquidity¹⁴. One can easily generalize these findings of Collin-Dufresne and state that not just the changes but also the absolute level of credit spreads are determined by other factors beyond the risk as assessed by official credit ratings. Academic research has focused on taxation and liquidity aspects in this respect, probably because these do lend themselves easily to standard empirical analysis¹⁵. While it would be beyond the scope of this paper to tackle this issue here, it is the professional work experience of this author that credit spreads in a particular case are difficult to understand, in many instances even lack immediate rational explanation. Here are some examples.

- Household names

So-called “household names” trade on much narrower spreads than indicated by their credit ratings. An extreme example was Porsche’s unrated 10-year bond issued in April 1997 which

¹⁴ While Collin-Dufresne et al. (2001) confirmed that factors important for example in Black-Scholes (1973) and Merton (1974) contingent claims framework such as a firm’s leverage, equity returns and volatility indeed had a significant correlation to spread changes, non-firm specific attributes like the return of the whole share market were a much stronger driving force. Overall their principal component analysis reveals that there is a large systematic component that lies outside the model framework.

¹⁵ e.g. Van Landschoot (2003) for the Euro corporate bond market

has often traded below the German government curve. A research hypothesis could posit that such anomalies will occur more frequently in markets with strong retail demand.

- Some spread levels may also be rooted in informational inefficiencies.

These are highlighted in Schultz (2001, p. 678) for the US corporate bond market where the potential buyer cannot observe all quotes in a central location. Similarly, if a bond of the same firm trades in different markets, one often observes differences which cannot possibly be explained by currency, taxation or other factors.

- Spreads are affected by new issue supply

A lead bank may be pressured to “move a transaction off their books” thus affecting secondary market spreads. Conversely, strong demand for a new issue will tighten spreads of existing deals.

- Rating assessment of market diverges from official agency ratings.

Official ratings are often not accepted by the gross of market participants who, colloquially speaking, consider agencies as “behind the curve”. Such cases have for instance been observed during the 1997 Asian crisis, but agencies also tend to be slow to recognize improvements in credit quality. Due to the formal internal processes involved, agencies frequently find it hard to react promptly. It has also been noted that conflicts of interest and the quasi-monopoly of some agencies may create rating distortions¹⁶. All in all, an official credit rating constitutes just a mostly qualitative assessment of a well informed but not

¹⁶ Such concerns are for instance reflected in a SEC concept release SEC (2003) for the oversight of credit rating agencies.

infallible party.

All these pricing aspects highlight that the particular decision of assessing a relative value of a bond involves a qualitative, respectively “market savvy” component not easily captured by any of the standard academic models.

3.2 *The shape of the term structure of credit spreads*

We have mentioned in the previous section that practitioners will turn to credit curves of comparable credit quality as a starting point for their relative value valuation. In this section, we review what is generally predicted and observed regarding the shape of this term structure of credit spreads.

Figure 6:

Term Structure of Interest Risky Discount Bonds in the Longstaff & Schwartz (1995) Model

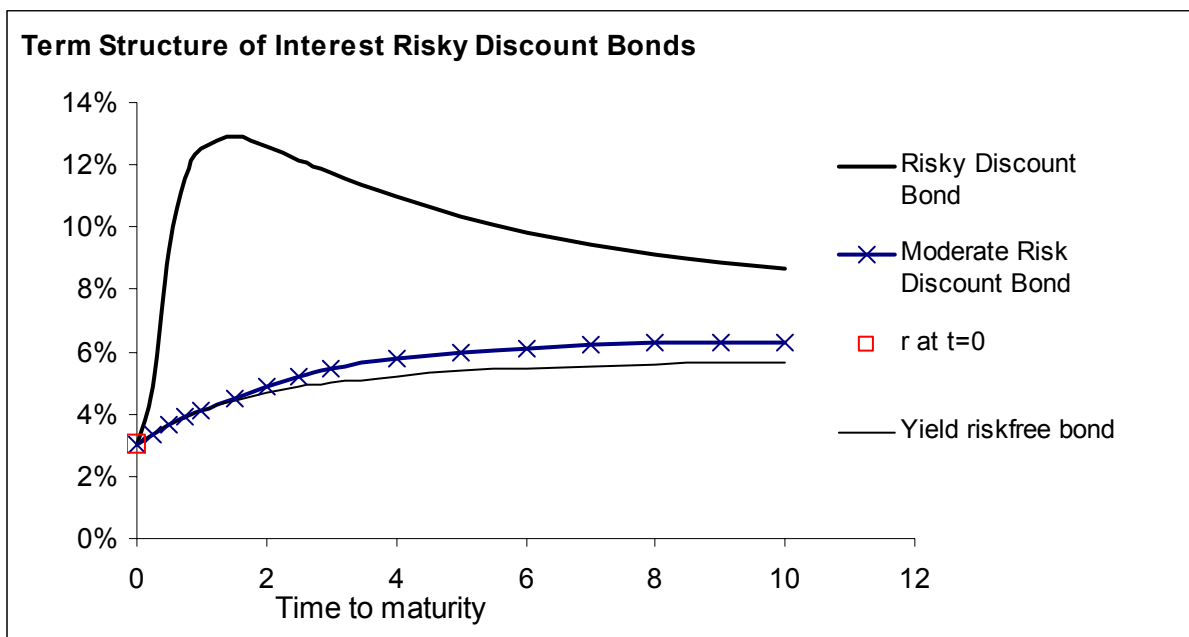


Figure 6 illustrates in generic ways the term structure of spot rates of a risky, respectively less risky discount bond as predicted by many of the mainstream credit risk models. In this instance, it was generated by the Longstaff & Schwartz (1995) model (L&S 95) which, for illustration, is explained in more detail in Appendix 2. L&S 95 is from the family of Merton's (1974) contingent claims models but one would find similar hump-shaped, downward-sloping credit yield curves for example in Jarrow, Lando & Turnbull's (1997) reduced form approach.

This theoretical prediction appears to be backed by empirical studies such as Fons (1994, p. 30) who estimates a cross-sectional regression of spreads on maturity and finds significantly negative coefficients for single B bonds. This result is also supported by Moody's default data where marginal default rates of speculative grade bonds (B rating) exhibit a declining trend with longer time horizons.

The intuition behind this result is that speculative firms, being very risky at issuance, have room to improve, i.e. the longer the time to maturity, the more likely the value of the firm will rise substantially. Another interpretation would be that speculative bonds obtain the character of an equity instrument and are thus traded on a break-up value instead of a yield basis. Conversely, high grade credit "can only become worse" through time and thus show an upward sloping term structure of credit spread.

These results are somewhat against the intuition of practitioners who observe that where a firm issues in two separate time tranches, it will be asked offer a higher yield premium for the longer maturity tranche of the transaction and this applies equally to weaker and stronger credits. Helwege & Turner (1999) indeed provide some empirical evidence for this observation. They argue that downward sloping credit curves are a result of "safer" speculative grade firms issuing longer-dated bonds which in turn leads to a sample selection bias.

What we can conclude is that there are two schools of thought for modeling term structure of credit spreads. A useful bond relative value tool for corporate bonds must thus be capable of accommodating both of them.

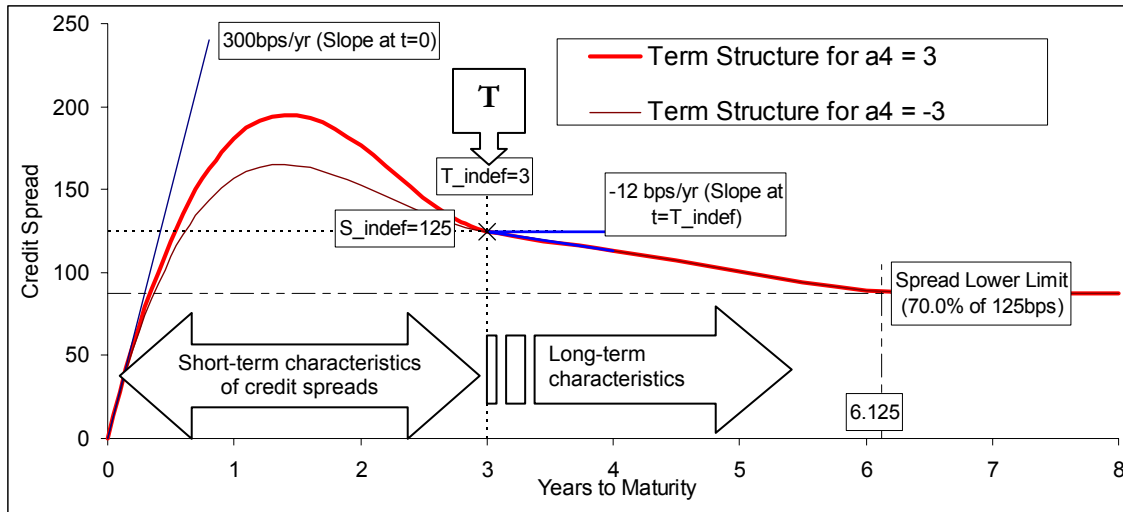
3.3 Heuristic fitting method for term structure of credit spreads

This section presents an approach for work tool so market practitioners can firstly, assess the relative price of a bond in view of less tangible pricing factors (as shown in section 3.1) and secondly, to prescribe a term structure of credit spreads they feel is appropriate for the particular instrument and market condition (as explained in section 3.2). The implementation of this model is shown in file “Cheap Rich List (Feb04).xls”. The file uses a sample of Swiss corporate industrial bonds to generate a list of cheap, respectively rich bonds. It employs a simple redemption yield based reference curve of the type that was derived in section 2.2.1 as a basis to calculate credit spreads. This yield to maturity based approach could easily be adapted into a spot rate framework using curve derived in section 2.2.2 or 2.2.3. Yet given the heuristic nature of the model, this is likely to add only limited value. The following describes the model with a simplified numerical example also documented in the Excel work book.

3.3.1 Defining the term structure of credit spreads in the model

The model lets the user choose the desired term structure of credit spreads for each rating category by means of shape parameters. This is illustrated through a numerical example in Figure 7 below. The term structure is basically broken down into two sub-periods. A short- to medium term period to T_{∞} (T_{indef}) which is followed by the long-term characteristics of the credit spread.

Figure 7: Explanation Shape Parameters Term Structure of Credit Spreads



As to the first period, a fifth order polynomial is fitted between zero and T_∞ .

$$Spread(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4$$

It must meet the following three boundary conditions:

- must be zero at time = 0 which means $a_0 = 0$
- must reach $S_\infty(S_indef)$ at time $t=T_\infty$.
- slope at point S_∞/T_∞ must equal the slope of the long-term curve beyond T_∞ . (more details on this slope are below)

The user affects the shape of the polynomial with two parameters.

- The initial slope (in bps per year) at time = zero may be specified
- The shape of the hump is also affected by parameter a_4 which corresponds to the fifth order coefficient of the polynomial. Figure 3 shows the term structure for $a_4= 3$, respectively minus 3.

In the long-term horizon, the user can specify a slope for the further development of credit spreads. In most cases it will be set to or slightly above zero to obtain constant or moderately increasing credit spreads beyond time T_{∞} . There is also the option to specify a lower limit below which the credit spread may never decline. This parameter is set as a percentage of S_{∞} . If this lower limit is specified as a number greater than one, it actually becomes an upper limit specification.

With above parameters, a wide range of shape specifications becomes possible. Figure 4 lists a range of potential shapes including some comments as the circumstances these could be appropriate. We again refer to section 3.2 for a discussion of research on this subject.

Figure 8: Selection of Term Structure Shapes

Chart 1: Simple, constant spread

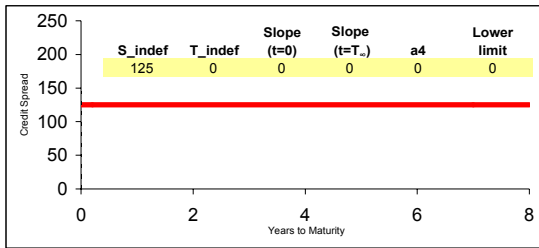


Chart 2: No hump, suitable for high quality bonds

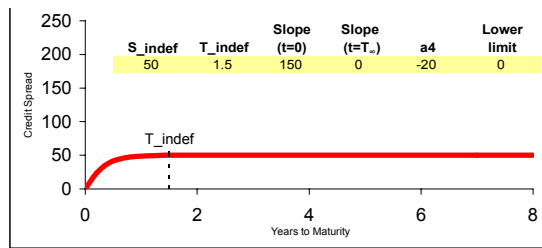


Chart 3: Small hump, suitable for medium quality bonds

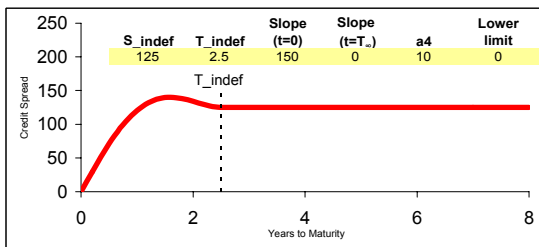
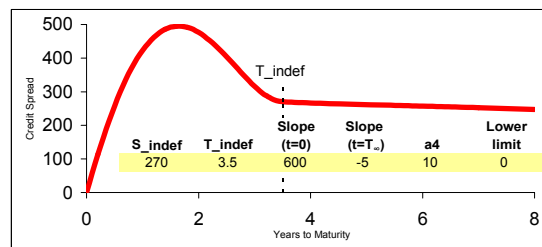


Chart 4: Pronounced hump, suitable for non-investment



3.3.2 Detecting cheap/rich bond with heterogeneous credit quality:

a simplified numerical example

For illustration, the following tables (Table 2 and 3) and figures (Figure 9) present a simplified numerical example of cheap/rich analysis. There are three bonds analyzed. Bond I has an AA rating while bonds IIa and IIb are rated BBB. A buy [sell] signal is generated if the model price based on the target spread is above [below] the market price.

Table 2: Data Example Bonds

	Bond	I	IIa	IIb
Rating		AA	BBB	BBB
Price (\$ per \$face value)*		102	99	106
Coupon (paid once annually)		4.00%	4.00%	4.00%
Time to Maturity		2.79 yrs	5.20 yrs	7.79 yrs
Yield to Maturity		3.24%	4.22%	3.12%
Spread to Benchmark		136 bps	208 bps	58 bps
Target Spread (derived from parameters in Table 3)		50 bps	122 bps	120 bps
Model Yield		2.37%	3.36%	3.73%
Model Price (MP) *		104.338	103.016	101.761
		MP > Price	MP > Price	MP < Price
Recommendation		Cheap Bond: Buy	Cheap Bond: Buy	Rich Bond: Sell

* Price excluding accrued interest

Figure 9: Generic Yield to Maturity Based Cheap Rich Analysis

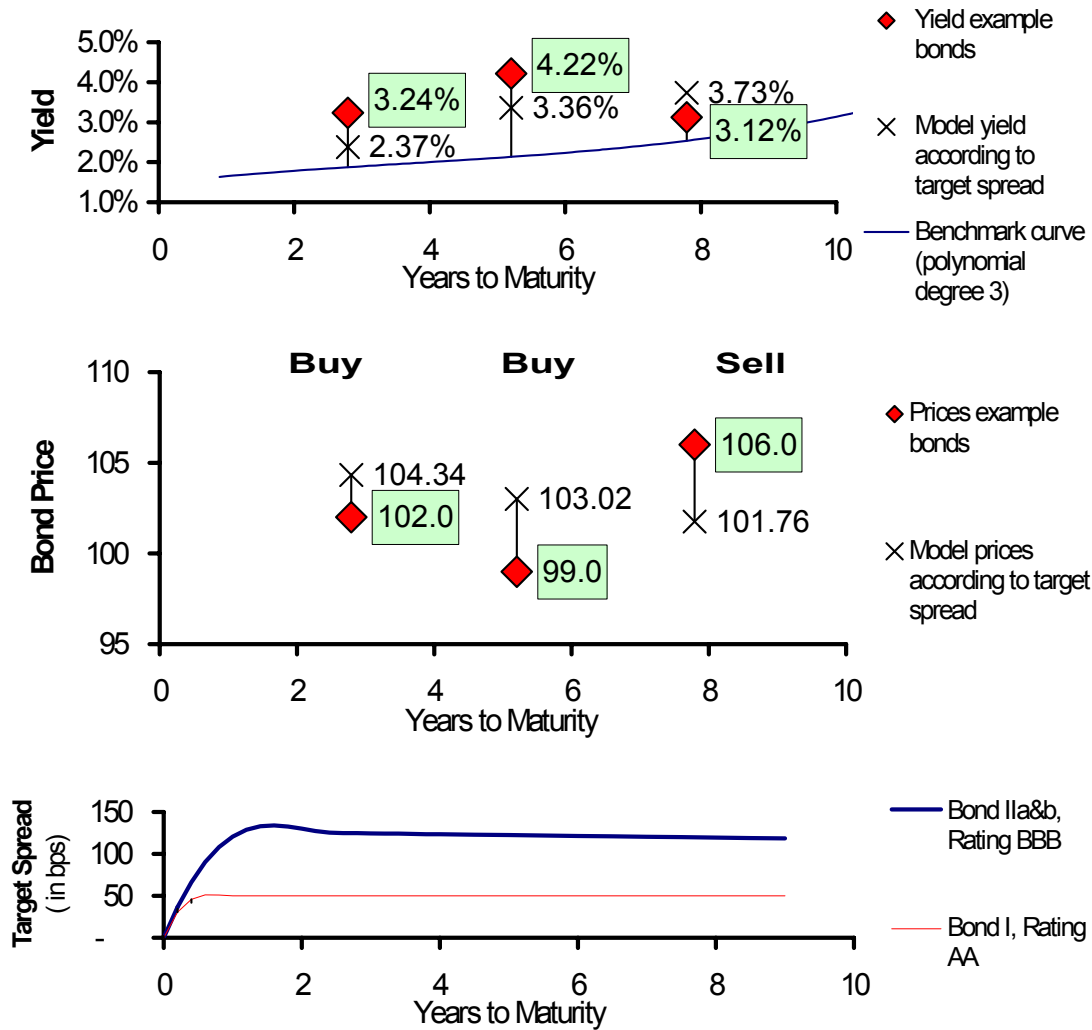
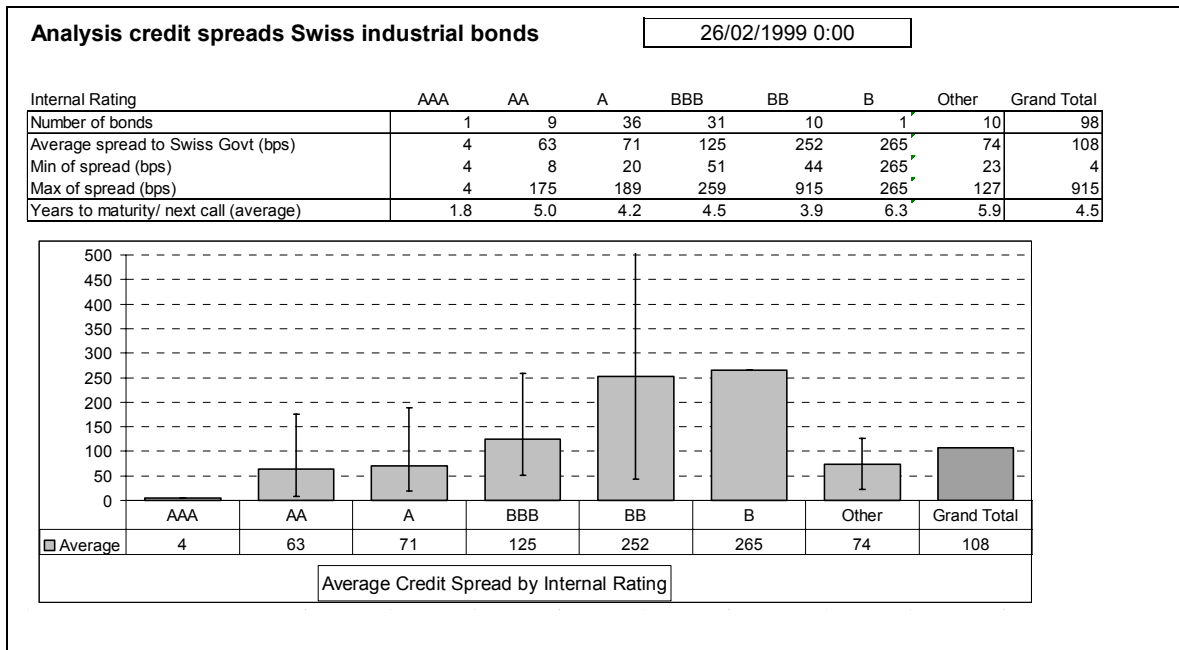


Table 3: Term Structure of Credit Spread Shape Parameters

	Rating	AA	BBB
S_∞ (in bps)		50	125
T_∞		1	2.5
Slope (t=0)		200	200
Slope (t= T_∞)		0	-1
“Hump parameter” a4		-6	2
Lower limit (as % of S_∞)		0.8	0.8

Compared to the main model, the version above is simplified in these aspects. The illustrative example does not take bid/ask spreads into consideration when generating buy/sell signals. To limit the number of recommendations the user may specify a sensitivity parameter to suppress recommendations where the price is very close to the model price. The sensitivity chosen could, for instance, take transaction charges into account. This the same filter described in Figure 2 for the model in section 2.2.1. For shorter bonds typical price changes in less liquid markets may lead to very erratic yield moves that do translate into meaningful buy/sell signals. The user may thus exclude the analysis for very short-term bonds. Finally, the main model provides statistics on the credit spreads observed in the reference baskets. An example is shown in Figure 10 below.

Figure 10: Generic Yield to Maturity Based Cheap Rich Analysis



3.3.3 *Some remarks on how to apply the model*

The model description has not addressed the issue of calibration yet. In other words, how should one determine the shape parameters for a particular instrument, respectively group of instruments? Given that this is not a fitting method for empirical research purposes, the following pragmatic method is recommended.

In a first instance, one would select a value for S_{∞} . If we leave all other parameters zero, this is nothing else than a flat credit spread we apply to a reference curve, i.e. parallel upward shift of the reference curve. For many users this is the ad-hoc method they will be used to and, reflecting on the empirical results of Fons (1994) discussed earlier¹⁷, this is not an unreasonable approach at all. To automate the selection of S_{∞} , one could derive it from a mean value for rating category as shown in the analysis Figure 10. Note that the rating category of a particular bond may be set by the user and would not necessarily coincide with an official agency rating.

With regard to the remaining shape parameters, it is not meaningful to set them for just one rating class. One would select, for example, common values for all BBB- to BBB+ rated bonds. As per section 3.2, there will be no hump for most rating categories and we would simply set a value for T_{∞} in the range of 1- 2 years, some generic slope at time $t=0$ (in bps per year) and a_4 to zero. The parameter of concern would be the slope at time $t= T_{\infty}$. This could be determined by a Fons (1994, p. 30) type regression analysis although it does most probably not make sense to update it as regularly as the main spread parameter S_{∞} .

¹⁷ Fons (1994) finds rather flat slopes for most rating categories and although t-statistics indicates values significantly different from zero, R^2 values are quite low.

Finally, in respect to speculative grade rating categories, there is obviously the option to set prescribe a hump-shaped structure unless the user prefers increasing term structure in line with Helwege & Turner (1999). In most bond market supply of these segment is rather sparse and a ballpark prescription of parameters without actual calibration of values may well be appropriate.

4 Conclusion

This paper illustrated the implementation of some models for yield curve fitting used in trading applications, something not yet formally documented in the academic literature. For bonds subject to credit risk it presented a heuristic model to obtain information on over-, respectively underpriced bonds. Both types of model implementations document the pragmatic nature of such solutions. In the absence of conclusive results by academic researchers, these models generate buy and sell signals as a result of the main pricing parameter for fixed income instruments which are interest rates, respectively credit spreads. The user can then evaluate them in view of his/her knowledge of local supply and demand aspects and other qualitative factors such as the ones listed in section 3.1.

There is another more general lesson one could learn from these models, which in terms of complexity are much simpler than most of the approaches currently advocated in quantitative literature. Even though their approach is static without the ambition of justifying them in a dynamic or non-arbitrage consistent theoretical framework, they “can be handled and understood” by the practitioners. More advanced approaches usually start from an idealistic premise about how the world should look like, respectively how rational investors should act. Researchers then find that reality is different and attempt to save the approach with progressively more sophisticated amendments and extensions. This could almost be compared to a “mathematical arms race” where increasingly complex quantitative theories are applied without markedly improving the predictive power of the models. Interest rate modeling is a good example where only models calibrated continuously to current market rates do have any meaningful applications in the market. It would perhaps be time that academic research in finance made the requirements of market practitioners a

starting point for improved models and not a futile chase for the ultimate true model that does not exist.

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Appendix 1:

Settlement Price Calculation New Zealand Domestic Bond Market

According to the Reserve Bank of New Zealand Formula (RBNZ 1997)

$$P = \frac{1}{(1+i)^{a/b}} \left[\sum_{k=0}^n \frac{C}{(1+i)^k} \right] + \frac{FV}{(1+i)^n}$$

where

- P: Market Value of Bond
- FV: Nominal or face value of bond
- i: Annual market yield / 2 (in %)
- c: Annual coupon rate in %
- C: Coupon Payment (= c/2 * FV) – semi-annual coupon
- n: Number of full coupon periods remaining until maturity
- a: Number of days from settlement to next coupon date
- b: Number of days from last to next coupon date

Appendix 2:

Longstaff & Schwartz 1995 Model

In their 1995 Journal of Finance paper, Longstaff and Schwartz (L&S 95) show a simple approach to value risky debt subject to both default and interest rate risk. In line with the traditional Black-Scholes (1973) and Merton (1974) contingent claims-based framework, default risk is modeled using option pricing theory. This means default occurs if the level of asset of a firm (V) falls below a bankruptcy threshold (K). V is assumed to follow the following stochastic process

$$dV = \mu V dt + \sigma V dZ_1$$

where σ is the instantaneous standard deviation of the asset process (constant) and dZ_1 is a standard Wiener process.

Similarly, interest rates are assumed to follow a standard Vasicek process (Vasicek 1997) as follows:

$$dr = (\zeta - \beta r) dt + \eta dZ_2$$

where

ζ is the long-term equilibrium of mean reverting process (constant)

β is the "pull-back" factor - speed of adjustment (constant)

η is the spot rate volatility (constant)

dZ_2 is a standard Wiener process.

The instantaneous correlation between dZ_1 and dZ_2 is ρdt

L&S 95 then derive the value of a risky discount bond as

$$P(X, r, T) = D(r, T) - wD(r, T)Q(X, r, T)$$

Thus the price of this bond is a function of X , which corresponds to the ratio of V/K , the interest rate r , and the time to maturity T . The terms to be calculated are explained below.

$D(r, T) = e^{A(T) - B(T)r}$ is the value of riskfree (no credit risk) discount bond according to Vasicek (1977) with

$$A(T) = \left(\frac{\eta^2}{2\beta^2} - \frac{\alpha}{\beta} \right) T + \left(\frac{\eta^2}{\beta^3} - \frac{\alpha}{\beta^2} \right) (e^{-\beta T} - 1) - \left(\frac{\eta^2}{4\beta^3} \right) (e^{-2\beta T} - 1)$$

$$B(T) = \frac{1 - e^{-\beta T}}{\alpha}$$

Here α represents the sum of the parameter ζ plus a constant representing the market price of risk, β is defined above

The $Q(X, r, T)$ term can be interpreted as probability - under risk neutral measure - that default occurs. It is the limit of $Q(X, r, T, n)$ as $n \rightarrow \infty$.

$Q(X, r, T, n)$ is calculated as follows:

$$Q(X, r, T, n) = \sum_{i=1}^n q_i \text{ with } \begin{cases} q_1 = N(a_1) \\ q_i = N(a_i) - \sum_{j=1}^{i-1} q_j N(b_{ij}), i = 2, 3, \dots, n \end{cases}$$

where $N(\cdot)$ denotes the cumulative standard normal distribution and

$$a_i = \frac{-\ln X - M(iT/n, T)}{\sqrt{S(iT/n)}} \quad b_{ij} = \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n) - S(jT/n)}}$$

Expressions $M(t,T)$ and $S(T)$ are defined as follows:

$$\begin{aligned}
M(t,T) &= \left(\frac{\alpha - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2} \right) t \\
&+ \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3} \right) \exp(-\beta T) (\exp(\beta t) - 1) \\
&+ \left(\frac{r}{\beta} - \frac{\alpha}{\beta^2} + \frac{\eta^2}{\beta^3} \right) (1 - \exp(-\beta t)) \\
&- \left(\frac{\eta^2}{2\beta^3} \right) \exp(-\beta T) (1 - \exp(-\beta t))
\end{aligned}
\quad \text{and} \quad
\begin{aligned}
S(T) &= \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2 \right) t \\
&- \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3} \right) (1 - \exp(-\beta t)) \\
&+ \left(\frac{\eta^2}{2\beta^3} \right) (1 - \exp(-2\beta t))
\end{aligned}$$

As a reminder, ρ is the instantaneous correlation between the asset and interest rate processes.

Finally, the constant parameter w is the write-down in case of a default in percent of the face value. In other words, it is one minus the recovery rate in case of a default.

Once the value of a pure discount bond is found, the value of a coupon bond is simply valued as series of discount bonds consisting of coupons and principal repayment. Note that L&S 95 also derive a closed form solution for perpetual floating rate debt in a similar fashion.

The authors conclude their work with an empirical model validation. They conduct a regression analysis of how historically observed spreads (sourced from Moody's) have correlated with the return of share indices as a proxy for the asset process, respectively change in interest rates. They indeed find significant negative correlations in most cases with both interest rate changes and development of asset prices. Just for high grade bonds (AAA and AA bond) they determined less significance for the asset correlation coefficient. This may be expected though because the "well cushioned" high grade credits will be less affected by downturns in the share market.

As an illustration of the L&S 95 model outputs, the charts below show the value and yield of a discount bond as a function of time to maturity T for the parameters in Table A2.1.

Figure A2.1: Risky discount bond prices as a function of bond tenor (time to maturity)

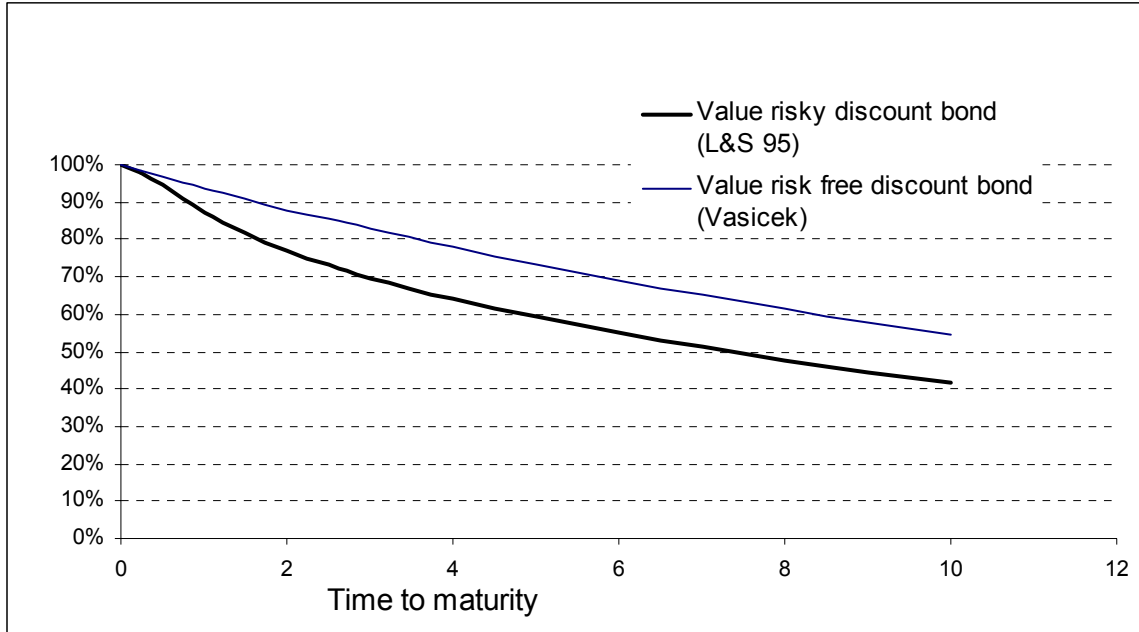


Figure A2.2: Term Structure of Interest Risky Discount Bonds

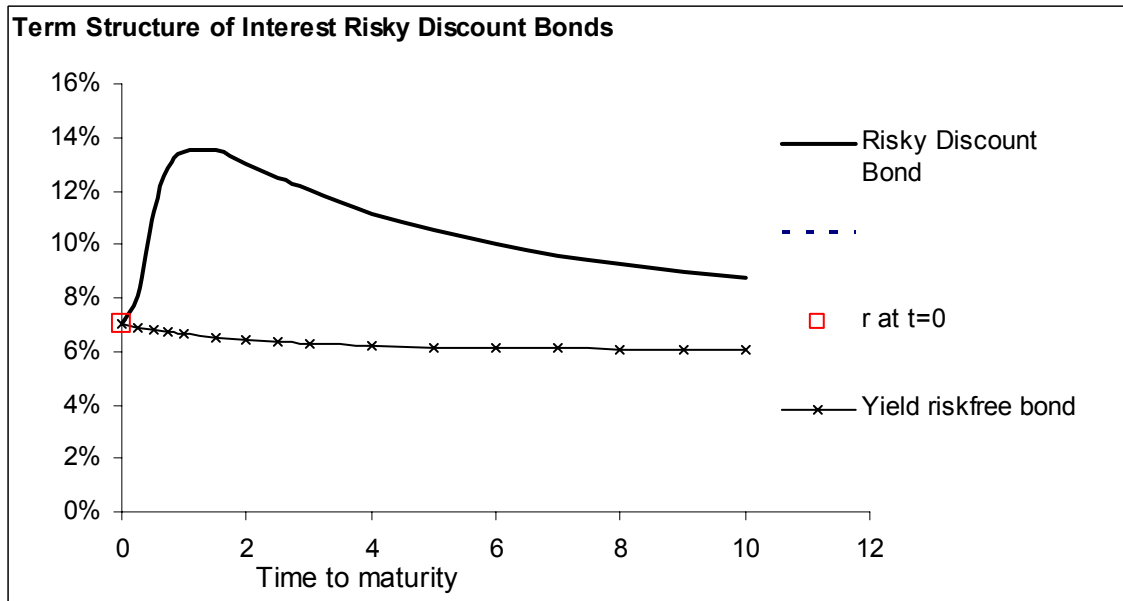


Table A2.1: Parameters L&S 95 Model Example

Parameter description	Symbol	Value
Rate r_0 at $t=0$	r	7.0%
"Pullback" factor interest rate	β	1.00
Instantaneous standard deviation of short rate	η	3.162%
	η^2	0.0010
α in L&S = ζ + constant	α	0.060
$V_0/K = X$ (measure of initial credit quality)	X	1.30
Writedown = 1 - Recovery Rate	w	0.50
Volatility of asset value process	σ	20.00%
	σ^2	0.0400
Instantaneous correlation asset/interest rate	ρ	- 0.25
Iterations for Q	n	100