Financial Engineering with FRONT ARENA

Problems and solutions

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Outline

Objectives

A broad array of common problems from financial world can be solved through certain logic that we generally call **Financial Engineering**. Financial engineers must be able to use specialised software for solving such kind of problems.
In this lecture, we give a survey of the course MT1460 — Financial and Risk Management Software (5 points, 60 hours, D level).
This course is given in the framework of the Master Programmes Analytical Finance (160–200 points) and MIMA Analytical Finance (60 points).

Aim

① To give a review of existing financial and risk management software.

② To refresh students' knowledge about typical problems of financial engineering and their mathematical solutions.

③ To teach students how to solve these problems, using FRONT ARENA software.

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Lectures, 3 points

Each lecture (4 hours), except the introductory one, starts with the presentation of solutions to exercises by some group of students. After the presentation all students take part in the discussion. Each student must be able to explain and discuss topics related to the presentation.
The presentation of solutions to exercises and the following discussion takes about one hour. During the next two hours the teacher will follow up the work of groups. During the last hour the teacher will present the outline of the material of the next lecture.
To pass the lectures, the student must solve all exercises.

Seminar, 2 points

In seminars (4 hours each), the students are expected to apply FRONT ARENA to solving real-world financial problems that are not included to the course material.
Every group will obtain a certain part of FRONT ARENA documentation, where a typical financial problem and methods of its solutions are described. The group's report must contain a title page, abstract, contents, introduction, where the problems are formulated, solutions, conclusions, and references.

Financial and Risk Management Software: course plan

1	24.01.05	A review of existing financial and risk management software.
2	27.01.05	Introduction to FRONT ARENA. Instruments.
3	31.01.05	Instruments with underlyings.
4	03.02.05	Interest rate models.
5	10.02.05	The LIBOR market model.
6	14.02.05	The Option Adjusted Spread model.
7	17.02.05	Credit derivatives.
	24.02.05	Seminar 1.
8	28.02.05	Repo instruments.
9	03.03.05	Desk risk management.
10	07.03.05	Consolidated risk management.
11	10.03.05	Limit management.
12	14.03.05	ARENA Data Model.
13	17.03.05	ARENA SQL.
	24.03.05	Seminar 2.

Contents of Lecture 5 — "The LIBOR Market Model"

- The LIBOR market model: theory.
- Context mapping.
- Valuation of caps and floors.
- Exercises.

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Discount bonds

Definition

A contract, which gives the holder an amount 1 at some future date T, is referred to as **discount bond**. 1 is called the **notional** or **face value** and T is referred to as the **maturity date**. The price at time t of a discount bond with maturity T and face value 1 is denoted by $P(t, T)$.

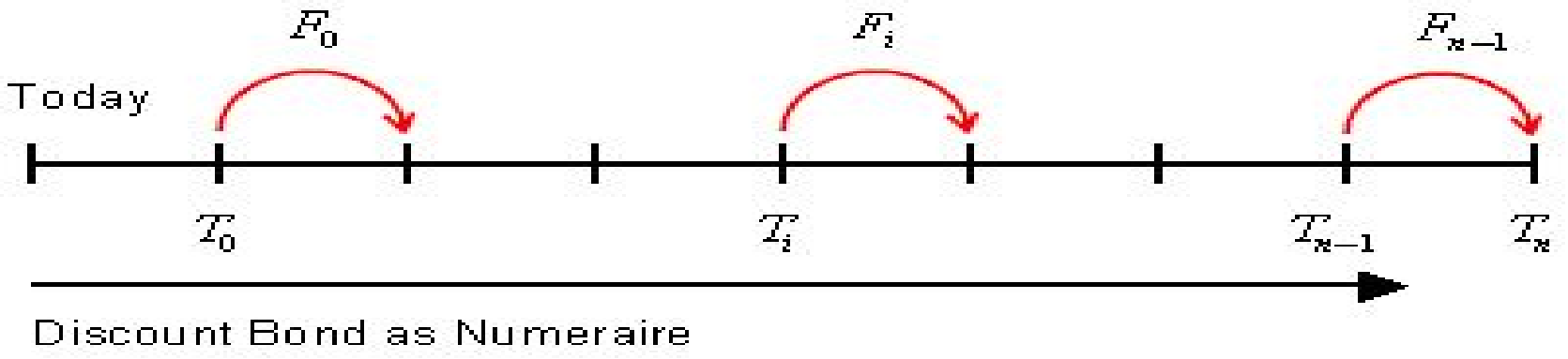
Forward rates

Definition

The **simply compounded forward rate** at time t spanning the future period $[T_1, T_2]$, $F(t, T_1, T_2)$ is defined by

$$\frac{P(t, T_2)}{P(t, T_1)} = \frac{1}{1 + F(t, T_1, T_2)(T_2 - T_1)}.$$

The following diagram illustrates a set of forward rates spanning the set of dates $\{T_i\}$:



The LIBOR market model: notation

Let the tenor structure be $0 = T_0 < T_1 < \cdots < T_n$ and i an integer ranging over the resets of the rates, e.g. $1 \leq i \leq n$.
We define $\eta(t)$ to be the unique index such that $T_{\eta(t)}$ is the next tenor date after t.

f_i	forward/swap rate at time i
μ_i	drift term
$\sigma_{i,k}$	the component of the volatility $\sigma_i^2(t)$ of $f_i(t)$ attributable to the kth factor: $\sum_{k=1}^{m} \sigma_{i,k}^2(t) = \sigma_i^2(t)$.
$z_k(t)$	Wiener processes.

The LIBOR market model: the drift terms

The drift terms μ_i depend on the choice of numeraire and can be determined by applying the assumption of no arbitrage. Suppose we have forward rates as the underlying rates and choose $P(T_0, T_{i+1})$ as the numeraire. Then the drift terms become

$$\mu_i(t) = \sigma_i(t) \sum_{k=\eta(t)}^{i} \frac{\tau_i f_i(t) \sigma_k(t)}{1 + \tau_i f_i(t)}.$$

The LIBOR market model: equations

The m-factor model is given by the following stochastic differential equation (SDE) for the underlying rates (swap or forward):

$$\frac{df_i}{f_i} = \mu_i(f_i(t), t)\,dt + \sum_{k=1}^{m} \sigma_{i,k}(t)\,dz_k(t).$$

The solution of the SDE is

$$f_i(t) = f_i(0) \exp\left(\int_0^T \left(\mu_i(s) - \frac{1}{2}\sigma_i^2(s) \right) ds + \sum_{k=1}^{m} \int_0^T \sigma_{i,k}(s)\,dz_k(s) \right).$$

Cap volatility equations

Assume that each underlying rate $f_i(t)$ has a lognormal distribution with variance equal to $\sigma_B^2 t$, where σ_B^2 is the implied Black volatility, which can be read from the market. Then the instantaneous volatility at reset for each rate is related to the above expression in the following way:

$$\int_0^{T_i} \sigma_i^2(t)\, dt = \sigma_B^2 T_i. \tag{1}$$

Cap volatility calibration

There are infinitely many solutions to equations (1), and our goal is to pick one that fits our needs. Let $\sigma(t) = (a + bt)e^{-ct} + d$ and $\sigma_i(t) = k_i\sigma(T_i - t)$. The calibration proceeds as follows.

① Find values on the constants a, b, c, and d such that equation (1) fit as close as possible.

② Set values of the k_i as

$$k_i = \sqrt{\frac{\sigma_B^2 T_i}{\int_0^{T_i} \sigma_i^2(t)\,dt}}.$$

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- Example: mappings for a swaption.
- Valuation parameters.

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The list of necessary mappings

In order to use the LIBOR Market Model when valuing instruments the following context mappings must be performed:

1. Map the instrument to the **Core Valuation Function** > **LIBOR Market Model**. This mapping tells FRONT ARENA to value the instrument with the LIBOR Market Model.
2. Map the instrument to an appropriate correlation matrix. The LIBOR Market Model requires a correlation matrix as input, and this mapping makes sure it gets one.
3. Map the instrument to an appropriate volatility Landscape. If the instrument is a Cap/Floor it suffices to map a volatility Landscape to the rate index.
4. If the instrument is a Swaption, we must, in addition, map a volatility Landscape to the instrument itself.

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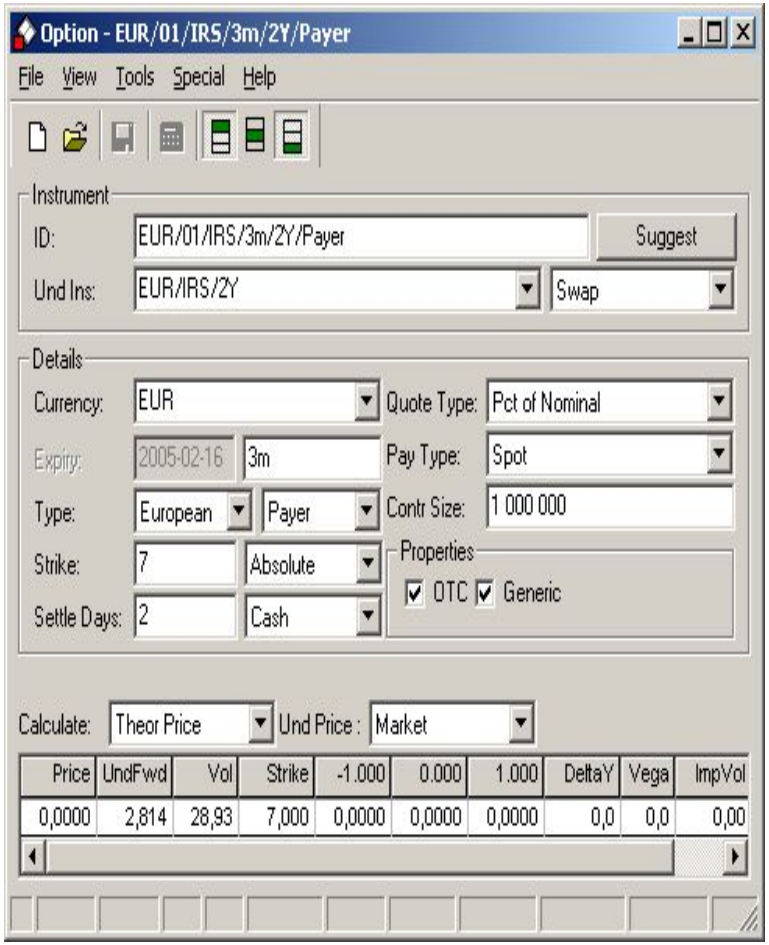
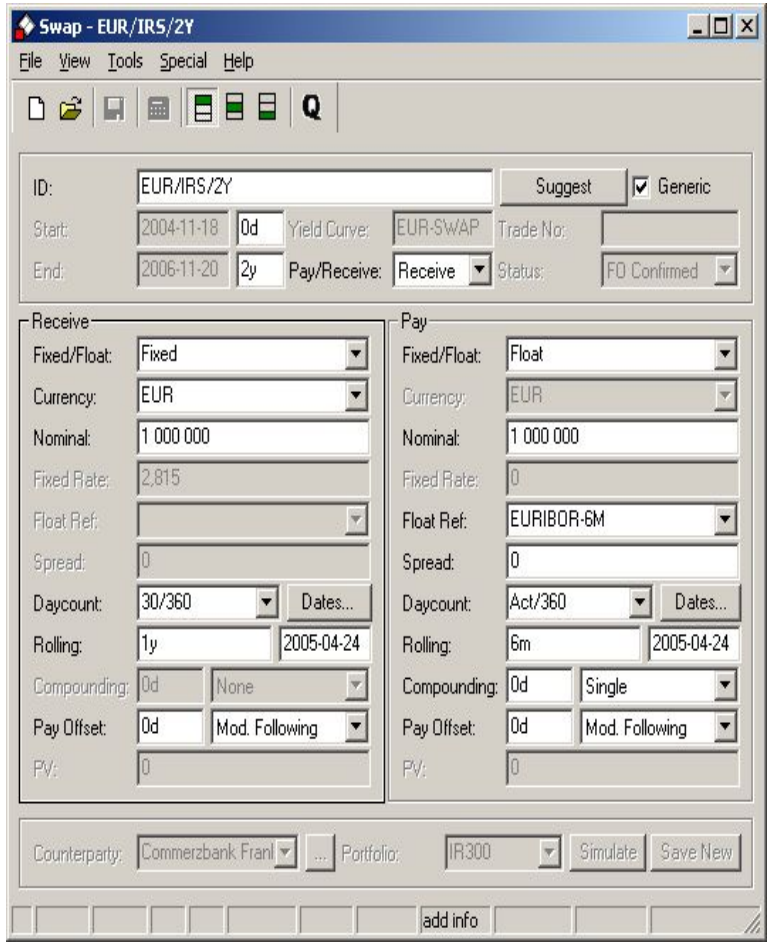
Mapping for a swaption

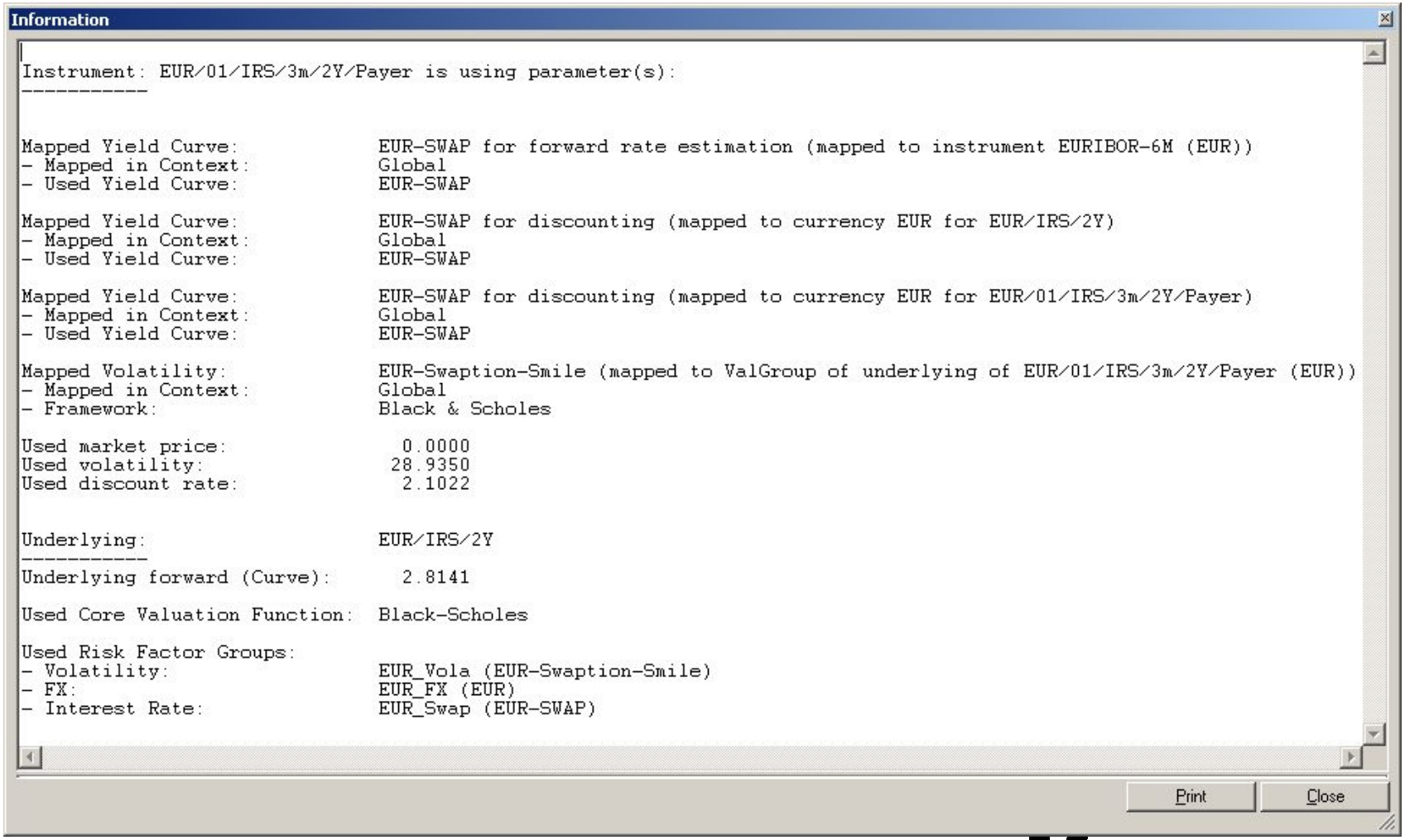
Consider the following swaption and underlying swap:





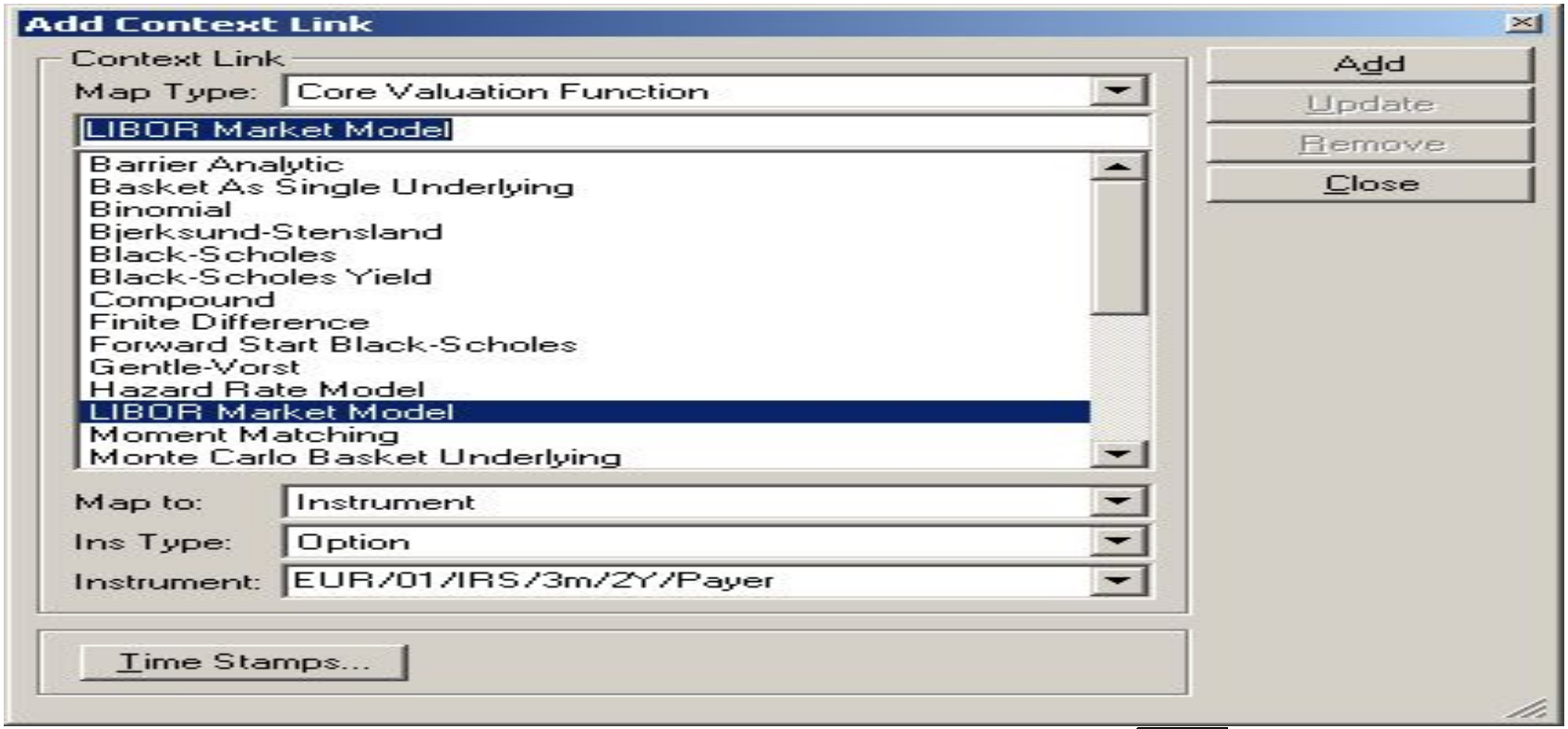
The mappings that apply

The mappings that apply are in the **Special** > **Information** window:



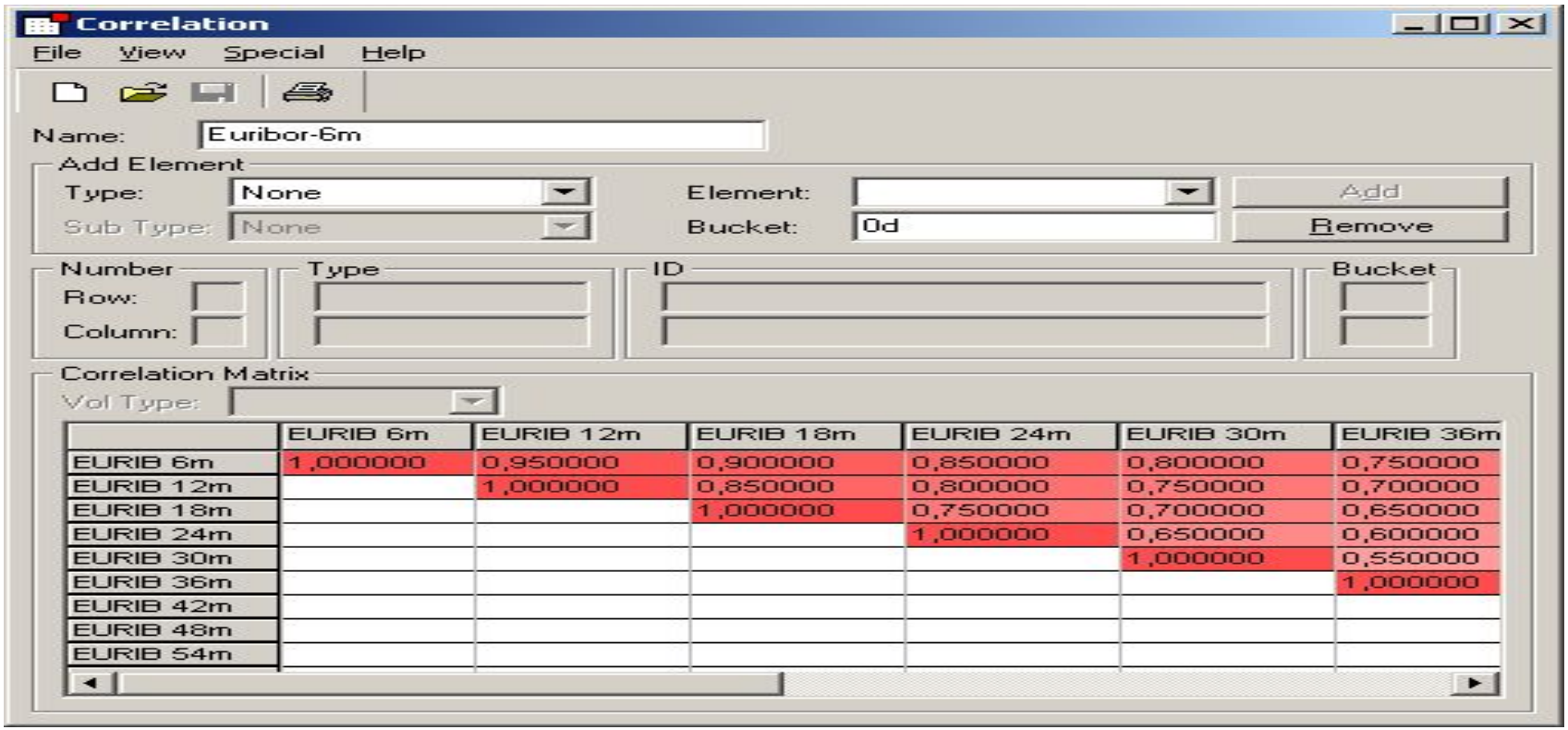
The first mapping

First, open an appropriate context in **File** > **Open** of the **Context** application. In this example it is 'Global'. To add a context link select **Edit** > **Add ContextLink** as follows:



The correlation matrix

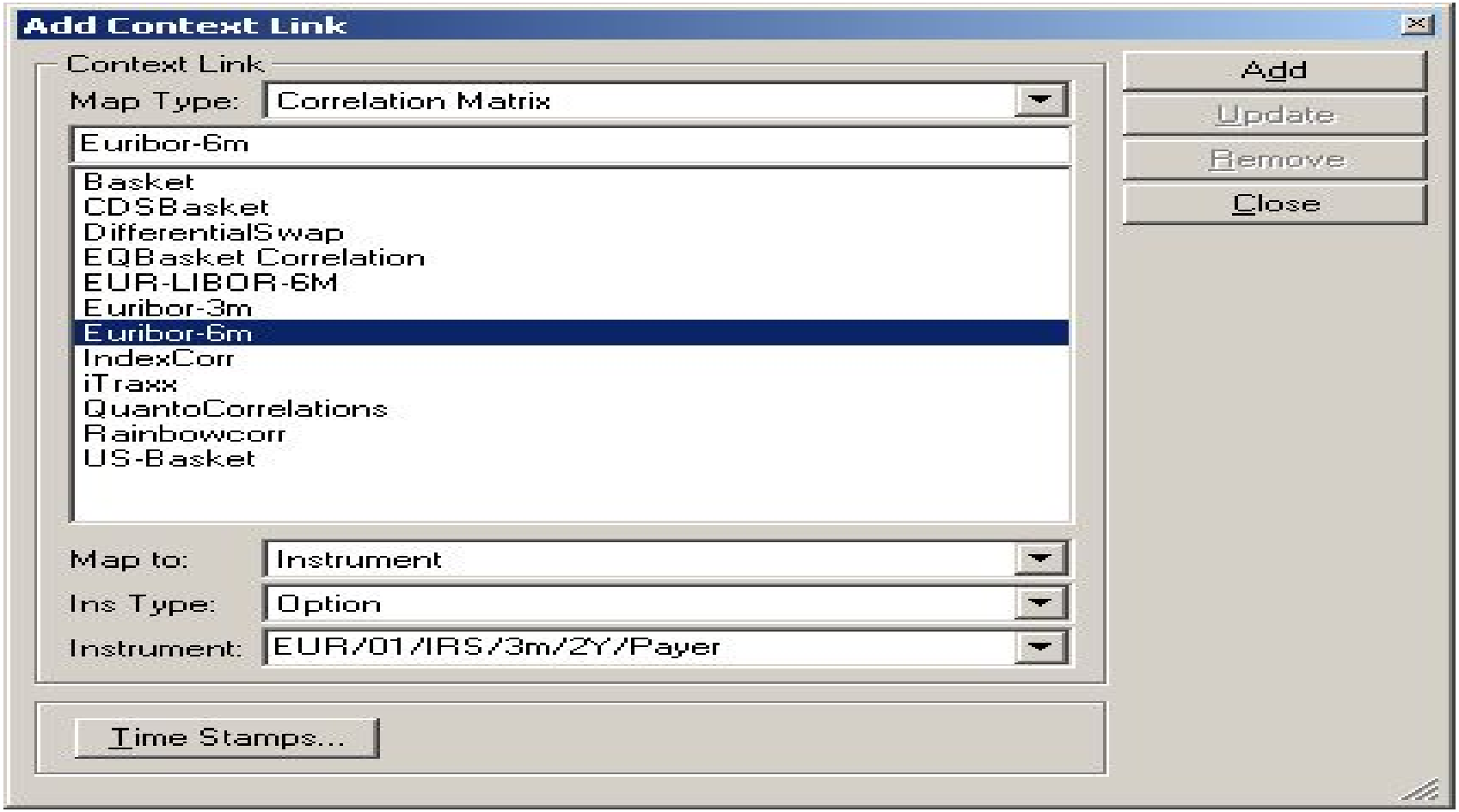
Correlations matrices are set up using the **Correlation** application, which is accessed by selecting **Data** > **Correlation** from the PRIME. The correlation matrix used by “EUR/01/IRS/3m/2Y/Payer" is illustrated in the figure below:



	EURIB 6m	EURIB 12m	EURIB 18m	EURIB 24m	EURIB 30m	EURIB 36m
EURIB 6m	1,000000	0,950000	0,900000	0,850000	0,800000	0,750000
EURIB 12m		1,000000	0,850000	0,800000	0,750000	0,700000
EURIB 18m			1,000000	0,750000	0,700000	0,650000
EURIB 24m				1,000000	0,650000	0,600000
EURIB 30m					1,000000	0,550000
EURIB 36m						1,000000
EURIB 42m						
EURIB 48m						
EURIB 54m						

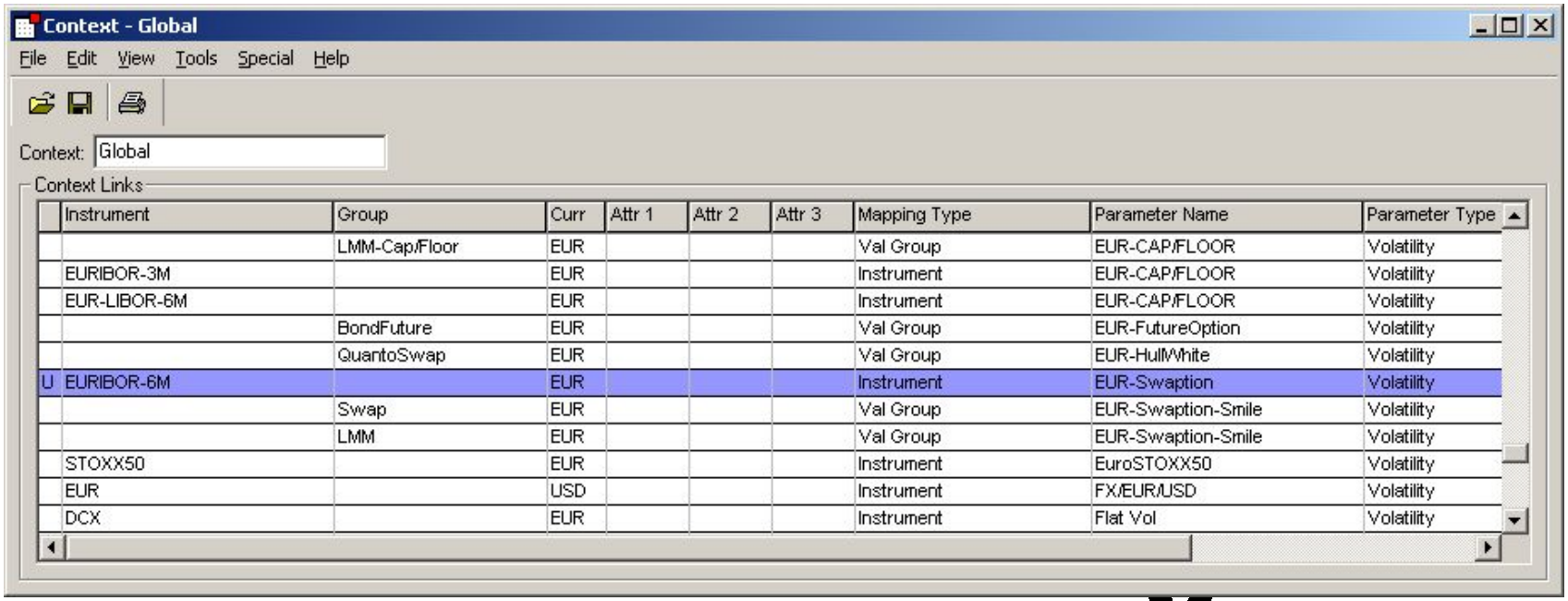
THE SECOND MAPPING

To add this context link select **Edit** > **Add ContextLink** as the following figure shows:



The third mapping

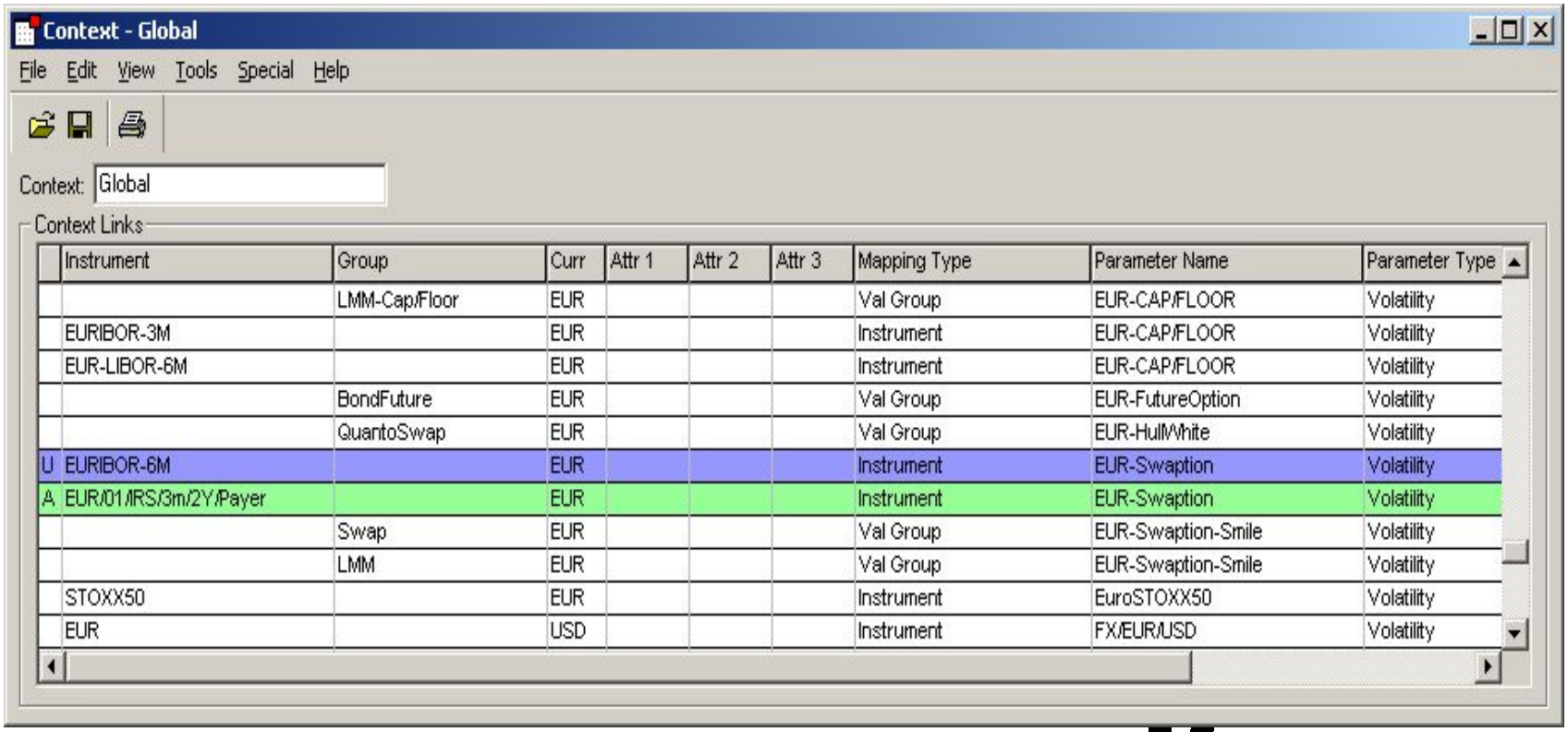
To enable the calibration process to take place it is necessary to map a volatility Landscape to the underlying rate index used by “EUR/01/IRS/3m/2Y/Payer", in our case “Euribor-6m". The mapping procedure is performed in the same way as for the correlation matrix mapping and the result is pictured in the following figure:



	Instrument	Group	Curr	Attr 1	Attr 2	Attr 3	Mapping Type	Parameter Name	Parameter Type
		LMM-Cap/Floor	EUR				Val Group	EUR-CAP/FLOOR	Volatility
	EURIBOR-3M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
	EUR-LIBOR-6M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
		BondFuture	EUR				Val Group	EUR-FutureOption	Volatility
		QuantoSwap	EUR				Val Group	EUR-HullWhite	Volatility
U	EURIBOR-6M		EUR				Instrument	EUR-Swaption	Volatility
		Swap	EUR				Val Group	EUR-Swaption-Smile	Volatility
		LMM	EUR				Val Group	EUR-Swaption-Smile	Volatility
	STOXX50		EUR				Instrument	EuroSTOXX50	Volatility
	EUR		USD				Instrument	FX/EUR/USD	Volatility
	DCX		EUR				Instrument	Flat Vol	Volatility

The fourth mapping

Once more, to enable the calibration to take place it is also necessary to map “EUR/01/IRS/3m/2Y/Payer" to an appropriate volatility Landscape. In this example the volatility Landscape ’EUR-Swaption’ is chosen as illustrated in the following figure:



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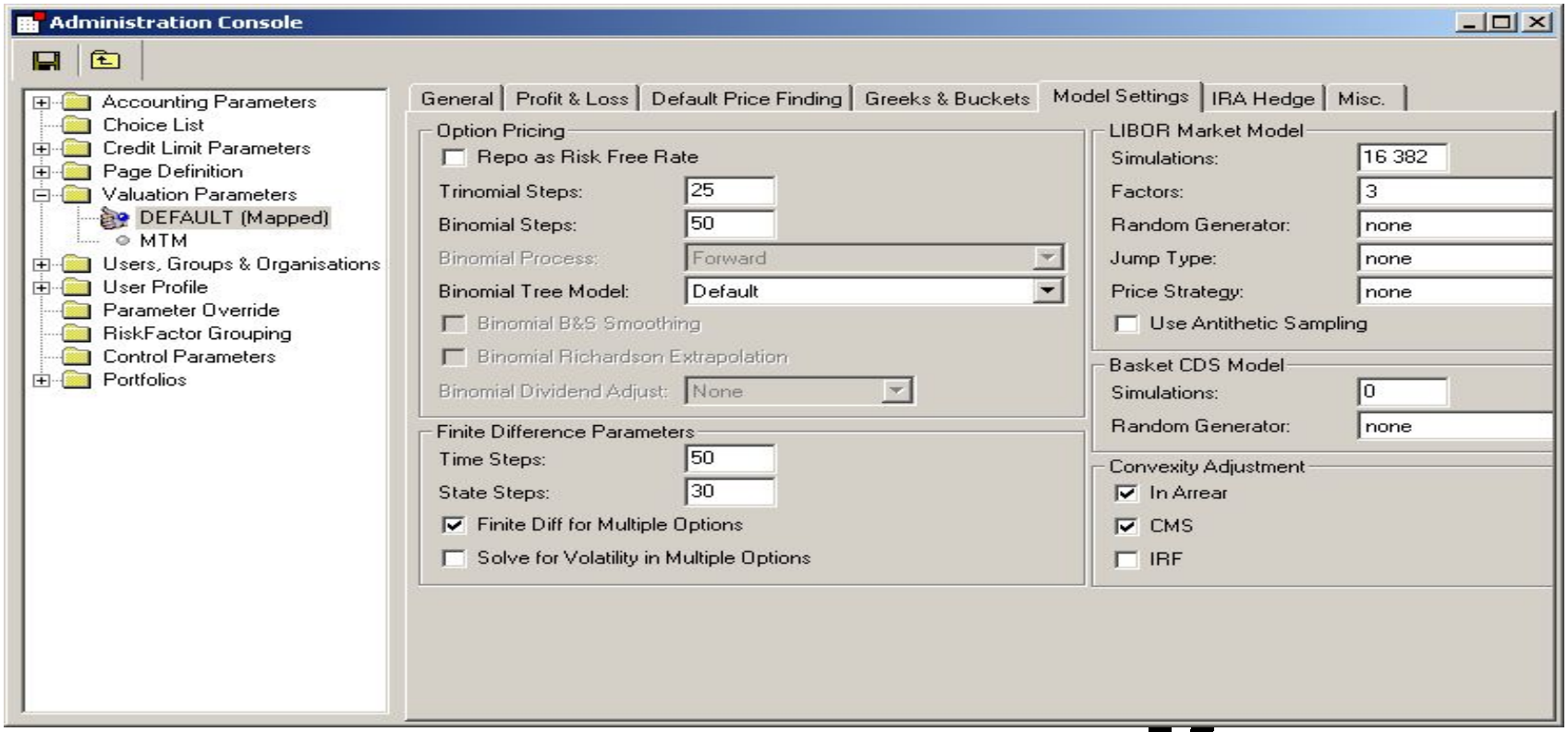
	EURIBOR-3M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
	EUR-LIBOR-6M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
		BondFuture	EUR				Val Group	EUR-FutureOption	Volatility
		QuantoSwap	EUR				Val Group	EUR-Hull/White	Volatility
U	EURIBOR-6M		EUR				Instrument	EUR-Swaption	Volatility
A	EUR/01/IRS/3m/2Y/Payer		EUR				Instrument	EUR-Swaption	Volatility
		Swap	EUR				Val Group	EUR-Swaption-Smile	Volatility
		LMM	EUR				Val Group	EUR-Swaption-Smile	Volatility
	STOXX50		EUR				Instrument	EuroSTOXX50	Volatility
	EUR		USD				Instrument	FX/EUR/USD	Volatility

Valuation parameters

Before running the LIBOR Market Model in FRONT ARENA, it is necessary to specify the number of Monte Carlo simulations and the number of factors to be used by the model. This is done in **Admin** > **Administration Console** application.



Types of caps and floors

- Plain vanilla caps and floors.
- Ratchet caps.
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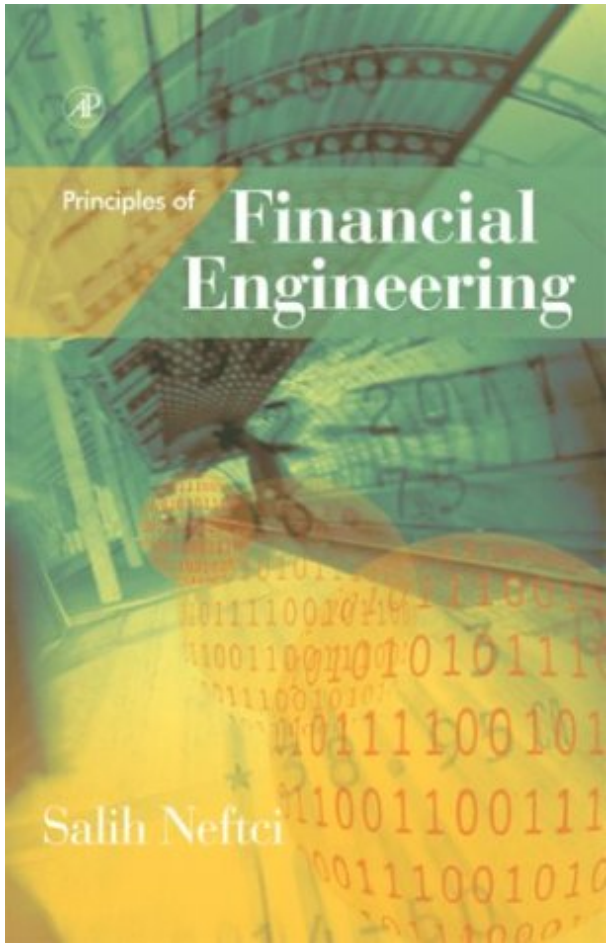
A TYPICAL EXERCISE

Calculate the theoretical price of the ratchet floor with the following parameters, using the LIBOR market model.

Currency	EUR
Strike	4
Start	2003-12-05
End	2006-12-05
Float Ref	EURIBOR-6M
Day Count	Act/360
Rolling	6m from 2006-12-05
Exclude 1st	Yes
Spread	0.5
Limit	3

Course literature



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Remarks and suggestions to FRONT ARENA

Simultaneously with preparing a course, we write a separate document containing our remarks and suggestions. It is supposed to contain three sections:

① realisation of methods of valuation different financial instruments;

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