Monte Carlo methods II

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Abstract

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Example: a down-and-out put option

Example 1. A *down-and-out* put option is a put option that becomes void if the asset price falls below the barrier S_b. Consider the European put option with $S_0 = \$50$, $X = \$50$, $r = 10\%$, $\sigma = 40\%$. This option has two months to maturity, each month consists of 30 days, and the barrier $S_b = \$40$ is checked every day. What is the price of this option?

Solution. Consider a multi-period binomial model with one-day period. We have $n = 60$ periods. Let S_k denotes the price of the asset at the end of the kth period. Then

$$S_{k+1} = \begin{cases} S_k e^{\sigma\sqrt{T/n}} \text{ with probability } p, \\ S_k e^{-\sigma\sqrt{T/n}} \text{ with probability } 1-p, \end{cases}$$

where $p = \dfrac{1 + rT/n - d}{u - d}$.

We will set up 100 experiments. In each experiment we generate 2000 trajectories of the pricing process. We estimate price as an average of the option's payoff with respect to trajectories.

MATLAB program, part 1

```
% File: MonteCarlo.m
% Estimating price of a down-and-out
% put option
% Author: Anatoliy Malyarenko
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S0=50;
X=50;
r=0.1;
T=2/12;
sigma=0.4;
Sb=40;
NSteps=60;
NRepl=2000;
NExperiments=100;
u=exp(sigma*sqrt(T/NSteps));
d=1/u;
p=(1+r*T/NSteps-d)/(u-d);
Price=1:NExperiments;
```
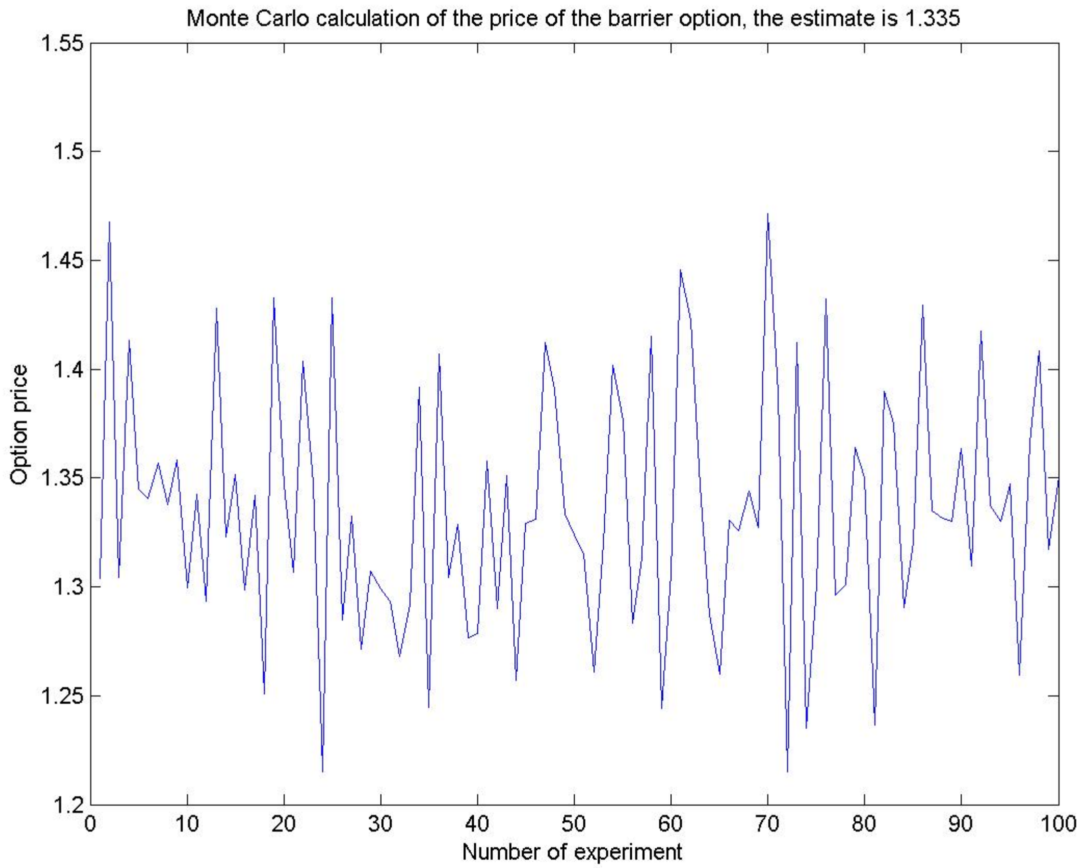

MATLAB program, part 2

```
for j=1:NExperiments
 Payoff=1:NRepl;
 for k=1:NRepl
  flag=0;
  Path=1:NSteps+1;
  Path(1)=S0;
  random=rand(size(Path));
  for l=2:NSteps+1
   if (random(l)<=p)
    Path(l)=Path(l-1)*u;
   else
    Path(l)=Path(l-1)*d;
   end;
   if (Path(l)<=Sb)
    flag=1;
```

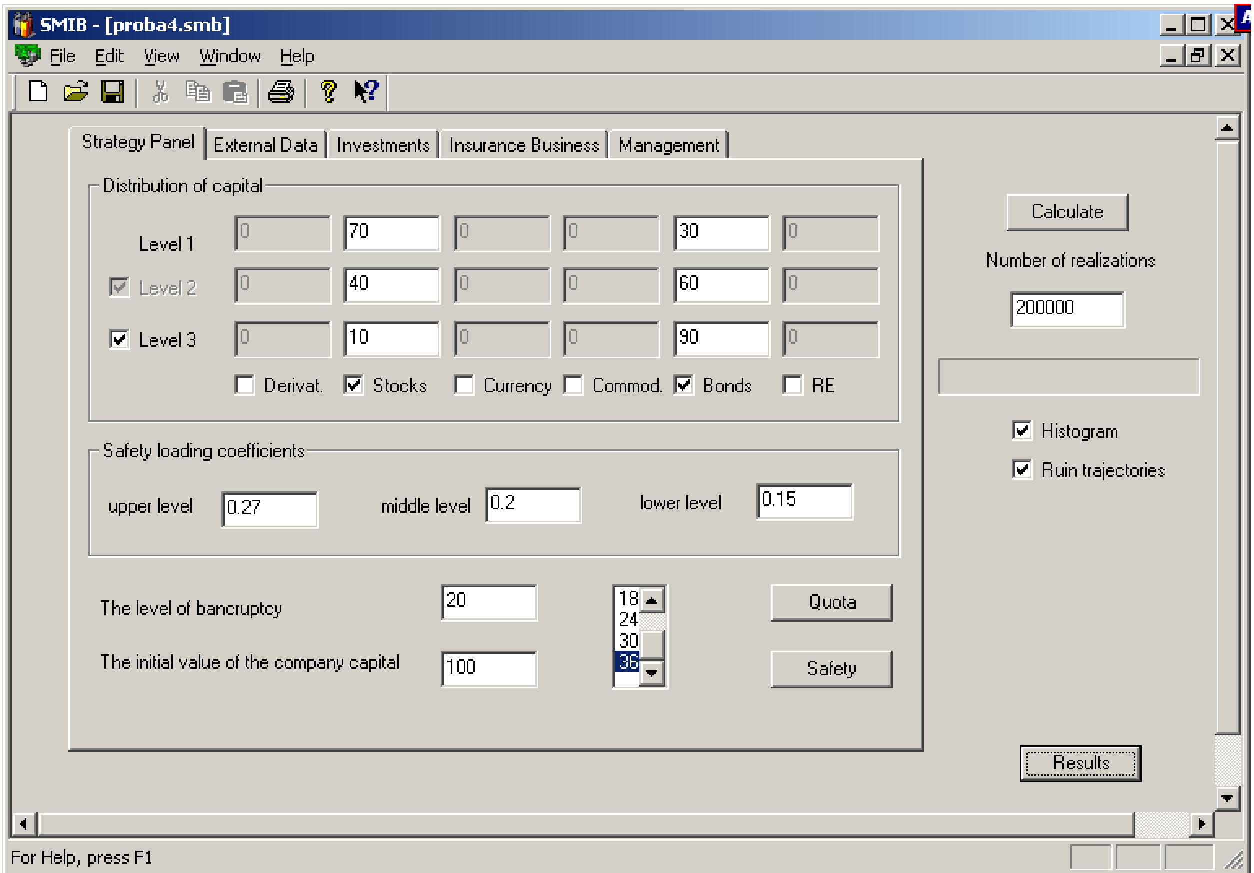
```
    break;
   end
  end
  if (flag==0)
   Payoff(k)=max(0,X-Path(NSteps+1));
  else
   Payoff(k)=0;
  end;
 end;
 Price(j)=sum(Payoff)/NRepl;
end;

price=sum(Price)/NExperiments;
plot(1:NExperiments,Price);
xlabel('Number of experiment');
ylabel('Option price');
```



□

A real program

Next pictures show the program that works using Monte Carlo methods. The authors are Dmitrii Silvestrov, Myroslav Drozdenko, and Anatoliy Malyarenko.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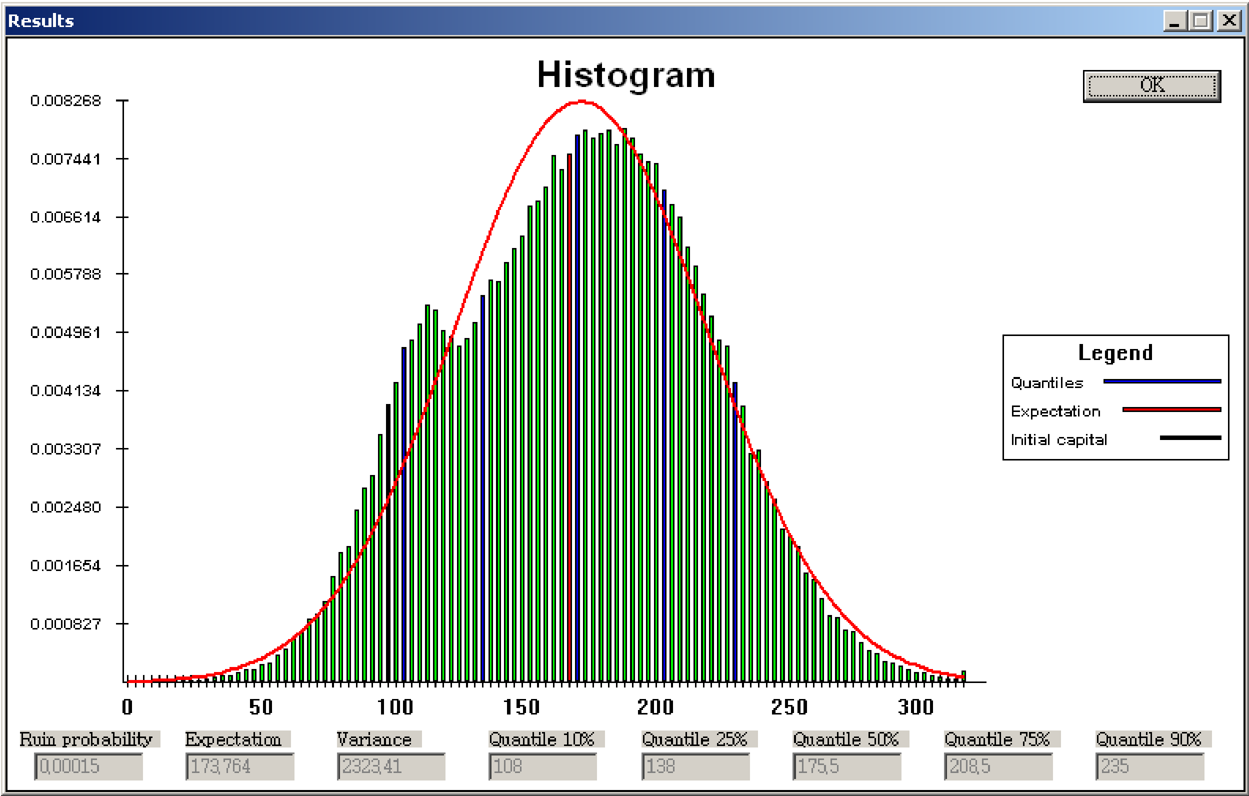
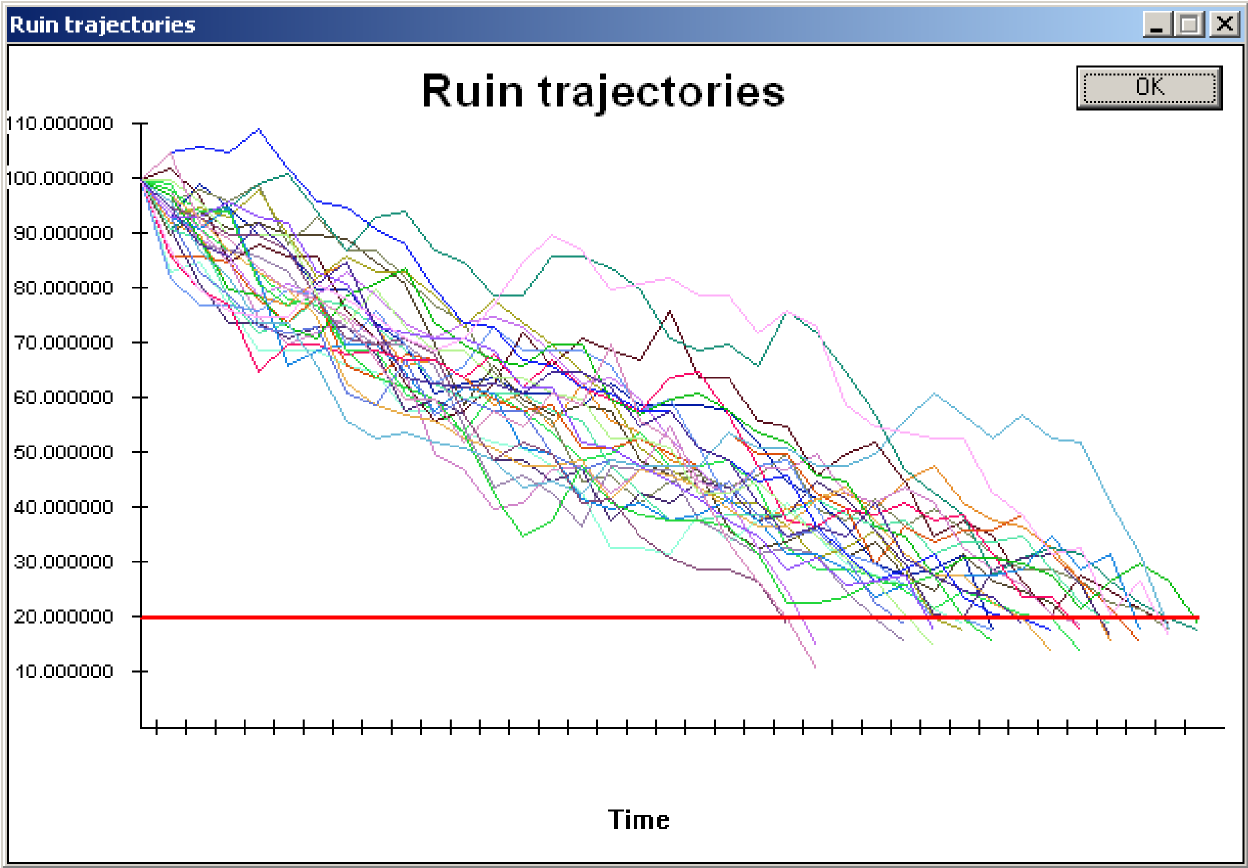
File Edit View Window Help
Strategy Panel
External Data
Investments
Insurance Business
Management
Distribution of capital
Level 1
Level 2
Level 3
Derivat.
Stocks
Currency
Commod.
Bonds
RE
Safety loading coefficients
upper level
0.27
middle level
0.2
lower level
0.15
The level of bancruptcy
20
The initial value of the company capital
100
Quota
Safety
Calculate
Number of realizations
200000
Histogram
Ruin trajectories
Results
For Help, press F1


Results
Histogram
OK
0.008268
0.007441
0.006614
0.005788
0.004961
0.004134
0.003307
0.002480
0.001654
0.000827
0
50
100
150
200
250
300
Legend
Quantiles
Expectation
Initial capital
Ruin probability
0,00015
Expectation
173,764
Variance
2323,41
Quantile 10%
108
Quantile 25%
138
Quantile 50%
175,5
Quantile 75%
208,5
Quantile 90%
235



Problems

1. The *down-and-in* put option is activated only if the barrier level S_b is crossed. Consider the three-month European down-and-in put option with the asset price \$50, strike price \$49, and barrier level \$39. The risk-free interest rate is 8%, the volatility is 30% per year. Assume that the price of the asset is checking each day at the close of the trading, and that each month consists of 30 days.

   Perform 100 experiments. In each experiment generate 1000 realisations of the pricing process. Draw the graph that shows the estimated price of the option in every experiment.