RisCura Fixed Income Series

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Calculating Credit Spreads Using the BEASSA Zero Curve

- We show how to determine the credit spreads for fixed rate bonds using the newly released BEASSA Zero Coupon Yield Curve.
- We also test the accuracy of the BEASSA Zero Curve when using two different interpolation methods and comment on the best method.
- We calculate the credit spreads, using the methods discussed, for a selection of non-government rated bonds and comment on the shape and slope of the credit curves over time.
- Finally, we comment on the liquidity assumptions of the credit curves as well as the bond market's liquidity in general.

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Calculating Credit Spreads Using the BEASSA Zero Curve

1 Introduction

As discussed in the previous report [1], which commented on the yield curve, the collection of rates from debt instruments with identical properties yet having varying maturity terms, was referred to as the "term structure" of interest rates. The report mentioned that the smooth, graphical interpolation of these rates is referred to as a yield curve. Although many different yield curves are in use in the SA market, the three most popular ones are the:

- The Government or risk-free yield curve;
- Bond Exchange/Actuarial Society of South Africa (BEASSA) Actuaries Yield Curve;
- BEASSA Zero Coupon Yield Curve.

The various advantages and disadvantages of each curve was discussed and it was shown how to construct a simple risk-free yield curve using the GOVI index bonds. This second report covers the following:

- How to determine the credit spreads for fixed rate bonds using the newly released BEASSA Zero Coupon Yield Curve.
- Testing the accuracy of the BEASSA Zero Curve when using two different interpolation methods and comments on the best method to select.
- Calculating the credit spreads, using the methods discussed, for a selection of non-government rated bonds and comment on the shape and slope of the credit curves over time.
- Comment on the liquidity assumptions of the credit curves as well as the bond market's liquidity in general.

2 Defining Credit Spreads

Investors buying risky bonds would demand lower prices or higher yields as compensation for the possibility of losing part or all of their invested funds. The arithmetic difference[1] between the risky bond's yield and the yield from a risk-free bond that has a similar maturity date and coupon rate is referred to as the credit spread. The spread should include a risk premium to reward investors for taking on the possibility of incurring losses[2].

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[1] While a compound difference may be more appropriate, in line with convention, we use an arithmetic difference for the spread calculation but will re-evaluate this position in a later paper.

[2] In theory, the higher the risk, the higher the yield on an instrument, or the discount rate, and the lower the discounted price. Hence to reflect higher risk, one increases the required rate of return,

The spread might either display sudden or continuous changes over time, the former as a result of market movements and liquidity while the latter due to sudden events like default events or credit migrations. While a variety of factors influence yields and spreads, views on four factors primarily determine the width of credit spreads. These factors are:

- Market risk;
- Liquidity risk;
- Default risk;
- Credit migration risk.

The last two types of risk can be collectively referred to as credit risk. Each of the above mentioned risks are explained in more detail below:

Market risk refers to the risk on levels of interest rates or absolute levels of rates. This risk arises due to, inter alia, changing demand and supply issues, sentiment, political and economic factors/expectations etc. In our discussion here we are more concerned about the influence of this on credit spreads. This is best described by observing that while a credit spread of 100 basis points on an absolute level of rates of say 3% may be suitable, the same arithmetic spread may not be suitable at an absolute level of interest rates of 20%, for example[3]. The impact of this debatable issue needs to be covered in more detail in a later paper, along with the multiplicative vs. additive spread debate.

Liquidity risk refers to the degree to which holdings can be liquidated and the effect that transaction pressure has on bonds' yields. Liquidity is primarily influenced by the supply and demand dynamics of the bond, e.g. high demand for a bond in short supply will drive prices up and the yield down to narrow credit spreads. Since both liquid and illiquid non-government bonds are compared to liquid government bonds, the effect of liquidity on credit spread is difficult to separate from credit issues. As a result, simple credit spreads also contain a liquidity premium.

Default risk refers to the ability of the bond issuer to pay the coupon and face value of the bond on the set dates. Failure on the part of the issuer to pay in a timely fashion will result in the bond holder being exposed to default risk. In the SA bond market, the default risk of government bonds is extremely small, since in theory government can address a

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which results in a lower price for the instrument. If negative cash flows exist, e.g. further financing of the bond, before additional interest is paid, such as the original CPI linked Toll Road (N3) stock, one would expect, by the same token, greater risk to manifest in a more negative cash flow, thereby lowering the discounted price. While higher yields shrink the positive cash flows more to reflect risk, negative cash flows, if such were to exist in the cash flow stream, unfortunately also shrink towards zero when yields increase. This results in a marginal increase in price, associated with these cash flows, when yields are increased, hardly accounting for risk. While fundamental to the approach, since most cash flows are positive, we will not delve into the problem in this paper, however it potentially points to the need for a far more complex pricing model. This model may involve using spreads around the yield curve depending on the source of credit risk or indeed a full stochastic process.

[3] Conclusive evidence exists that interest rate changes do influence credit spreads. This most likely happens through an indirect mechanism where interest rate changes affect economic growth, which in turn determines the default probabilities of companies. It has been suggested that spreads should be viewed as an insurance premium and not a mark to market concept.

shortfall through taxation. In contrast, non-government bonds that are not backed by the government do possess a real default risk since the sustainability of the bond issuer's profits is not guaranteed.

Linked to this issue is the concept of the recovery rate - when bonds default, investors rarely receive no payback. This payback may be a fraction of the contractual amounts and depends on the seniority of the debt, but all is generally not lost.

Credit migration risk captures the changes in an issuer's credit quality. To assist investors with judging the default risk associated with a bond, companies called rating agencies specialise in researching the financials of an issuer. This process is designed to dynamically assess the varying capability of the issuer to meet commitments for that particular bond. One issuer can have several bonds, all with different credit ratings. The rating agencies then publish their results as ratings that range from AAA to indicate an issuer with the highest credit quality, down to C to indicate an issuer that has a real possibility of defaulting on its financial commitments[4]. Corporate bonds with ratings down to BBB are generally referred to as investment grade, while bonds with ratings BB and below are referred to as speculative grade.

3 Elementary Methods to Calculate Credit Spreads

Assuming an efficient market, the width of a given credit spread (i.e. difference between the bond and risk-free alternative) is considered a good indication of the market's view regarding extra risks associated with that bond at any given time.

To determine a credit spread for a fixed rate bond, a proxy to use for a similar "riskless" bond is the liquid government issued bonds. Although there are a large number of issued government bonds, only a few of them trade frequently enough to be considered free of liquidity risk. Therefore, there is a rather small universe of government bond equivalents for all non-government bonds having a similar cash flow structure[5].

One solution is to approximate the spread by comparing the bond to a benchmark government bond that has more or less the same maturity term. As was shown in the previous report, the bond's credit spread can also be determined by comparing the bond's yield to an interpolated risk-free yield curve, over duration[6], to find the appropriate risk-free rate corresponding to that of the bond.

These two methods are illustrated in Figure 1 where the yields of a selection of four non-government bonds, one from each of the four investment-grade rating categories, are shown as at 30 September 2003. The credit spread for each bond was calculated using both methods mentioned above and the results are listed in Table 1.

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[4] Note that these rating agencies also use a modifier, either 1, 2 or 3 or + and -, to indicate relative quality within a certain rating category.

[5] The cash flow structure refers to the particulars of a bond like cash flow dates, book close dates, maturity dates and coupon rates.

[6] Arguably, spreads can also be calculated over a pure term to maturity basis, but this method does consider that bonds can have different coupon dates [18,19].

Figure 1. Determining credit spreads for a selection of four non-government bonds by either calculating the difference between each bond's market yield and a benchmark government bond or between it and an interpolated risk-free yield curve.

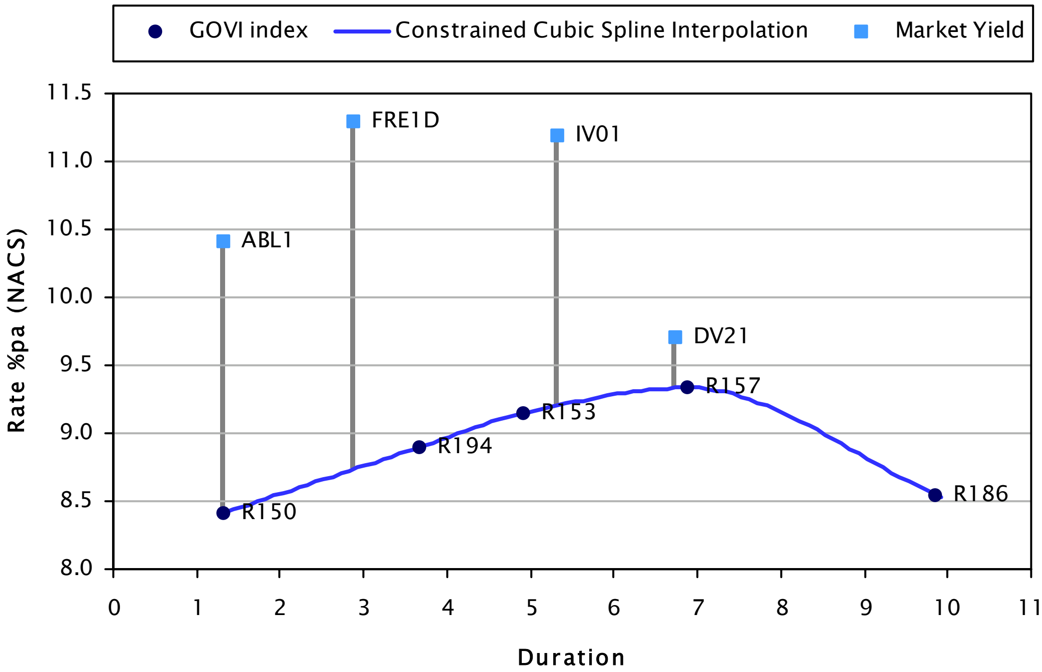

Table 1. The credit spread for a selection of non-government bonds calculated using two different methods as illustrated in Figure 1.

Bond Code	Rating	Benchmark Bond	Credit Spread to Benchmark	Credit Spread to Yield Curve
DV21	AAA	R157	37	37
IV01	AA	R153	204	199
ABL1	A	R150	200	200
FRE1D	BBB	R194	240	256

The risk-free yield curve used to determine the spread relative to this curve was determined by interpolating the term structure of the GOVI index bonds using a constrained cubic spline [2]. As expected, the width of each bond's credit spread increases as the rating decreases. For example, the AAA-rated DV21 bond from the Development Bank trades at a spread of 37 bps, while the BBB-rated FRE1D securitised bond[7] from First

[7] According to FitchRatings, the implicit element of diversification in securitised bonds is considered when assigning credit ratings to these bonds. This means that a AAA-rated securitised bond is directly comparable to an AAA-rated corporate bond, for example.

Rand trades at a spread of between 240 and 256 bps. Although both methods rely on approximations – one using the government benchmark bonds and the other the interpolated yield curve based on the benchmark bonds – the results are very similar. This might be attributed to the narrow band that the yields for the GOVI index bonds traded in during this period and may change when the bond market becomes more volatile and the yield curves change shape at the short end.

4 Calculating Credit Spreads Using Zero Curves

A better solution to approximate credit spreads is to compare the bond's yield with that of a "created" synthetic risk-free bond that has the exact same cash flow structure as the non-government bond being scrutinised.

4.1 Different Types of Zero Curves

Various types of zero coupon yield curves (or zero curves) exist that can be either directly observed or mathematically derived from instruments that trade in the market. Early in 2002, investors were able to buy and sell stripped government bonds through STRIPCO, a vehicle created by CorpCapital Bank. These bonds' coupons and principal payments were stripped into separate cash flows and traded as zero coupon bonds on BESA [3].

In theory, using the traded instruments, a risk-free market-based term structure and zero curve can be constructed. However, in practice these bonds trade infrequently and therefore their market yields include an implicit liquidity premium, making these bond yields unsuitable to use when determining a term structure for a liquid risk-free zero curve.

Another alternative is to mathematically derive the zero curve from other instruments that trade more frequently. One way is to model the swap curve[8] as a par bond yield curve paying quarterly coupons. Another method is to derive the zero curve from a collection of liquid government coupon-bearing bonds by a mathematical process known as bootstrapping [4,5].

A prototype of such a bootstrapped zero curve was introduced to the SA market late in 2001 by BESA. In response to input from the industry, the zero curve has seen many improvements and refinements since its release nearly two years ago and was only officially released as the BEASSA Zero Coupon Yield Curve early in November last year. The curve is generated using the daily market yields of the bonds in the current GOVI index as well as the rates from money market instruments to define the curve's short-term structure [6]. The GOVI index is constructed from the most liquid government bonds and in turn forms a (large) component of the All Bond Index (ALBI) maintained by BESA. The entire ALBI is reviewed and updated on a quarterly basis BESA [7].

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[8] A swap position is equivalent to a long position in a par bond that pays quarterly coupons and a short position in a floating rate note that pays JIBAR on a quarterly basis. It is then possible to convert this curve into a zero curve [19,20].

Two versions of the zero curve are derived on a daily basis, as shown in Figure 2. One is labelled the Perfect Fit Zero Curve and the other the Best Decency Zero Curve. The former is recommended when subjectivity should be kept to a minimum. For example, in pricing instruments or when determining credit spreads. Unfortunately, these curves are also more volatile on a day-to-day basis. Therefore, for studying topics like market trends or underlying market structures, the Best Decency Zero Curve is recommended. Although both curves are derived from the same GOVI index bonds and short-term money market instruments, the main difference between them is the different optimisation constraints used during the bootstrapping process.

4.2 The Evolution of the BEASSA Zero Curve

The data points of the zero curve initially released, labelled the Mark I curve, were the exact cash flow dates and corresponding bootstrapped zero rates of the GOVI index bonds. These data points totalled over a hundred separate cash flow dates and made it possible to smoothly interpolate the term structure of zero rates using a method referred to as quartic spline interpolation [8]. Although mathematically complex and calculation intensive, this interpolation method represents the industry standard when interpolating zero curves[9].

After the Mark I release, some minor improvements and adjustments were made in the forthcoming Mark II and Mark III releases. A major change in the Mark IV, released in December 2002, was that the curve dates were changed to uniformly spaced dates instead of previous cash flow dates. In the new version, the curve's dates were spaced yearly with some shorter spacings at the short-dated side of the curve. This meant a reduction to 42 curve dates and zero rates, instead of the previous hundred or so data points. Furthermore, since the new curve dates were arbitrarily selected, the corresponding zero rates were calculated using the quartic spline interpolation method prior to the distribution by BESA. This makes it impossible to re-use this interpolation method to obtain any other zero rate for a given cash flow date. An example of the Perfect Fit and Best Decency versions of the Mark IV Zero Curve for 31 December 2002, is shown in Figure 2. Note the equal spacing between the dates on the axis. Any attempt to accurately re-price the GOVI index bonds using either linear or cubic spline interpolation will fail because the data points are too widely spaced to allow accurate interpolation.

Following the Mark IV release, some minor changes were made to the curve with the Mark V release. The Mark VI curve, released in August this year, was the next major revision to appear since the Mark IV curve's release. In this release the data points for the curve were increased from 42 to 165, at intervals of one month up to 5 years and then quarterly thereafter. Both Perfect Fit and Best Decency versions of this curve for 30 September 2003 are shown in Figure 3. Note the difference in spacing between these curves and those shown in Figure 2. Although these curves still do not allow the use of the quartic spline interpolation, the increased number of curve dates and zero rates at least allows the use of simple interpolation methods to obtain reasonable accurate zero rates.

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[9] A brief explanation on how the quartic spline interpolation method works is given in Appendix C.

Figure 2. The Mark IV version of the BEASSA Zero Curve as on 31 December 2002, showing both the Perfect Fit and Best Decency Curves.

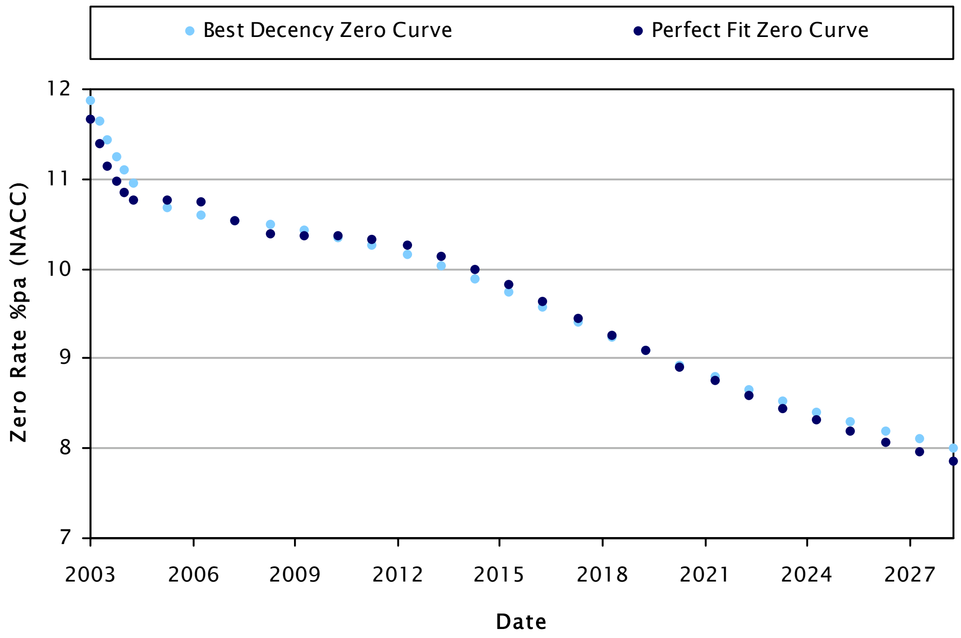
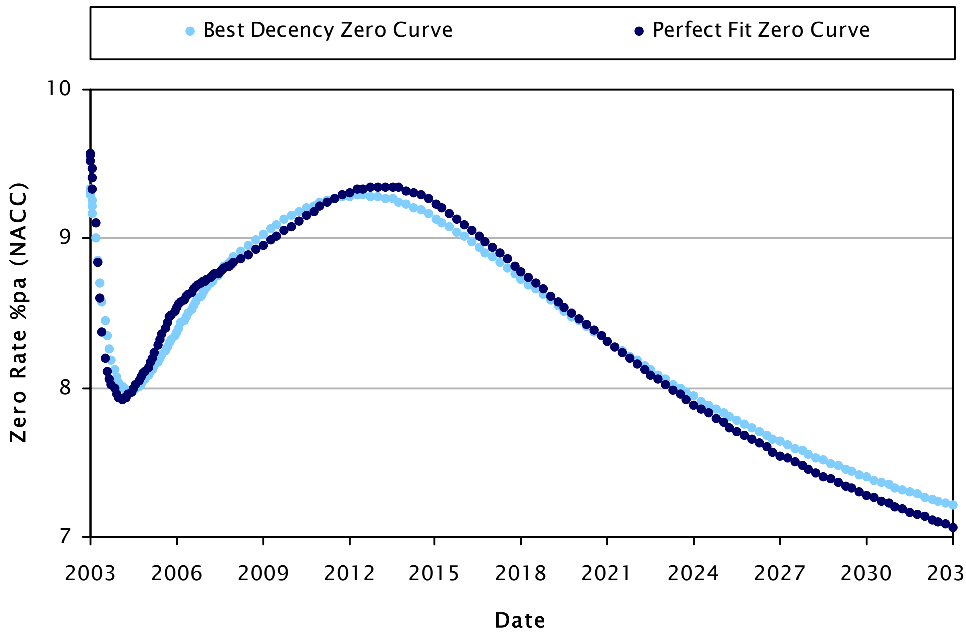


Figure 3. The current Mark VI version of the BEASSA Zero Curve as on 30 September 2003, showing both the Perfect Fit and Best Decency Zero Curves. For this curve a total number of 165 data points is used to compare the older version of the zero curve, like that shown in Figure 2, where 42 data points were used.



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4.3 Using the Current BEASSA Zero Curve to Calculate Credit Spreads

Determining the credit spread for a fixed rate bond using the current Mark VI version of the zero curve involves four steps:

- Firstly, the zero curve has to be interpolated to find the zero rates corresponding to the cash flow dates of the instrument.
- Next these zero rates are used to obtain the risk-free all-in-price[10] of the bond.
- Having the risk-free price, the implied (single) risk-free yield is then calculated for the bond using the Newton-Rhapson method.
- The last step is then to calculate the credit spread by taking the arithmetic difference between the bond's market yield and this synthetically calculated risk-free yield.

Details concerning the last three steps are contained in Appendix A, including an example showing how the price, yield and credit spread are calculated. The first step regarding the different types of interpolation and the accuracy thereof is dealt with in Appendix B.

Based on the detailed discussion and example calculations shown in Appendix A and B, the credit spreads for all available zero curves that have been distributed by BESA since their release on 24 July 2003 up to and including 30 September 2003 (a total of 41 zero curves) were calculated. From these results, some basic success ratio statistics were obtained that should help to motivate a selection of the best methods to use when interpolating the current Mark VI version of the zero curve.

4.4 Accuracy of the Current BEASSA Zero Curve

In Table 2, for each of the two interpolation methods as discussed in Appendix B, the number of times each one of the five bonds had a yield difference of zero is expressed as a ratio to the total number of yield differences calculated. These success ratios indicate that (recognising the error margin of the calculated bond price) the yield calculated with the natural cubic spline interpolation (NCSI) method consistently matched BESA's value for the R150, R194 and R186 bonds. In total, all five bonds matched BESA's value 98% of the time using the NCSI method. The success ratio for the linear interpolation (LI) method was not as good with a total success ratio of 86%. Of all five bonds, only the R194 bond's calculated yield consistently matched BESA's values.

Table 2. The success ratio indicates the number of times each one of the five bonds had a yield difference of zero for two different interpolation methods.

	R150	R194	R153	R157	R186	Total
LI	49%	100%	93%	95%	95%	86%
NCSI	100%	100%	98%	90%	100%	98%

[10] The bond's all-in-price is the price of the bond per R100 nominal, including accrued interest.

Table 3. The maximum absolute yield difference of each bond for the two interpolation methods, expressed in basis points.

	R150	R194	R153	R157	R186
LI	0.2	0.0	0.1	0.1	0.1
NCSI	0.0	0.0	0.0	0.1	0.0

However, closer inspection of the yield differences used to compile Table 2 reveals that even though both the LI and NCSI methods have non-zero yield differences, these values are very small. In Table 3 the maximum of the absolute yield difference for each bond, as determined using either one of the two interpolation methods, is listed. From the table, all yield differences are 0.002 or less, which translates into a maximum difference of 0.2 basis points for any given interpolation method and GOVI index bond. Since yield differences are usually only quoted as integer values (e.g. 0.203% as 20 basis points), this will not influence the yield difference and can therefore be ignored for all practical purposes.

From the results presented in Table 2 and Table 3, it is obvious that both interpolation methods, albeit that the LI method is at a slight disadvantage, are accurate when interpolating the curve. The reason for this is that the older versions of the zero curve have utilised much fewer data points than the current curve and subsequently linear and cubic spline interpolations gave very different results. The increased number of data points gives a much better indication of the shape of the true curve that is being interpolated. If the number of data points in the current zero curve were to be increased even further with any future release, the two interpolation methods would give almost identical results.

Since the two interpolation methods investigated in this report give very similar results, the choice of interpolation method therefore ultimately depends on a trade-off between resources and accuracy.

- If a calculation involving zero curves calls for yields to be as accurate as possible, the NCSI method would be the preferred choice. However, the downside to this would then be that the mathematical routines for these types of interpolation may not be readily available in an application like Excel and might be difficult to implement.
- If the calculation only requires the yield to be accurate when quoting in basis points, the LI method would be the preferred choice due to its simplicity and ease of implementation in applications like Excel.

5 Other Methods to Determine Default Probabilities

Excluding the current method of using bond prices to determine default probabilities, there are two other methods to determine default probabilities [9]. The one method uses historical data and the other uses equity prices.

The historical method uses actual default experiences to compile default rate statistics for different time periods. Combined with recovery rates, which measure the final amount a creditor would receive once an issuer defaults, these statistics can be used to determine the possible losses in a bond's value over a given time period. The use of this method is limited in the SA market due to the lack of a proper history of default events. Internationally compiled statistics [10] can however be used as a proxy for the local market, but the relevance of applying a developed market's statistics to an emerging market is questionable.

The other method to use is a bond issuer's equity prices as a more up-to-date indicator of its default probability. By definition a bond defaults if the issuer's value is less than the face value of the bond. According to Merton [11], the bond-holder therefore effectively has a put option on the assets of the issuer with a strike price equal to the face value of the bond. Using standard option pricing theory - like the well-known Black-Scholes equation - a price can then be determined for the default-risky bond. This approach contains some flaws, the most important being that not all issuers of bonds have equity, hence traded prices to observe and analyse. The model also only allows for default at the maturity of the bond, not before[11].

6 Comparing Credit Spreads Calculated Using Zero Curves and Risk-Free Yield Curves

Using the same risk-free yield curve shown in Figure 1, which was calculated using the yields from GOVI index bonds as on 30 September 2003, the credit spreads for a selection of fixed rate non-government bonds were calculated. This collection was made up from the 48 bonds that had a credit rating from either Fitch, Moody's or CA Ratings. Since neither of the rating agencies provide ratings for all the fixed rate bonds currently traded on BESA, the universe of rated bonds was maximised by ignoring each bond's rating modifier and simply grouping them into the relevant main rating category, e.g. AAA, AA etc. Bonds with less than six months to maturity were excluded since the pricing formula shown in Equations 1 and 3 only applies[12] to bonds with more than six months to maturity.

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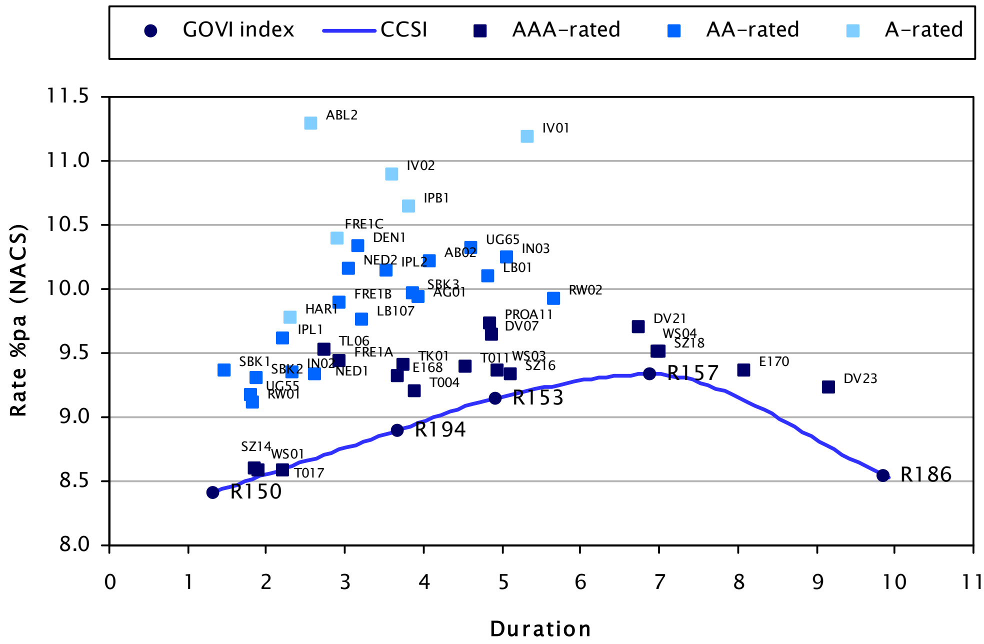
[11] The Moody's KMV risk model is based on this approach [21].

[12] For more information regarding the pricing of bonds and the determination of yields, refer to Appendix A.

Figure 4. The same risk-free yield curve that was shown in Figure 1, is shown here along with the distribution of the market yields of the selected non-government fixed rate bonds. Each bond is labelled and the credit rating for all the non-government bonds is also shown.



The risk-free yield curve as well as the market yields of the selected bonds, including credit rating, are shown in Figure 4. The BBB and BB-rated securitisations by First Rand, FRE1D and FRE1C, are not displayed on the graph due to the wide spreads at which they trade and the effect this would have on the scaling of the vertical axis. Also, non-government bonds with durations less than that of the R150 are excluded since the structure of the risk-free curve to the left of the R150 bond is not very well defined due to the lack of proper risk-free short term instruments. We have already dealt with this issue of the shorter duration instruments in a previous paper.

As noted in the previous report, the AAA-rated bonds seem to be closely spread out in a reasonably well-defined band above the yield curve along the duration axis. The AA and A-rated bonds also form similar bands, each rough band higher than that of the bond grouping of a better credit quality. These rating bands at different distances from the yield curve indicate a strong correlation between a bond's credit rating and its spread, and also illustrate how this measure of default risk is perceived and priced by the market. Using the same bond universe, the credit spreads for these bonds were calculated using their corresponding market yields and the BEASSA Zero Curve from the same date that the risk-free yield curve was constructed. In Figure 5 the distribution of these spreads, calculated using the two different methods, are compared on a bond-by-bond basis. The bonds were grouped according to their credit rating and ranked within each group based on the credit spread, which is calculated using the zero curve. This is similar to the method followed in the previous report, where the credit spreads for bonds were calculated using the risk-fee yield curve.

Figure 5. A comparison between the credit spreads calculated using the risk-free yield curve and the BEASSA Zero Curve, as on 30 September 2003, for a selection of non-government fixed-rate bonds. The spreads are also grouped according to credit rating, with the ZC or YC labels indicating that the calculation was either performed using the zero curve or risk-free yield curve, respectively.

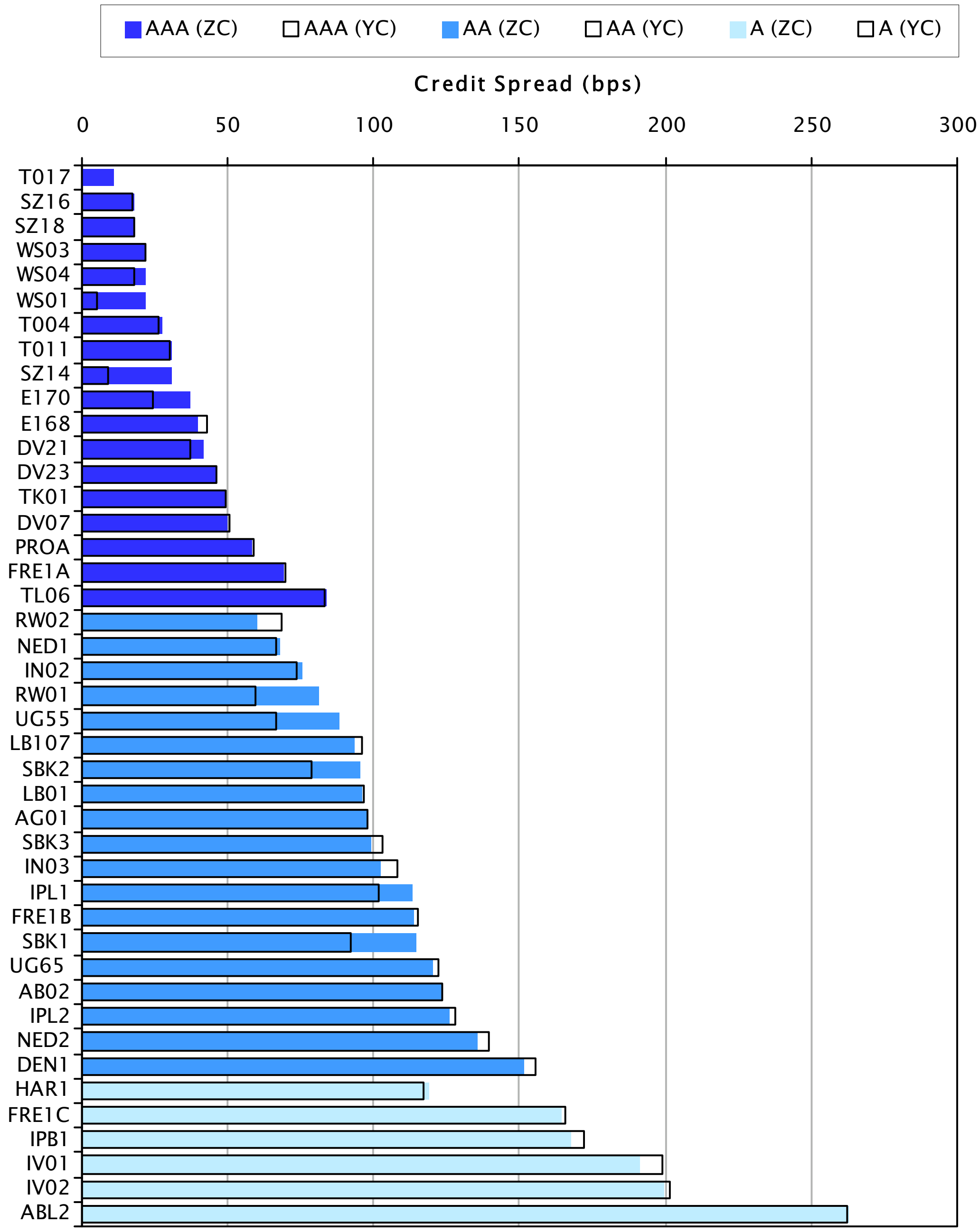
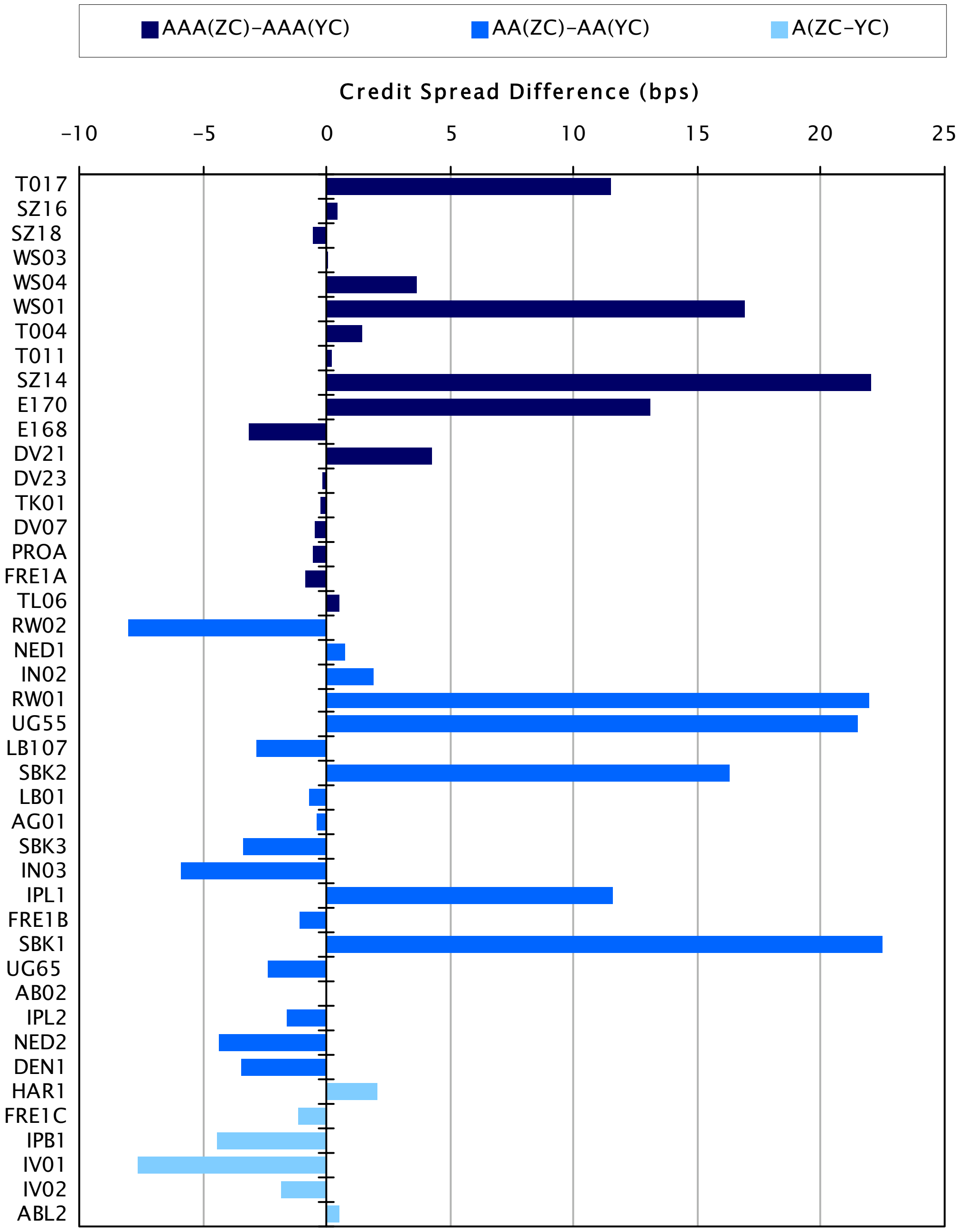


Figure 6. The difference in credit spreads calculated using the zero curve relative to those values calculated using the risk-free yield curve as on 30 September 2003, grouped by credit rating category and ranked in descending order based on each bond's duration.



In Figure 5 the spreads calculated using the BEASSA Zero Curve (risk-free yield curve) are indicated by solid (transparent) coloured bars, or the label ZC (YC) in brackets after the credit rating. Comparing these credit spreads, both methods give similar results for the majority of bonds. Those bonds that have large differences are easily identifiable in Figure 6 where the difference between credit spreads from these two methods is shown bond-by-bond, ranked by duration in ascending order and colour-coded by credit rating.

With the spread differences ranked in this way, it is apparent that the largest differences occur for the very short dated bonds. The four bonds that have a spread difference in excess of 20 bps are the AAA-rated SA Roads Agency's SZ14, AA-rated Umgeni Water Board's UG55, AA-rated Rand Water Board's RW01 and AA-rated Standard Bank's SBK1 bonds. Referring back to the risk-free yield curve and market yield distribution, shown in Figure 4, these four bonds lie more or less on the same vertical line at the very short side of the yield curve, which is anchored by the market yield of the R150 bond. In fact, most of the bonds exhibiting a large difference in spread, calculated using the two methods, are located around the short side of the yield curve. For example, both the Standard Bank SBK2 and Trans-Caledon Tunnel Authority WS01 bonds have differences in spreads in excess of 15 bps. The reason for this may be due to the way the R150 bond is used in the generation of the current version BEASSA Zero Curve.

The R150 is the most liquid of all SA domestic bonds[13] and should therefore, certainly trade at a premium. This will be especially important to analyse in our next report which deals with liquidity issues, as it seems to be a factor at the short end, where all other comparable bonds trade very infrequently. However, as referred to earlier, the R150 has multiple redemption dates and therefore its cash flows are modelled differently compared to the other generating bonds in the bootstrapping process used to generate the zero curve. This would change the shape of the curve in this region and influence the way that other fixed rate bonds are priced from this curve. Therefore, when calculating the implied yield and credit spread from the price, some degree of error would be introduced in the spreads.

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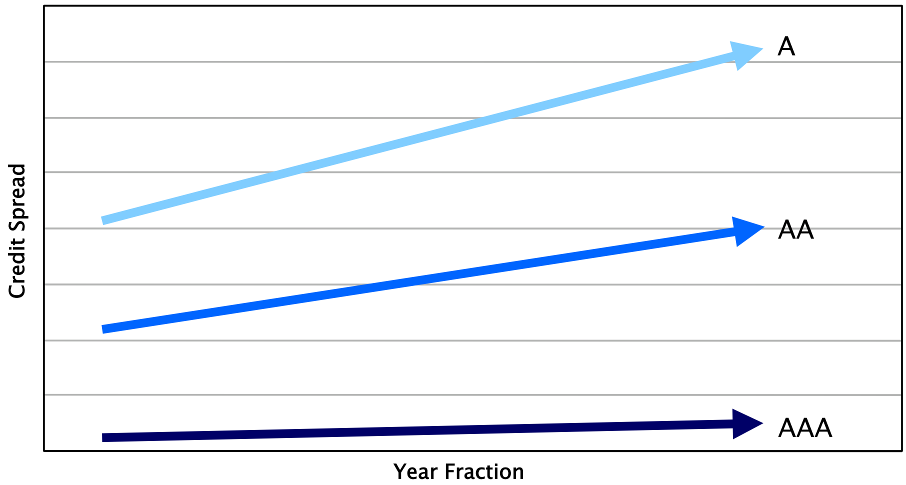
[13] As measured by the daily average volume traded over the one-year period from 1 October 2002 to 30 September 2003. Since this analysis was done, the R150 in fact lost its position as the most liquid government bond.

Figure 7. A conceptual diagram of the term structure of the bonds' yields for each credit rating category, showing how the credit spread of a bond supposedly increases with its maturity term.



7 Analysing the Term Structure of Credit Spreads

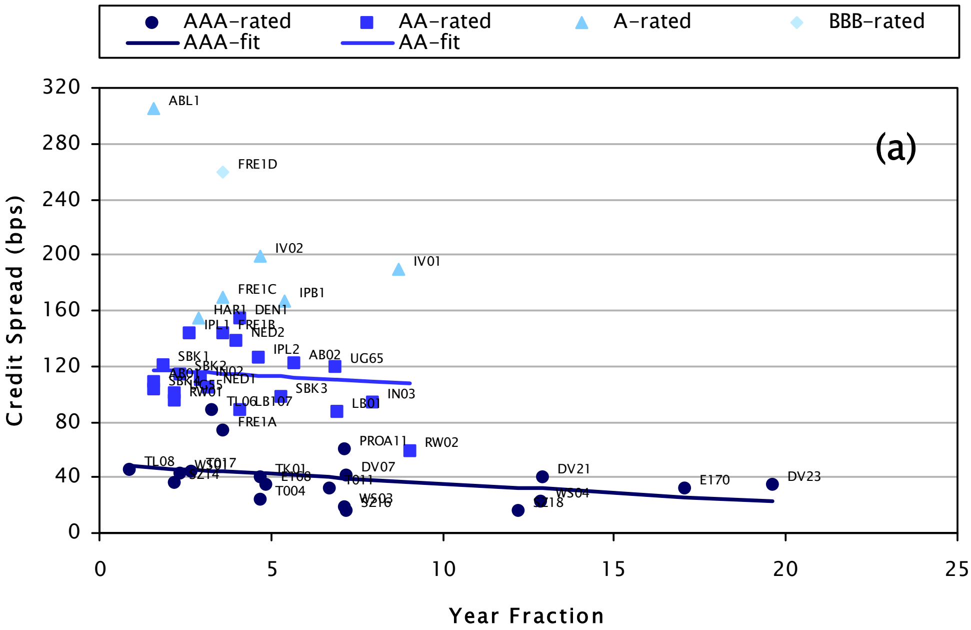
Theoretically, there is a relationship between the term to maturity of a bond and its credit spread, which is referred to as the term structure of credit spreads or the credit curve [12] In Figure 7 a credit curve is shown that conceptually illustrates how the credit spread of bonds with different credit ratings should change in relation to their maturity term.

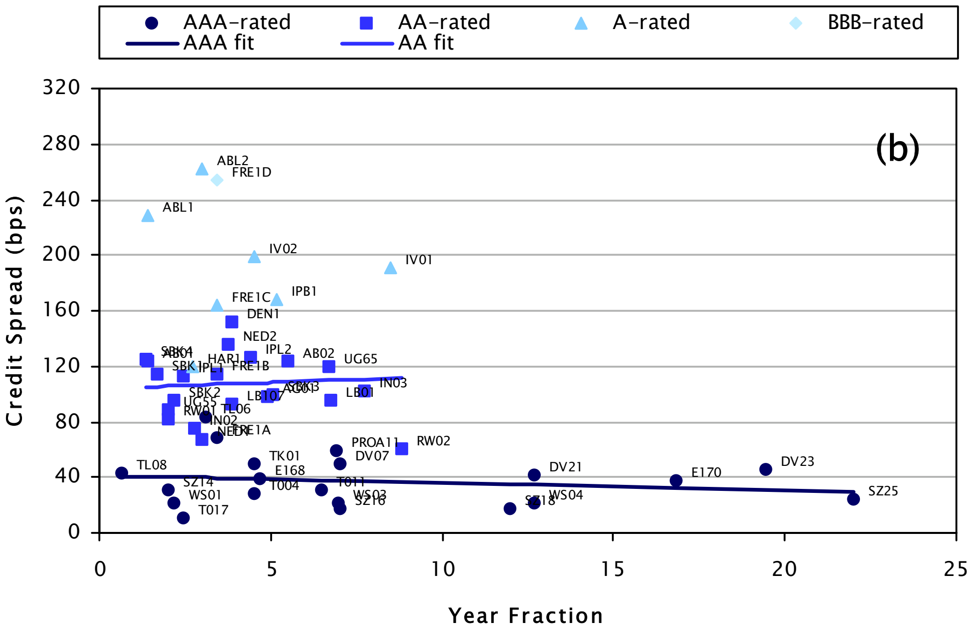
In general, assuming similar liquidity levels, the spread for all bonds would tend to increase for longer maturity terms since the year-on-year probability of a bond defaulting at some point in the future increases with time. Viewing each rating category separately, the spread for AAA-rated bonds would increase at a lower rate than lower credit quality bonds since the a lower rated bond is more likely to default than a AAA-rated bond, over the same time period.

Another risk that influences the short side of the credit curve, is credit migration risk. This risk captures the short-term changes in a bond's credit quality and the effect this has on the value of the portfolio. Since AAA-rated bonds can only be down-rated, the spreads for short maturity term bonds may be wider than mid and longer maturity term bonds. This in turn would change the flat upward sloping AAA credit curve into a downward sloping curve. The effect of credit migration risk on the other rated bonds would not be that noticeable since bonds could be either up- or down-rated. The effect of diversification risk might also impact on the slope of the AAA credit curve. The high degree of correlation between rating and term implies that high credit quality bonds with longer maturity terms would have narrowing credit spreads [13].

Liquidity risk also influences credit spreads. When a bond's liquidity is measured by the size of the bond's issue, many authors report a negative relationship between the spread and the issue size: the larger the size, the larger the issue's liquidity and the lower the spread [14]. When reflecting on the effect these risks have on credit spreads, the credit

Figure 8. The calculated credit spread of a selection of non-government rated bonds on (a) 24 July and (b) 30 September 2003 as a function of their maturity term expressed as a year fraction.





curve shown in Figure 7 can look very different with real data when liquidity issues come into play. This will be investigated in the next paragraph using the term structure of credit spreads calculated on two different dates.

Using the credit spreads calculated in the previous paragraph with the BEASSA Zero Curve and NCSI method, each bond's spread as a function of maturity year fraction is plotted in

Figure 8(a). A similar graph using credit spread data for 24 July 2003 is also plotted in Figure 8(b). As was done in the previous paragraph, BBB and BB-rated securitisations were also left out of both graphs due to the wide spreads at which they trade and the effect this would have on the scaling of the vertical axes. In both graphs, bonds are colour-coded to indicate their credit rating.

The banding effect, referred to in the previous paragraph, is clearly observable for the AAA and AA-rated bonds. Too few bonds in the lower credit categories, like the A-rated category, make it difficult to observe any trend. Based on this banding effect, a trend line can be fitted to the AAA and AA-rated bonds using linear regression. These regression lines are indicated in both graphs in Figure 8. In the AA-rated category the Rand Water RW02 bond was taken as an statistical outlier in the regression analysis based on its narrow spread compared to the rest of the AA-rated bonds.

For both the AAA and AA-rated bonds on both dates the R-square values are very small, indicating that neither one of the credit curves qualifies as a true indication of the term structure of either one of the two credit rating categories. For both credit curves on 24 July 2003, the R-square value is approximately 0.145, while for the curves on 30 September 2003, the R-square value is worst at approximately 0.03. This implies that these credit curves can in fact either be upward, flat or downward sloping and that no conclusive comments can be made on these curves.

In the literature [14], various arguments and evidence exists for both downward and upward sloping credit curves. Supposing the credit curve for the AAA-rated bonds is really downward sloping, possible reasons for this could be as follows:

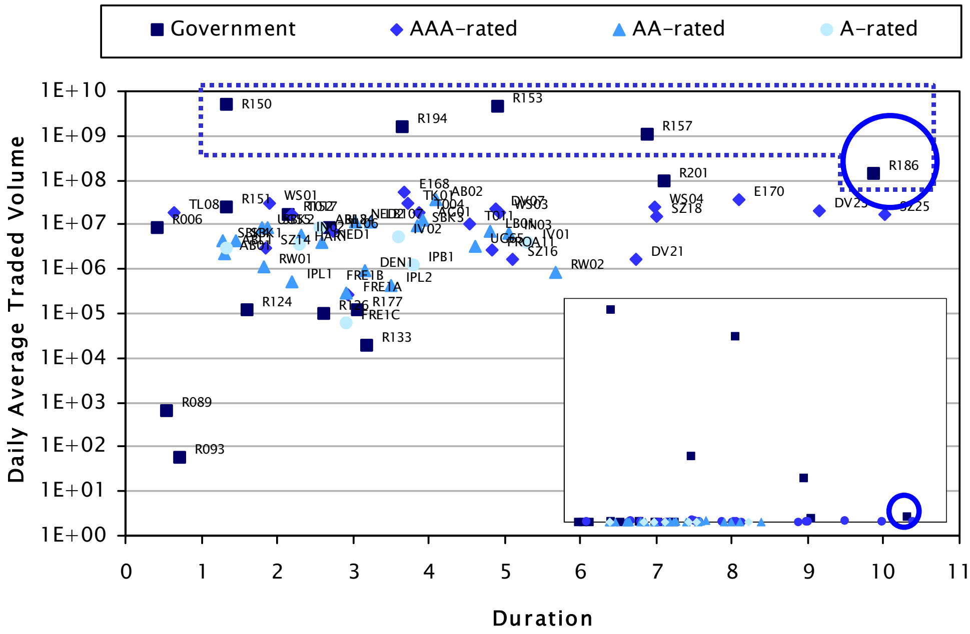
Firstly, since safer firms issue longer-dated bonds, these would trade at a lower yield than the shorter-dated bonds. Examples of these safer long bonds are the Development Bank's DV23 and Eskom's E170 bonds. Although these bonds are not guaranteed by the government, the issuers of these bonds are regarded as highly credit-worthy and subsequently both bonds would trade at a narrower spread since credit risk is minimal.

Secondly, our bond market is renown for not being very efficient, especially due to low liquidity. Many of the large life insurers purchase long-dated government and AAA-rated bonds, on a buy-and-hold basis for the purposes of their liability matching. As a result of this, the liquidity of these bonds is low, subsequently leading to a narrowing in spreads with buyers willing to pay a high premium for a limited number of these bonds when they become available in the market. Another factor influencing liquidity, is also the age of the bond. For example, the DV23 bond from the Development Bank was only issued in mid-February 2003 and would therefore be bought in high volumes by life insurers, therefore pushing down the liquidity premium of the credit spread.

In Figure 9 the Daily Average Traded Volume of all government as well as non-government fixed rate bonds as a function of each bond's duration is shown. The daily average of the traded volume was calculated by aggregating the volume that each bond traded between 30 September 2002 and 2003, and dividing this number by 252 (the number of business days in a year).

Each bond's duration was taken as on 30 September 2003. Using the traded volume as a measure of how liquid each bond was over a one-year period, the graph in Figure 9

Figure 9. The Daily Average Traded Volume of all government as well as non-government fixed rate bonds as a function of each bond's duration as on 30 September 2003.



illustrates the liquidity of each bond as a function of its duration. In the graph there is a distinction made between government bonds and the three main credit rating categories for non-government bonds. Also note that the vertical axis of the graph is calibrated in a log-scale. This is done since the volume of traded government bonds is of a higher magnitude than the non-government bonds. For example, compared to the most liquid bond (the R150) which traded at a daily average of 5.3 billion, the most liquid non-government bond (the Eskom E168) only traded at a daily average of 53 million, a factor of a hundred times less. At the bottom right hand corner of Figure 9 an inlay graph is shown where the log scale of the vertical axis has been replaced by a linear one. Presenting the liquidity of the different bonds in this way, clearly illustrates just how liquid the five GOVI index bonds (enclosed by the dashed line in Figure 9) are compared to other bonds.

In Figure 9 almost all AAA-rated bonds traded in volumes of between 10 and 100 million, indicate the same level of liquidity. Even the long-dated E170 bond was very liquid and did not agree with the buy-and-hold argument mentioned earlier. However, the DV23 bond confirms the earlier statement that newly-issued bonds are generally very liquid.

Focussing on the non-government bonds, the credit rating of a bond also influences its liquidity. Bonds with high credit ratings are more liquid since these bonds are popular for investment managers to include in their portfolios. As the bond's credit rating decreases, it becomes less popular and therefore less liquid since only managers specifically looking to pick up some credit exposure in their portfolios trade these bonds. Many investment managers' mandates also excludes them for holding any lower-rated bonds.

An interesting point is the low trade of the R186 (indicated by the circle in Figure 9). Most of the "talked about" inefficiency in the yield curve is at longer durations particularly related to the R186, where demand is far higher than government supply. A liquidity adjustment may be appropriate when including this bond in the yield curve due to this inefficiency. We cover this point in more detail in our next paper. The point, however, highlights, that a risk-free curve may well have liquidity bias away from the more commonly assumed liquid bonds.

8 Conclusions

In general, it was shown in this report how the newly-released BEASSA Zero Coupon Yield Curve can be used to calculate credit spreads for non-government bonds and analyse the term structure of these spreads.

In order to determine the credit spread, the zero curve was used to determine the price of a fixed rate non-government bond. This was done by breaking each bond down into a series of zero coupon bonds, discounting each cash flow separately using the zero curve and then aggregating them to determine a price. Since the zero curves are derived from a set of liquid government bonds, any non-government bond priced using these curves will reflect the risk-free price as opposed to the bond's market price. These prices can then be transformed into yields using the standard bond pricing formula. The implied yield therefore also reflects the risk-free yield, making it possible to calculate a credit spread using the bond's market yield.

The original version of the BEASSA Zero Curve was such that the government bonds that were used to generate the zero curve could be re-priced exactly from the curve's given data, using the quartic spline interpolation method. Subsequently, various other versions of the curve were released that did allow the use of this advanced interpolation method, but forced users to employ more basic methods like linear or cubic spline interpolation. Whereas these curves initially had too few data points to accurately re-price the generating government bonds, in the latest release this number has been increased enough to allow for accurate interpolation using either linear or cubic spline interpolation. Both methods give similar results - price errors of tens of Rand per R1 million nominal and yield errors in the order of tenths of a basis point. In determining credit spreads, these errors are quite acceptable since these values are usually quoted in basis points. Other applications may require the price and yield errors to be smaller. In these cases, users are advised to obtain the zero curve in its original, non-interpolated, format and to calculate prices or yields using the quartic spline interpolation method.

In an application using the credit spread calculation method illustrated in this paper, the credit spreads for a selection of non-government rated bonds were calculated on two separate dates. These spreads were then graphed as a function of each bond's maturity term to reveal the term structure of the credit spreads as grouped by credit rating. However, linear regression on the term structure of the AAA and AA-rated bonds, revealed no statistically significant trend and was unable to confirm whether the observable credit curve for both rating categories was in actual fact either upward or downward sloping, as predicted by various authors.

On closer inspection of these bonds, liquidity risk seems to introduce uncertainty in accurate pricing at both the short end of the yield curve (primarily influenced by the R150) and the long end (primarily influenced by the R186) for vastly different reasons. Liquidity risk will need to be separated from credit risk to allow for more accurate pricing. We will carry out this analysis in our next report.

A Pricing Bonds, Determining Yields and Calculating Credit Spreads

Ignoring the fact that interpolated zero rates from the current Mark VI curve may lead to inaccuracies when calculating the prices for GOVI index bonds, this appendix only focuses on the mathematical and technical details behind pricing these bonds and calculating yields and credit spreads. Interpolation issues regarding the Mark VI curve are dealt with separately in Appendix B. To avoid any interpolating, the older Mark III Perfect Fit version was used for the examples in this Appendix. Even though bootstrapped zero rates, instead of interpolated zero rates, are used in these example calculations, the methodology to calculate prices, yields and credit spreads remains the same.

The SA bond market has quite a few peculiarities making it unique compared to international bond markets. The most important ones, that directly influence the way a fixed rate bond is priced, are given here.

- Firstly, all bonds listed on BESA are traded on a yield-to-maturity (YTM) basis and settled three business days after the deal date at the bond's all-in price. If the deal date is T, the settlement date is then generally referred to as the $T+3$ date.
- Furthermore, the bond's all-in price includes an accrued interest portion, which is the interest that has accumulated since the last interest payment on the previous coupon date, but not including the settlement date. Subtracting this accrued interest from the all-in price, gives what is called the bond's clean price.
- Also, if the settlement date is in the period between the bond's previous coupon date and the book close date prior to the next coupon date, the settlement date is referred to as being in the cum-interest period. The buyer then pays the seller the all-in price, consisting of the market price plus the accrued interest, reimbursing the seller for the loss of interest on the issuer's next coupon date. If however, the settlement date is in the period between the book close date prior to the next coupon date, the settlement date is referred to as being ex-interest period. In this case the seller, and not the buyer, receives the full interest payment on the issuer's next coupon date. This loss of interest for the buyer is however reflected in the bond's price. These two different interest periods are graphically illustrated in Figure 10.

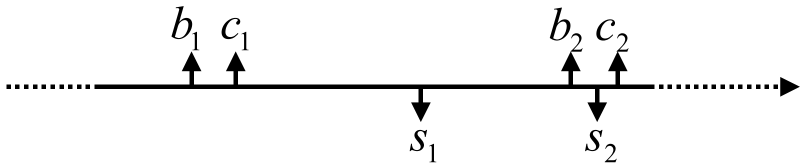
A.1 Pricing Bonds Using the Zero Curve

According to the JSE's Gilt Clearing House (GCH) formula, the all-in price P of a fixed rate bond traded with more than six months to maturity at an yield in NACS format of y, paying a fixed coupon rate c per year is given by [15]:

$$P = \left[\sum_{i=1}^{n} \frac{cf}{a} D(y)^{-i} + fD(y)^{-n} + \delta \frac{cf}{a} \right] D(y)^{-\frac{e}{j}} \qquad (1)$$

where

Figure 10. Consider the timeline with c_1 and c_2 indicating a bond's coupon dates and b_1 and b_2 the bond's book close dates. The settlement date s_1 is referred to as being in the bond's cum-interest period, while the settlement date s_2 is in the ex-interest period.



$$D(y) = 1 + \frac{y}{a} \tag{2}$$

with n the number of full coupon periods between the settlement and maturity dates, f the face value of the bond, a the number of coupon payment per year, e the number of days from the settlement date to the next coupon payment and j the number of days in a coupon period. According to the GCH formula, bonds are priced on R100 nominal, rounded to five decimals, to get prices like 85.77155 for example. The factor δ takes a value of one when the settlement date falls in the cum-interest period, or zero when it coincides with the ex-interest period. As a matter of interest, Equation (1) can also be simplified by substituting the summation with an annuity factor [16]. If the bond has less than six months to maturity left, it is priced according to money market standards using different formulas, which is beyond the scope of this report.

For zero rates quoted in the NACC format, the pricing formula given in Equation (1) should be transformed as follows:

$$P = \delta \frac{cf}{a} D(t'', t_1) + \frac{cf}{a} \sum_{i=2}^{n} D(t'', t_i) + fD(t'', t_n) \tag{3}$$

where

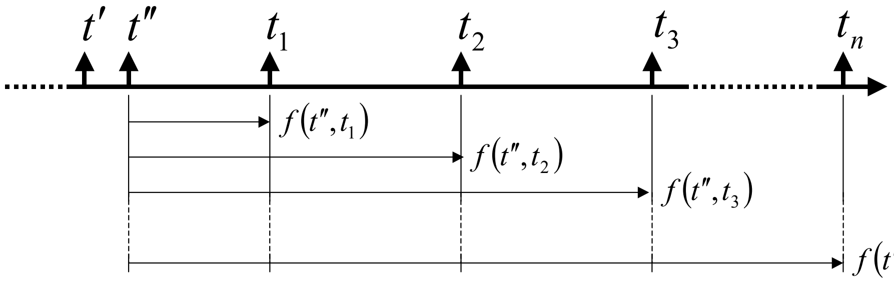
$$D(t'', t_j) = \exp\left(-(t_j - t'') f(t'', t_j)\right), \quad j = 1:n \tag{4}$$

The parameters given in Equation (3) remain the same as those originally defined in Equation (1) except that t'' now represents the bond's settlement year fraction and t_j, $j=1:n$ corresponds to the cash flow year fraction. The discounting factors are calculated using the year fraction difference and the period forward rate for the corresponding period, given by:

$$f(t'', t_j) = \frac{r_0(t_j) t_j - r_0(t'') t''}{t_j - t''}, \quad j = 1:n \tag{5}$$

The year fraction is calculated with reference to the bond's deal date t' as base date and is obtained by dividing the serial day difference between the settlement/coupon/maturity date d and deal date d' by 365:

Figure 11. A graphical illustration of the method in which forward rates are used to discount future cash flows to the settlement date t'' and not the deal date t'.



$$t = \frac{d' - d}{365}, \quad t \in [t', t'', t_j] \tag{6}$$

In theory, future cash flows can be discounted using the zero rates directly. In practice however, this is not possible since the bonds are only settled in price three business days after the deal date. Therefore the future cash flows are actually being discounted to the near-future value of the bond on its settlement date and not to the present value on its deal date. Figure 11 illustrates how these different period forward rates relate to the cash flow and settlement dates.

Example

Consider determining the all-in price of the R194 government bond using the Perfect Fit zero curve on 15 November 2002. According to BESA, the R194 bond traded at a yield of 11.290% on that day and was settled on 20 November, three business days later, at a price of 97.17836. Using Equations (3) to (6), the bond price can then be calculated as illustrated in Table 4.

The first entry in the first column is the settlement date of the bond, 20/11/2002, given the deal date. The rest of the entries in the first column correspond to individual cash flow dates, including both coupon and principal payments. The second column contains the zero rates that correspond to the cash flow dates given in the previous column. In the third column the period forward rates, calculated using Equation (5), are shown. In the next column these forward rates are then used to calculate discount factors using Equation (4). These factors discount each cash flow to its value on the settlement date of the bond, as shown in the last column where the present value of the cash flow is shown, rounded to five decimal places according to the specifications of the GCH formula. Aggregating these cash flows at the bottom of column five gives the all-in price of the bond as 97.17836, matching BESA's value.

Table 4. An example illustrating how to price the R194 bond for 15 November 2002 using the BEASSA Perfect Fit Zero Curve.

Date	Zero Rate (%)	Period Forward Rate (%)	Discount Factor	Discounted Cash Flow
20/11/2002	12.69700			
28/02/2003	12.37880	12.36289	0.96670	4.83348
31/08/2002	11.88000	11.86562	0.91181	4.55905
28/02/2004	11.52710	11.51452	0.86356	4.31780
31/08/2004	11.32956	11.31904	0.81744	4.08722
28/02/2005	11.28018	11.27166	0.77366	3.86830
31/08/2005	11.31905	11.31227	0.73010	3.65050
28/02/2006	11.35915	11.35356	0.68934	3.44669
31/08/2006	11.33181	11.32686	0.65165	3.25825
28/08/2007	11.22151	11.21678	0.61896	3.09482
31/08/2007	11.05101	11.04629	0.58972	2.94861
28/08/2008	10.89212	10.88740	0.56299	59.11365
			All-in price	97.17836

A.2 Determining Yield Given a Bond's Price

In Equations (1) and (3) two different formulas to value a fixed rate bond were shown, one where the yield is in NACS format and the other in NACC. By equating these two expressions the implied yield in NACS format can be numerically solved, given the zero rates in NACC format, coupon rate, face value, settlement as well as the maturity date. This is done using an appropriate root-solver method like the Newton-Rhapson iteration algorithm [17]. The GCH formula does not specify the terms of this conversion, but market practice defines the implied yield as that yield that produces an unrounded all-in price equal to the target all-in price. The implied yield is then rounded to the three decimal places in percentage format according to BESA's specifications.

The Newton-Rhapson method works as follows. Returning to Equation (1), the bond pricing formula in NACS units can be expressed as a simple function, assuming that all other variables in the formula, except the yield y, can be treated as constant values:

$$P = f(y) \tag{7}$$

with P the all-in bond price as calculated using Equation (3). Now define a new function $h(y)$ such that:

$$h(y) = f(y) - P \tag{8}$$

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having a derivative $h'(y)$:

$$h'(y) = f'(y) \tag{9}$$

In general the Newton-Rhapson method is then given by the following iterative algorithm:

$$y_{i+1} = y_i - \frac{h(y_i)}{h'(y_i)} \tag{10}$$

with y_i is an initial guess for the yield $h(y_i)$ and $h'(y_i)$ the function and function derivative values at the yield y_i. Iteratively repeating this calculation improves the approximation of the yield, y_{i+1} with regards to the previous approximation y_i for the yield. The iteration algorithm in Equation (10) is repeated until y_{i+1} equals y_i, both rounded to three decimal places. Yield values obtained from this are referred to as the bond's implied yield since they were calculated indirectly from its price.

Example

The risk-free price that was calculated in the previous example is now used to reverse the yield of the R194 bond out from the GCH bond pricing formula using the iterative Newton-Rhapson root solver. Given the risk-free all-in-price of 97.17836 for the R194 bond on 15 November 2002, as well as the relevant bond details like the coupon rate, maturity, coupon and book close dates, the implied yield for that day is calculated using the algorithm in Equation (10). A summary of the results from this iterative calculation is given in Table 5.

Table 5. An example illustrating how the implied yield for the R194 bond on 15 November 2002 is calculated from the bond's all-in-price by using the Newton-Rhapson iterative method.

i	y_i	y_{i+1}	$\lvert y_{i+1}-y_i \rvert$
1	1.000	8.943	7.943
2	8.943	11.155	2.212
3	11.155	11.290	0.135
4	11.290	11.290	0.000

Taking the first row of entries, the second column contains the initial root guesstimate, the third column the results of the first iteration and the last column the absolute difference between the two values. Applying the iterative algorithm a second time with the first row value in the third column as input, results in the second row of entries. The iteration is continued until the absolute difference in the last column is zero. In this example this happens after the fourth iteration, converging to an implied yield of 11.29%.

A.3 Calculating Credit Spreads

Having calculated the implied risk-free yield for a bond, all that is left to obtain the credit spread S_i for trading day i is to subtract the implied yield y_i from that bond's corresponding market yield Y_i :

$$S_i = Y_i - y_i \tag{11}$$

Since the bond's implied yield was calculated using a zero curve derived from highly liquid government bonds that are considered free of credit risk, it implies the bond's yield had it also been a very liquid and risk-free instrument. It will therefore have a lower value than the market yield of the bond. This difference then reflects the yield premium for credit and liquidity risk as viewed by the market.

Example

The credit spread for the R194 bond on 15 November 2002, is calculated using the market yield and the implied risk-free yield calculated in the previous example. According to BESA, the bond traded at a yield of 11.29%. This is the same value that was obtained when the implied yield was calculated in the previous example. Furthermore, from Equation (11) the credit spread is zero as would be expected since the implied yield was calculated from a risk-free price that was in turn calculated from a zero curve that was bootstrapped from the GOVI index bonds.

B Interpolating the Zero Curve

Whereas the Mark IV version had too little data points to properly interpolate for unknown zero rates, the Mark VI version has enough data points to allow for the use of simple interpolation methods to obtain reasonably accurate zero rates. Although many different interpolation methods exist to determine unknown function values between given values, the two most popular piecewise methods are linear and cubic spline interpolation. In general all interpolation methods work by fitting a curve to two or more known data points and then applying this function to the required input to obtain the function value.

B.1 Linear Interpolation

The simplest form of interpolation is where a zero rate $z_i(t)$ is estimated by fitting a straight line of the form:

$$z_i(t) = a_i + b_i t \qquad (12)$$

to any given pair of known zero rates and curve dates. The coefficients a_i and b_i are calculated from the values of the known zero rates. Although this is very simple method to use, one disadvantage is that it is impossible to fit a smooth curve through the given zero rates. Mathematically, the reason for this is that the derivative of the linear segments at the end-points of these segments is discontinuous. To obtain a smooth curve, a more advanced interpolation method should be used that fits a higher order polynomial between each pair of zero rates.

B.2 Cubic Spline Interpolation

As opposed to linear interpolation where a straight line, or first order polynomial, is fitted between each pair of zero rates and curve dates, a third order polynomial is used in cubic spline interpolation to estimate the zero rate $z_i(t)$:

$$z_i(t) = a_i + b_i t + c_i t^2 \qquad (13)$$

The coefficients a_i , b_i and c_i are calculated from the values of the known zero rates. Various types of cubic splines exist, all of them differentiated by specific mathematical constraints placed upon them. The most popular member of this family is the natural cubic spline since it generates a very smooth curve because both the first and second derivatives at the end-points of every curve segments are continuous. If there are n data points, the above statement would be mathematically expressed as:

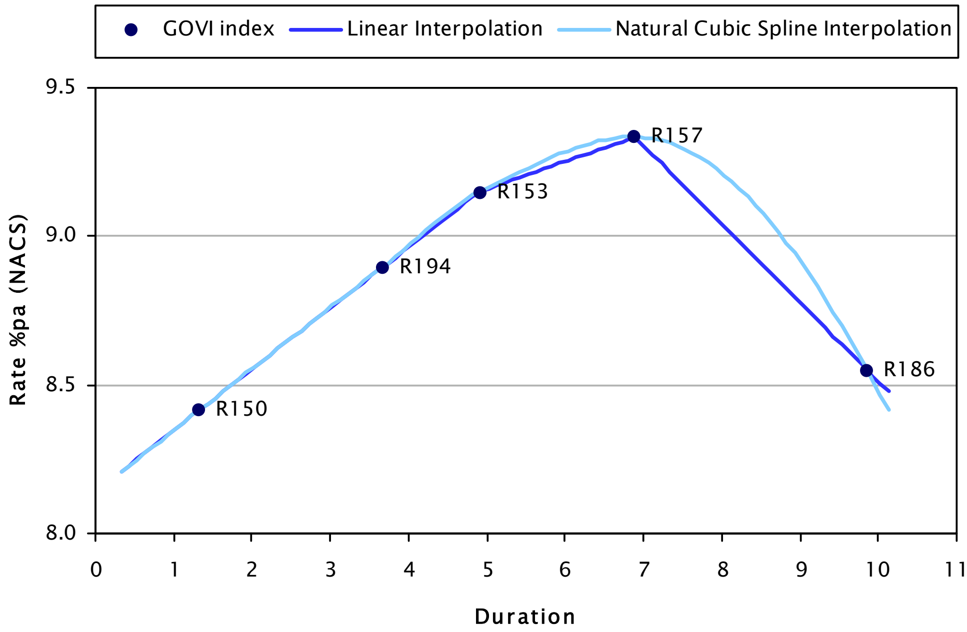
$$z_i'(t_i) = z_{i+1}'(t_i) \ ; \ z_i''(t_i) = z_{i+1}''(t_i) \qquad (14)$$

Furthermore, the second order derivatives at end-points of the spline are also set to be zero:

$$z_1''(t_0) = z_n''(t_n) = 0 \qquad (15)$$

In Figure 12 a comparison is shown between linear and cubic spline interpolation for the yields of the GOVI index bonds on 30 September 2003. Note that both methods only give

Figure 12. A comparison between the linear and natural cubic spline interpolation methods for the yields of the GOVI index bonds on 30 September 2002 illustrating the differences between these two interpolation methods.



agreeable results when the majority of data points are either uniformly increasing or decreasing. The largest differences between the two methods occur at turning points where the linearly interpolated curve has discontinuities and the cubic spline interpolated curve fits smoothly, albeit over shooting, through the data points. Another cubic spline that can be obtained by adding and adjusting mathematical constraints is the constrained cubic spline [2]. Although the natural cubic spline is a very smooth curve, its inherent smoothness also makes it prone to oscillation between data points. The constrained cubic spline solves this problem by sacrificing its smoothness in return for stability. This type of spline is useful when there is not a large number of data points, resulting in them being widely spaced apart which increases the risk for oscillation.

However, since the current format of the zero curve contains a large number of data points, interpolation methods for the Mark VI curve will be limited to linear and natural cubic spline interpolation. Since the terms "linear interpolation" and "natural cubic spline interpolation" will be frequently referred to for the remainder of this report, the acronyms LI and NCSI will be used instead for these two interpolation methods.

B.3 Calculating Prices and Yields Using the Zero Curve and Different Interpolation Methods

In Table 6 and Table 7 the price and implied yield is calculated for each bond in the GOVI index using both the LI and NCSI methods and the zero curve on 30 September 2003. In the fourth and seventh column of both tables, these calculated prices and yields are compared to BESA's values by calculating the arithmetic difference between them. The results are referred to as the price difference and the yield difference.

Comparing the price differences for the two tables, the biggest price difference occurred when the R157 bond was priced using zero rates obtained by the NCSI method. This difference of 0.00140 is in the order of R14 per R1m nominal, which is negligibly small and may be admissible for most pricing purposes. When comparing the yield differences, it is noticeable that for both interpolation methods the yield difference is zero for all bonds, even though the corresponding price differences are not. The reason for this is twofold. Firstly, since yields are rounded to three and prices to five decimal places, some margin of error on the bond price as input to the Newton Rhapson iteration algorithm is allowed. Secondly, every bond's price has a certain sensitivity to yield changes and therefore these error margins change from one bond to the next, making one bond price more sensitive and the other less.

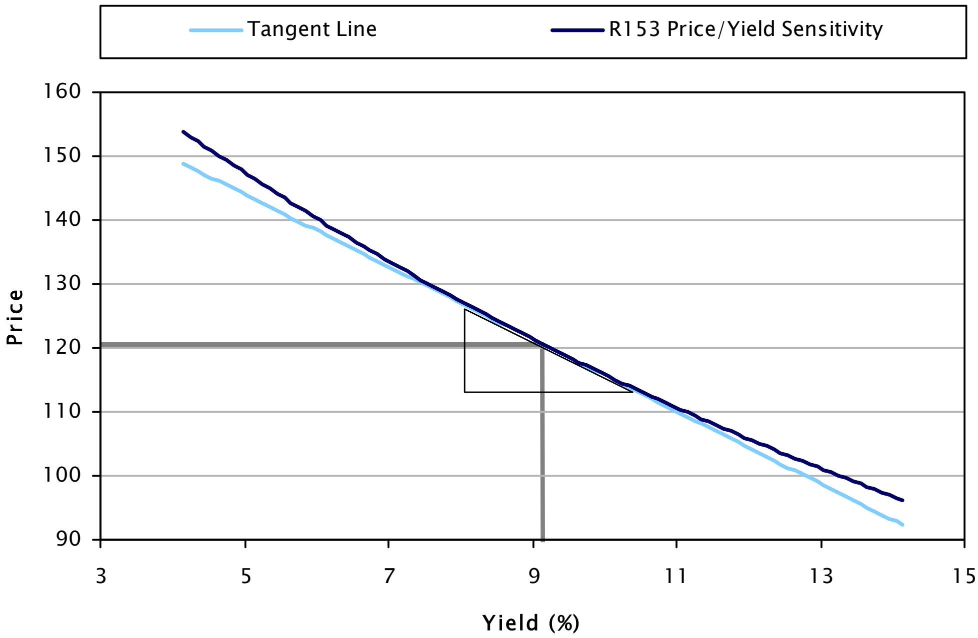
Table 6. Pricing and yield results using the linear interpolation (LI) method.

GOVI index	Calculated All-in-price	BESA's All-in-price	Price Difference	Calculated Yield	BESA's Yield	Yield Difference
R150	105.74635	105.74644	0.00009	8.415	8.415	0.000
R194	104.85415	104.85410	0.00005	8.895	8.895	0.000
R153	120.56288	120.56270	0.00018	9.150	9.150	0.000
R157	130.27561	130.27496	0.00065	9.335	9.335	0.000
R186	122.49886	122.49828	0.00058	8.550	8.550	0.000

Table 7. Pricing and yield results using the natural cubic spline interpolation (NCSI) method.

GOVI index	Calculated All-in-price	BESA's All-in-price	Price difference	Calculated Yield	BESA's Yield	Yield difference
R150	105.74640	105.74644	0.00004	8.415	8.415	0.000
R194	104.85411	104.85410	0.00001	8.895	8.895	0.000
R153	120.56410	120.56270	0.00140	9.150	9.150	0.000
R157	130.27478	130.27496	0.00018	9.335	9.335	0.000
R186	122.49842	122.49828	0.00014	8.550	8.550	0.000

Figure 13. A graph of the sensitivity of the R153 bond's all-in-price to a change in yield as on 30 September 2003.



B.4 Calculating Error Margins

In Figure 13 a graph is shown illustrating the change in the R153 bond's all-in-price as a function of its yield on 30 September 2003. Considering that a bond price is the summation of a set of discounted cash flows, an increase in yield decreases the discounting factors and therefore further discounts future cash flows. The summation of these discounted values therefore gives a lower bond price, which clarifies the negative correlation between prices and yields. The sensitivity of the bond's price to a change in yield can be approximated by calculating the slope of a tangent line to the price vs. yield curve, as illustrated in the graph. Since the price/yield function is not a linear one, the curve is not a straight line, but has curvature. Therefore the tangent line is only a good approximation of the slope of this function for small changes around the given yield.

This slope can be determined in numerous ways. One method is to analytically differentiate the bond price formula given in Equation 1 with respect to the yield and then calculate the function value of this derivative. The second method is to calculate the bond price for a small interval around the given yield using Equation 1. Provided that this interval is small enough and that curvature in this interval is negligibly small, the curve would approximate a straight line of which the slope can be determined using linear regression analysis. The last method involves transforming the value of the bond's modified duration into the slope of the tangent line. The modified duration MD gives the percentage price change of a bond for a one unit change in yield and is defined as:

$$MD = -\frac{1}{P}\left(\frac{\Delta P}{\Delta Y}\right) \tag{16}$$

with ΔP the bond's price change, P the all-in-price and ΔY the change in yield. For small changes in the yield, the above equation can be rewritten as follows:

$$\frac{dP}{dY} = -MD \times P \tag{17}$$

where dP/dY is then the slope of the tangent line. For example, from BESA's data, the R153 bond was priced at 120.56720 and had a modified duration of 4.697 on 30 September 2003. Inserting these values into Equation 17, gives a slope value of -5.663. Therefore, a change of 0.001 in the yield would change the bond's price by 0.005663 or 0.00566, considering the rounding conventions for the bond prices. Reversing this argument it indicates that for the R153 bond, the calculated bond price can differ by as much as 0.00566 from the price before this would actually be reflected as a change of 0.001 in the calculated yield.

This error margin agrees with the results presented in Table 6 and Table 7 where the price differences for the R153 bond using the LI and NCSI methods were 0.00018 and 0.00140 respectively. Both these values are less than the error margin of 0.00566 and therefore explains why these price differences did not reflect in the calculation of the implied yield. Similar error margins can also be calculated for the other bonds in the GOVI index. These calculated values, including that of the R153 bond, are shown in Table 8 and compared to the price differences of both interpolation methods. For every bond in the GOVI index, the price difference from both the linear and cubic spline interpolation method is well below the error margin and confirms that yield differences for all bonds should be zero even though corresponding price differences are not.

Table 8. A comparison between the prices differences and the error margins on 30/09/2003

GOVI index	LI method	NCSI method	Error margin
R150	0.00009	0.00004	0.00135
R194	0.00005	0.00001	0.00367
R153	0.00018	0.00140	0.00566
R157	0.00065	0.00018	0.00857
R186	0.00058	0.00014	0.01158

C Interpolating the Zero Curve Using Quartic Splines

From bootstrapped zero rates and cash flow dates, an instantaneous forward rate curve can be calculated that is then interpolated using a quartic, or fourth order spline. This is done since it is generally accepted that a zero curve should be interpolated in such a way as to obtain the smoothest possible forward curve [6,8]. This is considered vital since the forward curve magnifies blemishes in the zero curve and a smooth forward curve therefore necessarily results in a very smooth zero curve. The parameters from the forward curve interpolation are then used to construct a smooth continuous zero curve.

Below the process for fitting a quartic spline to bootstrapped is illustrated in three steps. In the first step, the zero rates are transformed into instantaneous forward rates. In the next step these forward rates are then interpolated using a quartic, or fourth order, spline. In the last step, the parameters from this interpolation are then used to construct the interpolation for the given zero rates.

Step 1: Transforming Zero Rates to Instantaneous Forward Rates

The zero rates z_j between $j=0$ and $j=n$ that are obtained from the bootstrapping process are transformed into approximated instantaneous forward rates q_j using the following expression:

$$q_j = z_j + t_j\left(b_j + c_j t_j\right) \tag{18}$$

where t_j refers to the time period and q_j has a value between $j=2$ and $j=n-1$. At $j=0$ the forward rate q_0 is simply equal to the zero rate z_0. At $j=1$ the forward rate is approximated by the slope of the zero curve between z_1 and z_2:

$$q_1 = z_1 + \frac{\Delta z_2}{\Delta z_1} \tag{19}$$

The same applies to determining the forward rate at $j=n$ using the slope of the zero curve between z_{n-1} and z_n:

$$q_n = z_n + t_n \frac{\Delta z_n}{\Delta t_n} \tag{20}$$

The coefficients b_j and c_j in Equation (18) are determined from fitting a quadratic polynomial $z_j(t)=a_j+b_jt+c_jt^2$ to a set of three successive zero rates. In matrix format this calculation can be expressed as:

$$\begin{bmatrix} 1 & t_{j-1} & t_{j-1}^2 \\ 1 & t_j & t_j^2 \\ 1 & t_{j+1} & t_{j+1}^2 \end{bmatrix} \begin{bmatrix} a_j \\ b_j \\ c_j \end{bmatrix} = \begin{bmatrix} z_{j-1} \\ z_j \\ z_{j+1} \end{bmatrix} \tag{21}$$

The above matrix equation is in the format $\mathbf{Ax=b}$ for which the roots can be quickly and efficiently solved.

Step 2: Interpolating the Instantaneous Forward Rates

Having the forward rates, the next step is to interpolate these by fitting the quartic polynomial $q_j(t)=a_j+b_jt+c_jt^2+d_jt^3+e_jt^4$ to each pair of forward rates. For $n+1$ forward rates it is therefore necessary to solve for $5n$ unknown variables. To explicitly solve these unknown variables, $5n$ equations are obtained by the appropriate boundary and continuity conditions with the latter applying to the polynomial itself as well as its first, second and third derivatives.

As a simple example, consider a forward curve that only consists of three forward rates. The coefficients for the two quartic polynomials making up the spline between these three node points can then be easily determined by solving for the roots of the following matrix equation.

$$\begin{bmatrix} t_1-t_0 & \frac{1}{2}\left(t_1^2-t_0^2\right) & \frac{1}{3}\left(t_1^3-t_0^3\right) & \frac{1}{4}\left(t_1^4-t_0^4\right) & \frac{1}{5}\left(t_1^5-t_0^5\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t_2-t_1 & \frac{1}{2}\left(t_2^2-t_1^2\right) & \frac{1}{3}\left(t_2^3-t_1^3\right) & \frac{1}{4}\left(t_2^4-t_1^4\right) & \frac{1}{5}\left(t_2^5-t_1^5\right) \\ 1 & t_1 & t_1^2 & t_1^3 & t_1^4 & -1 & -t_1 & -t_1^2 & -t_1^3 & -t_1^4 \\ 0 & 1 & 2t_1 & 3t_1^2 & 4t_1^3 & 0 & -1 & -2t_1 & -3t_1^2 & -4t_1^3 \\ 0 & 0 & 1 & 3t_1 & 6t_1^2 & 0 & 0 & -1 & -3t_1 & -6_1^2 \\ 0 & 0 & 0 & 1 & 4t_1 & 0 & 0 & 0 & -1 & -4t_1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3t_2 & 6t_2^2 \\ 0 & 0 & 0 & 3 & 4t_1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -t_1 & -t_1^2 & -t_1^3 & -t_1^4 & 1 & t_2 & t_2^2 & t_2^3 & t_2^4 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \\ e_2 \end{bmatrix} = \begin{bmatrix} z_1t_1-z_0t_0 \\ z_2t_2-z_1t_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -q_1-q_2 \end{bmatrix} \quad (22)$$

It is also in the format $\mathbf{Ax=b}$ and can be solved using LU factorisation to avoid matrix non-singularities.

Step 3: Interpolating the Zero Rates

After the coefficients for the quartic polynomials are determined, these can then be used to interpolate the zero curve between any two given zero rates, z_{j-1} and z_j, using the expression:

$$z_t = \frac{1}{t}\begin{bmatrix} 1 & a_j & b_j & c_j & d_j & e_j \end{bmatrix} \begin{bmatrix} z_{j-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} t_{j-1} \\ t-t_{j-1} \\ t^2-t_{j-1}^2 \\ t^3-t_{j-1}^3 \\ t^4-t_{j-1}^4 \\ t^5-t_{j-1}^5 \end{bmatrix} \quad (23)$$

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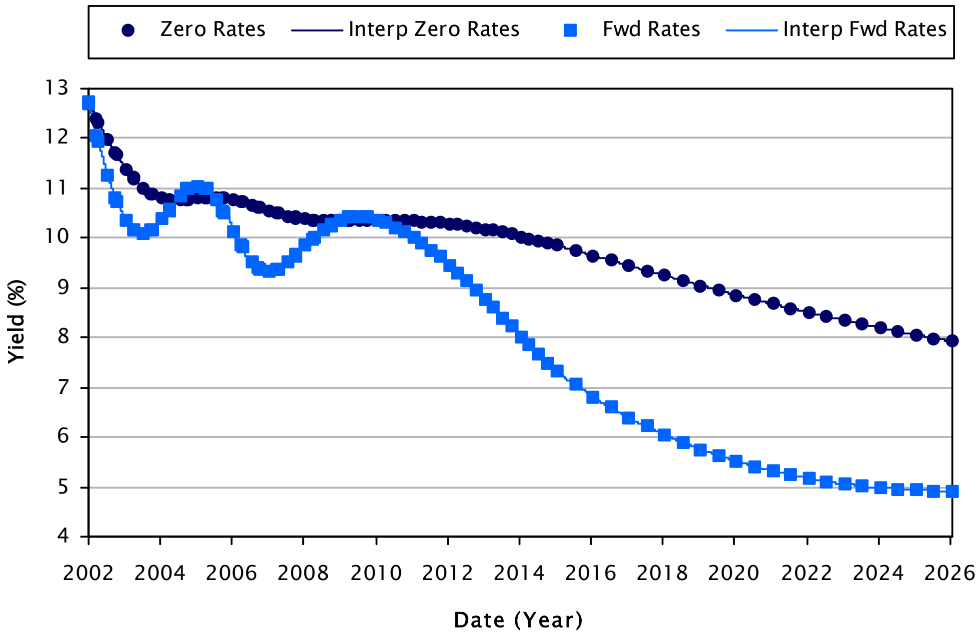
Figure 14. With the instantaneous forward rates, the quartic spline can be fitted to these rates. Using these fit parameters, the interpolated zero curve can then be determined.



In Figure 14 an example of the Mark III BEASSA zero curve is shown, with cash flow dates and corresponding bootstrapped zero rates. As shown in Step 1, the individual zero rates (dark blue circles) are then transformed into instantaneous forward rates (light blue squares) that correspond to the zero rates' same cash flow dates. These forward rates are then interpolated using the quartic spline, as shown in Step 2, to obtain a continuous instantaneous forward curve (light blue line). From the interpolation parameters, a curve can then also be fitted to the zero rates to give a smooth, continuous zero curve (dark blue line).

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