Preface

Objectives and Audience

In the past two decades, we have witnessed the revolutionary period in the trading of financial derivative securities in financial markets and the phenomenal surge in research activities in derivative pricing theory. Leading edge banking and financial firms around the globe are hiring science experts who can use advance analytical and numerical techniques to price financial derivatives and manage portfolio risks, a phenomenon coined as "Rocket Sciences on Wall Street". Such developments have spurred the growth of new degree programs in mathematical and computational finance in both North America, Europe and the Far East. This book is thus written to meet the demands of these students.

In this book, the derivative products are modelled as partial differential equation models from the perspectives of an applied mathematician. Analytical solutions to these derivative pricing models are sought, together with solution by numerical techniques when appropriate. This book includes quite a comprehensive coverage of derivative products commonly traded in financial markets, and the latest development of option pricing methodologies and algorithms. Fundamental concepts in financial economics which are necessary to the understanding of the derivative pricing theory are included, but possibly in a less formal style compared to other similar books on mathematical finance. Advanced concepts in probability theory which are essential for more in-depth understanding of derivative pricing theory, like the martingale theory and stochastic differential calculus, are briefly introduced in the book. The level of mathematics in this book is tailored to readers with preparation at the advanced undergraduate level of science and engineering majors. Other audience of this book include practitioners in financial engineering who seek the latest development in option pricing techniques, and scientists and mathematicians who would like to start exploring into the theory of derivative pricing.

The readers are assumed to have some basic knowledge of linear partial differential equations, probability and statistics, and numerical methods. No prior knowledge in finance theory and options is assumed.

Guide to the Chapters

This book contains seven chapters, each divided into a number of sections. Every chapter is ended with a set of well thought-out exercises. On one hand, the exercises provide the stimulation for refreshing concepts and knowledge acquired from the text; and on the other hand, they help lead the readers to results and concepts found scattered in recent journal articles in the field of derivative pricing.

The first chapter introduces general characteristics of financial derivatives. Fundamental concepts in financial option theory, like arbitrage, hedging, self-financing strategy, are discussed and definitions of these terms are presented. It is then followed by the discussion of the assumptions of asset price dynamics and the mathematical formulation of stochastic processes, in particular, the Geometric Brownian process. The Ito lemma, a basic tool used for the evaluation of stochastic differentials, is derived. The riskless hedging principle and risk neutrality are the cornerstones of the Black-Scholes-Merton pricing theory. The precise meaning and implication of these concepts are examined in the last section. The chapter is ended with the derivation of the renowned Black-Scholes equation.

In Chapter 2, the Black-Scholes equation is solved for different European-style derivative securities, which are distinguished by their contractual specifications. Under certain idealized assumptions, closed form solutions for the Black-Scholes equation can be found. The greeks of these Black-Scholes formulas are derived and their financial interpretations are discussed. The extension of the Black-Scholes model, incorporating the effects of discrete dividends, transaction costs, time-dependent interest rate and volatility are considered. Some of the issues of pricing biases of the Black-Scholes model are addressed. The last section deals with the discussion of pricing models for options on futures contracts.

The pricing of options that are multivariate in nature are considered in Chapter 3. Examples of such multi-state options include the index options, basket options, cross currency options, exchange options and options on the extremum of several risky assets. The general Black-Scholes equation for options with multiple underlying assets is developed. Analytic formulas for a variety of multi-state options are derived.

Chapter 4 is concerned with the pricing of the American-style options. The characterizations of the optimal exercise boundary associated with the American option models are presented. Special attentions are paid to examine the behaviors of the exercise boundary right before and after a discrete dividend payment, and immediately prior to expiry. The optimality conditions for the determination of the early exercise boundary are discussed. The early exercise premium is shown to be expressible in terms of the exercise boundary in the form of an integral. Several analytical approximation methods are discussed for the valuation of American options.

Examples of option models which lend themselves to closed form solutions are limited; and frequently, option valuation must be resorted to numerical procedures. The common numerical methods employed in option valuation are the binomial schemes, finite difference algorithm and Monte Carlo simulation. These numerical methods are discussed in Chapter 5. The primary essence of the binomial models is the simulation of the continuous asset price movement by a discrete random walk model. The finite difference approach seeks the discretization of the differential operators in the Black-Scholes equation. The Monte Carlo method simulates the random movement of the asset prices. It provides a probabilistic solution to the option pricing problems. An account of option pricing algorithms using these approaches is presented.

Path dependent options are option contracts where their payouts are related to movements in the price of underlying asset during the life of the option. The common examples are the barrier options, Asian options and lookback options. Chapter 6 presents the mathematical approaches for their valuation, including analytic approximation techniques and numerical procedures when analytic formulas do not exist.

Chapter 7 deals with the pricing of bonds and interest rate derivatives. Various classes of interest rate models are introduced, starting with the Vasicek mean reversion model, Cox-Ingersoll-Ross model, and extending to other multi-factor models. The discussion is extended to no-arbitrage models where the initial term structures are taken as inputs into the models. Pricing models of swaps, caps, floors, commodity-linked bonds and convertible bonds are also considered.

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