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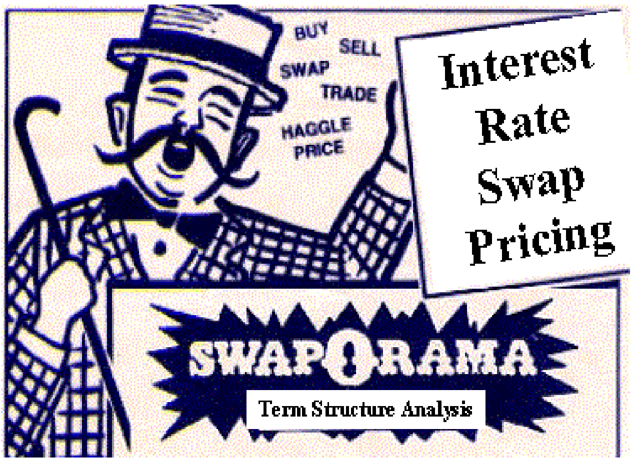
Group: 4B

SEMINAR 1



Term Structure Analysis and Interest Rate Swap Pricing

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1 Introduction

In this report we will explore term structure analysis of interest rates in which we will describe what a term structure is and how it is shaped. Also we will go into the main theories of the term structures which will be explained with the help of examples.

The main focus of this report will be the interest rate swaps. Like other derivatives, swaps are tools that firms can use to easily change their risk exposures. We will specifically go into and try to explain the “plain vanilla swap” and the swap valuation. We will show how this can be done using Matlab.

We hope this report will give you sufficient basic knowledge and understanding of term structure analysis and interest rate swap pricing.

2 The term structure of interest rates

The term structure of interest rates is a yield curve showing the relationship between the term to maturity of a bond and its yield to maturity. Yield to maturity is the rate of return on a bond investment if the bond is held from the current time until its maturity date. The yield to maturity is seen as a long term bond yield expressed as an annual rate. The calculation takes into account the current market price, par value, coupon interest rate and time to maturity. It also assumed that all coupons are reinvested at the same rate. The term to maturity is the number of years until the last promised payment.

From the resulting term structure an interest rate pattern can be determined. The interest rate pattern can be used to discount cash flows appropriately, because most bonds have coupons, the term structure must be determined using the prices of these bonds.

The figure of the term structure changes as time passes. When the short-term yields are higher than the long-term yields the term structure is downward sloping. When the long-term yields are higher then the short-term yields the term structure is upward sloping.

When plotting the yields and terms to maturity of bonds, the curve running through the scatter (=the resulting yields with their corresponding maturities) is the estimate of the term structure. The shape of the term structure gives valuable clues to market expectations about future interest rates and future monetary policies.

There are three possible factors influencing the shape of the term structure[1]:

1. The market's expectations regarding the future direction of interest rates
2. The possible presence of liquidity premiums in expected bond returns
3. Market inefficiency or possible impediments to the flow of funds from the long or short term end of the market to the short or long term end.

[1] Modern Investment Theory (Fourth Edition), Robert A. Haugen 1997 (page 367)

Seminar 1
Term Structure Analysis and Interest Rate Swap Pricing

There are three main theories concerning the forces that cause the term structure to change its shape as time goes by of the term structure[2]:

1. The market expectations theory. This theory states that the term structure is solely determined by the market's expectations regarding future yields on 1-year bonds. The yield to maturity of a 5 year bond is the average of the yields expected on 1 year bonds over the next 5 years. The distinctive feature of the market expectation theory is that for a given period of time, the market expects to get the same rate of return on all bonds, regardless of their term of maturity.

Example[3]

	1-yr bond	2-yr bond	3-yr bond	4-yr bond	5-yr bond	6-yr bond
Now	3 %	3	3	3	3	3
1 year from now	7%	7	7	7	7	7
2 year from now	10%	10	10	10	10	10
3 year from now	12%	12	12	12	12	12
4 year from now	10%	10	10	10	10	10
5 year from now	8%	8	8	8	8	8
Yield to maturity	3%	5	6,67	8	8,4	8,33

Expect
annual
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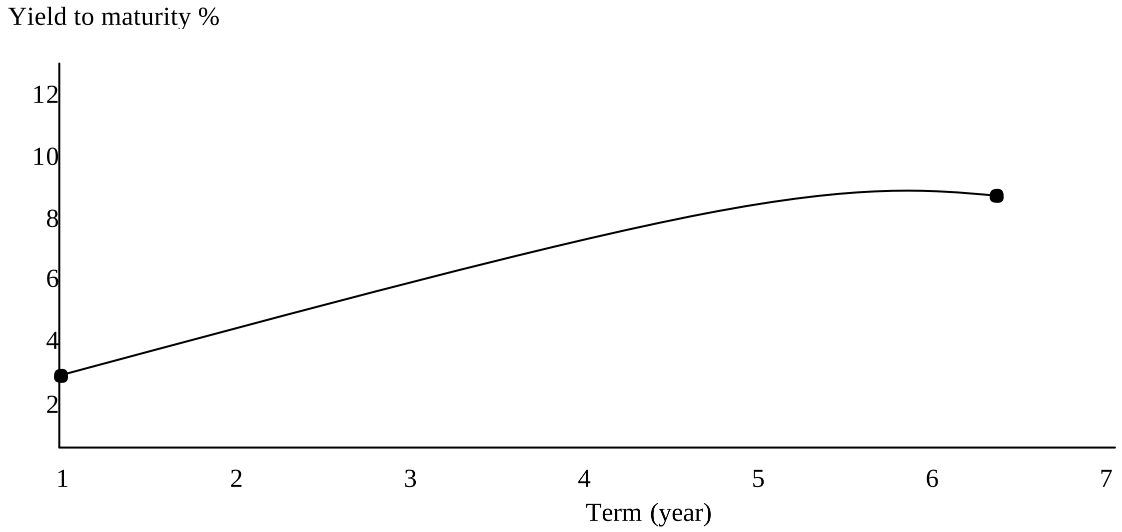
Yields
maturi

We selected:

a. 1-year bond with an interest rate of 3%,
b. 2-year bond with an interest rate of 7%,
c. 3-year bond with an interest rate of 10%,
d. 4-year bond with an interest rate of 12%,
e. 5-year bond with an interest rate of 10%,
f. 6-year bond with an interest rate of 8%.

So to compute the yield to maturity for a 3-year bond, we average the three returns in the three cells moving diagonally from the first row, the thrid column to the third row, first column. The current yield to maturity of a 3-year bond is the average of 3 %, 7% and 10 % (= 6,67 %).

[2] Modern Investment Theory (Fourth Edition), Robert A. Haugen 1997 (page 376-386)
[3] Modern Investment Theory (Fourth Edition), Robert A. Haugen 1997 (page 378)



2. The liquidity preference theory of the term structure. The liquidity preference theory is a hypothesis that forward rates offer a premium over expected future spot rates. The liquidity preference theory takes into account possible existence of risk or liquidity premiums, which where non existence for the the market expectations theory. For 1-year bonds the market expectations regarding future interest rates are the same as the 1-year bond under the the market expectations theory, but there is a difference when we have 2-year or more bonds. When investing in a bond that matures within two years or more, investors want an additional x percent premium in their expected return over what they require to invest in a 1-year bond. Investors are risk-averse and will demand a premium for bonds with longer maturities, because the prices of these bonds are more volatile than that on short-term bonds A premium is offered in the form of greater forward rates in order to attract investors to longer-term bonds. The effect of liquidity premiums is to make the term structure, if the rates are expected to rise, more upward sloping or less downward sloping, if the rates are expected to fall.

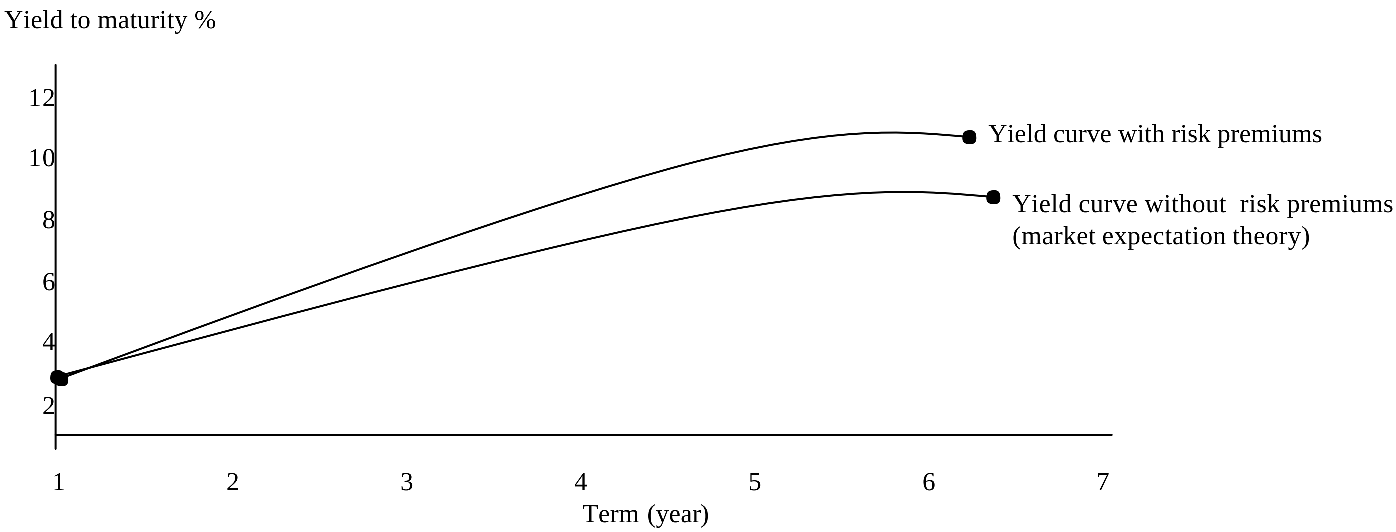
Example[4]

Assuming for a 2-year bond that investors require an additional 2 percent premium in their expected return over what they require to invest in a 1-year bond. A 3-year bond is assumed to have a 3 percent premium over the 1-year bond and the same premium is assumed to hold for bonds with maturities in excess of 3 years.

	1-yr bond	2-yr bond	3-yr bond	4-yr bond	5-yr bond	6-yr bond
Now	3 %	5	6	6	6	6
1 year from now	7%	9	10	10	10	10
2 year from now	10%	12	13	13	13	13
3 year from now	12%	14	15	15	15	15
4 year from now	10%	12	13	13	13	13
5 year from now	8%	10	11	11	11	11
Yield to maturity	3%	6	8,33	10	10,6	10,67

Expected annual rates of return

Yields to maturity

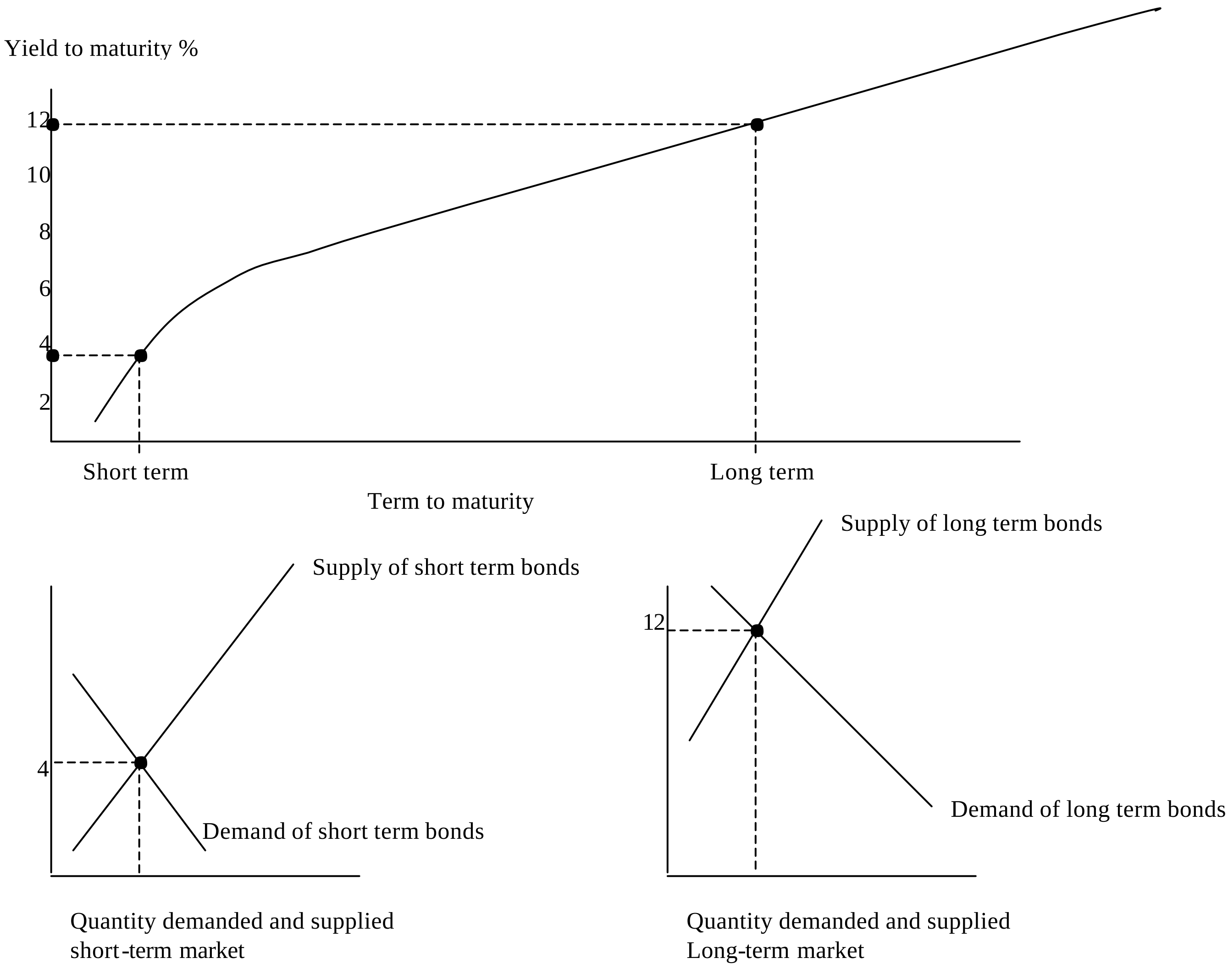


3. The market segmentation theory of the term structure. This theory assumes the market consists of large investors who are extremely risk averse, corporations and financial institutions for which survival is of paramount importance. The investors prefer to make bond purchases that match their needs. Everyone seeks to immunize his or her

[4] Modern Investment Theory (Fourth Edition), Robert A. Haugen 1997 (page 382)

portfolio. The portfolio is immunized by minimizing risk. The market can be divided into segmented or contained submarkets on the basis of maturity. When segmenting the market we divided the term structure in two parts, a short term segment and a long term segment. Each segment has a schedule of supply and demand for bonds. At the intersection of supply and demand we establish the yield. The term structure is shaped by the direction of bonds flows from one financial institution to another and by the intensity and nature of economic investment by business firms. In other words the yield curve is primarily determined by the interaction of the large investors in the market. The term structure is upward sloping because of the nature of bond flows and investment demands.

Example



3 Interest Rate Swaps

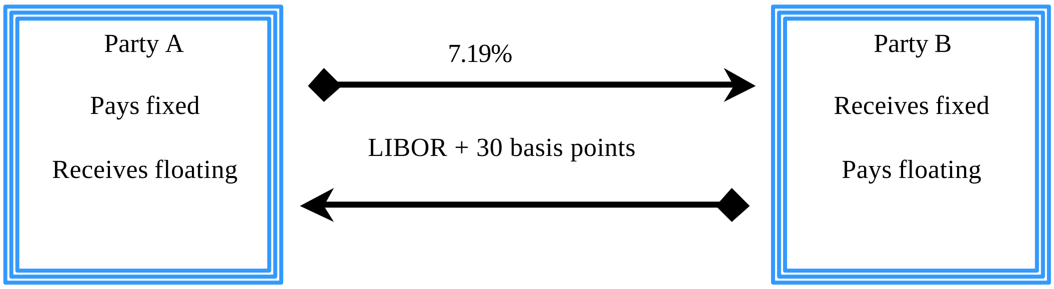
Like other derivatives, swaps are tools that firms can use to easily change their risk exposures. Variable interest rate borrowers would use Swaps to hedge their interest costs in a market where variable interest rates are expected to rise while fixed rate borrowers would use a Swap to take advantage of lower interest rates in a market where variable interest rates are expected to fall.[5]

A financial swap is a contract between two counter parties to exchange a series of cash payments. Contained in the swap contract is a specification of the rate of interest applicable to each cash payment, the currency in which each cash payment will be made, the time table for the payments, provisions to cover the contingency that a counter party might default, and other issues that affect the relationship between the counter parties.

3.1 Plain Vanilla Swap [6]

Let us consider a simple example to illustrate the different issues involved in plain vanilla interest rate swaps.

Consider the following swap. Party A pays a fixed rate 7.19% per annum on a semi-annual basis, and receives from party B LIBOR[7] + 30 basis points.



[5] http://www.stgeorge.com.au/corporate/transaction/int_rate_rm/swap.asp?orc=institution 2004-01-09
[6] Derivative Securities, Jarrow & Turnbull, South-Western College 2 edition ISBN-0538877405
[7] London Inter-Bank Offer Rate. The interest rate that the banks charge each other for loans (usually in Eurodollars). This rate is applicable to the short-term international interbank market, and applies to very large loans borrowed for anywhere from one day to five years. The LIBOR is officially fixed once a day by a small group of large London banks, but the rate changes throughout the day.

The current 6-month LIBOR rate is 6.45% per annum. The notional principal is 35 million $.

The fixed rate is a swap is usually quoted on a semi-annual bone equivalent yield basis. Therefore, the amount that is paid every six months is:

$$(\text{Notional Principal}) \left(\frac{\text{Days in period}}{365}\right)\left(\frac{\text{Interest Rate}}{100}\right)$$

$$= (\$35{,}000{,}000) \left(\frac{182}{365}\right)\left(\frac{7.19}{100}\right)$$

$$= \$1{,}254{,}802.74$$

Where it is assumed that there are 182 days in this particular period.

The floating side is quoted on a money market yield basis. The difference between the two rate computations is in the number of days in a year convention employed. Therefore, the payment is

$$(\text{Notional Principal}) \left(\frac{\text{Days in period}}{360}\right)\left(\frac{\text{Interest Rate}}{100}\right)$$

$$= (\$35{,}000{,}000) \left(\frac{182}{360}\right)\left(\frac{6.45 + 0.30}{100}\right)$$

$$= \$1{,}194{,}375$$

In a swap, the payments are netted. In this case, Party A pays Party B the net difference. $(1,254,802.74 – 1,194,375) = $60,427.74.

This example illustrates four points. First, payments are netted in interest rate swaps. In the example, Party A sent Party B a payment for the net amount. Second, principal is not exchanged. This is why the term “notional principal” is used. Third, Party A is exposed to the risk that Party B might default, and conversely, Party B is exposed to the risk of Party A defaulting. If one party defaults, the swap usually terminates.

Fourth, on the fixed-payment side a 365-day year is assumed, while on the floating payment side a 360-day year is used. The number of days in the year is one of the issues specified in the swap contract.

3.2 Swap Valuation[8]

Lets look at an example of a two year interest rate swap. A financial institution is receiving fixed payments at the rate 7.20% per annum and paying floating payments in a two year swap. Payments are made semi-annually. Details about payment dates, the term structure of Treasury rates and term structure of LIBOR rates are shown in the table below.

Payment Dates	Days Between Payment Dates	Treasury Bill Prices, B (0,T)	EuroDollar Deposit, L (0,T)
$t_1 = 182$	182	0.9712	0.9698
$t_2 = 365$	183	0.9399	0.9358
$t_3 = 547$	182	0.9088	0.9075
$t_4 = 730$	183	0.8801	0.8758

B(0,T) denotes the present value of receiving for sure one dollar at date T.
L(0,T) denotes the present value of receiving for sure one Eurodollar at date T.

First lets look at the fixed side of the swap. At the first payment date, t_1, the dollar value of the fixed payment is

$$N_p \times 0.0720 \times (182/365).$$

Where N_p denotes the notional principal. The present value today of receiving one dollar for sure at date t_1 is $B(0,t_1) = 0.9712$. Therefo r the present value of the first fixed payment is

$$N_p \times 0.9712 \times 0.0720 \times (182/365).$$

By repeating this analysis, the present value of all fixed payments is

$$\begin{aligned} V_r(0) = N_p & \times 0.9712 \times 0.0720 \times (182/365). \\ & + 0.9399 \times 0.0720 \times (183/365). \\ & + 0.9088 \times 0.0720 \times (182/365). \\ & + 0.8801 \times 0.0720 \times (183/365). \\ = & \ N_p \times 0.1332. \end{aligned}$$

8 Partly taken from: Derivative Securities, Jarrow & Turnbull, South-Western College 2 edition ISBN-0538877405

Low lets look at the floating side of the swap. The pattern of payments is similar to that of a floating rate bond, except that there is no principal payment in a swap. At time zero, the present value of the sequence of floating rate payments is the notional principal, N_p. But given that there is no principal payment in a swap, we must subtract the present value of principal repayment that would normally occur at time t_4. Thus, the present value of the floating rate payment is

$$\begin{aligned} V_F(0) &= N_p - N_p \times L(0, t_4) \\ &= Np[1 - 0.8758] \\ &= Np \times 0.1242\,, \end{aligned}$$

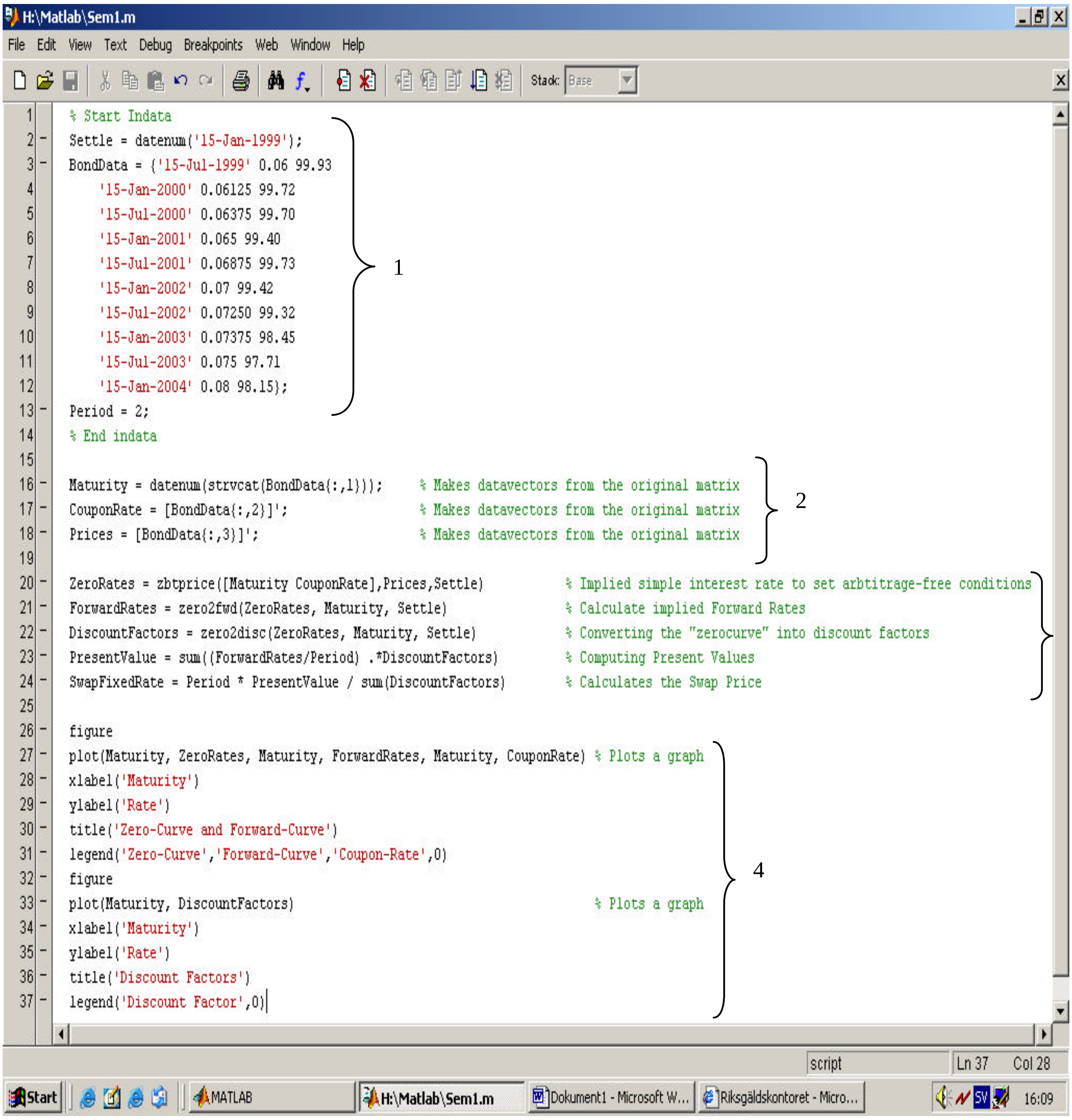
where $L(o,t_4)$ is the present value of receiving one Eurodollar at date t_4.
The value of the swap to the financial institution receiving fixed and paying floating is

$$\begin{aligned} \text{Value of Swap } &\equiv V_R(0) - V_F(0) \\ &= N_p \times [0.1332 - 0.1243] \\ &= N_p \times 0.0089 \end{aligned}$$

If the notional principal is 20 million dollars, the value of the swap is \$178,000.
In this example, the Treasury bond prices are used to discount the cash flows based on the Treasury note rate, and the Eurodollar discount factors are used to measure the present value of the LIBOR cash flows. This incorporates the different risks implicit in these different cash flow streams.

4 Matlab Application of Interest rate swap pricing

Here we are showing a way to price swaps in Matlab.



1: The inputs of this Matlab program are :

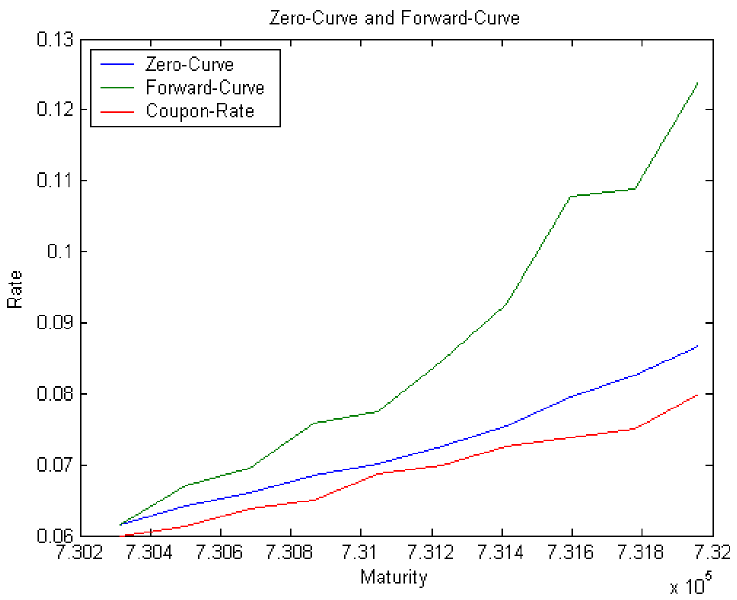
- Settle date 'Settle'. The settle date is the date when the contract is made.
- Maturity dates, coupon rates and market prices wich are all defined in 'BondData'.
- Number of periods 'Period'. The number of periods defines the lifespan of the coupons.

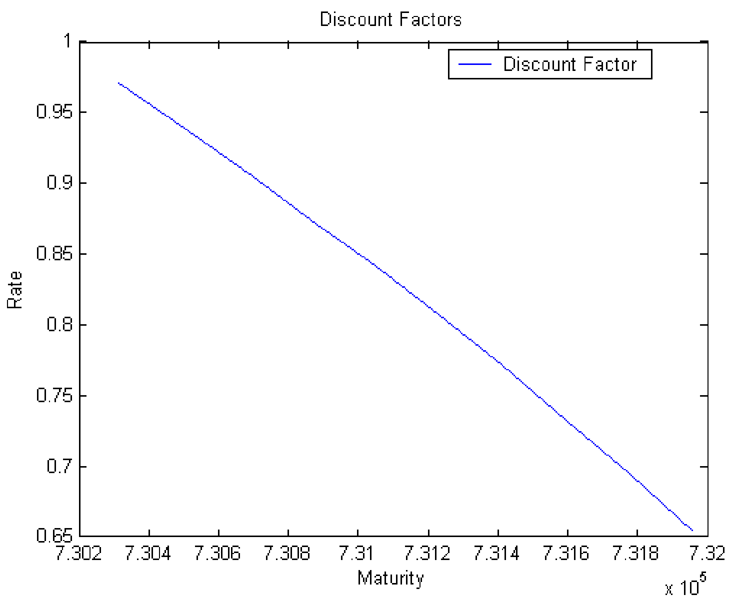
2: Here we let the program sort the 'BondData' into single vectors.

3: Here we let the program do the actual calculations to get the swap price with the help of term structure functions:

- 'zbtprice' , this bootraps the coupon bonds and gives us an implied zero curve.
- 'zero2fwd', this produces the implied forward rate from the implied zero curve.
- 'zero2disc', this produces the discount factors again using the implied zero curve.
- 'sum', this sums up all values.

4: This is the Matlab code of showing the results graphically. The results from this Matlab code is shown here:





5 Conclusion

Interest rate swap pricing and term structure analysis are closely related.
Interest rate swap pricing is a tool widely used to control the risk exposures of market fluctuations. When making major investments they normally lasts for several years and involve borrowed money. You can minimize your risk exposure, to not being able to meet your due payments during bad economic times, by entering into a swap contract. Interest rate swap pricing involves spending a lot of time analysing economic data and have a good ability to use and link various sophisticated financial tools. If using a developed software, like Matlab, configured to your business situation you can free up resources, which you then can allocate to other business practises.

6 References

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Matlab 6.5.0.180913a Release 13 June 18, 2002 & Financial Toolbox