Credit derivatives

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Abstract

Contents of the lecture.

- Credit Default Swaps.
- Credit Default Swaptions.
- Credit Spread Options.

Background

Credit derivatives include a range of instruments designed to transfer credit risk without requiring the sale or purchase of bonds and/or loans. They were originally designed in the early 1990s by U.S. banks to manage credit risk in their loan portfolios while preserving important customer relationships. The evolution of credit derivatives was prompted by the increased demand for asset-backed deals backed by credit instruments. The credit derivatives market has been growing rapidly since the early 1990s.

The evolution of credit derivatives allows domestic banks to provide the following benefits to their clients.

- Customised exposure to credit risk.
- Enable users to take short positions in credits previously not possible in the underlying securities.
- Provide institutional investors access to the bank loan market, generally on a leveraged basis.

Increase diversification in concentrated credit portfolios.

Extract and hedge specific sections of credit risk.

Of the instrument group termed "Credit Derivatives" the most commonly traded are

- **Credit Default Swaps.**
- **™** Total Return Swaps.
- Credit Default Swaptions.
- **Credit Spread Options (Eurobond Options).**

Credit Default Swaps (CDS): definition

The credit default swap market consists of buyers and sellers of credit protection. In its simplest form, the "protection buyer" pays a fixed periodic fee (as a percentage of notional) and receives, in turn, a contingent payment in the event a default occurs on the underlying credit. For default swap transactions the credit default payment must be clearly defined; thus it is triggered by a material event such as a scheduled payment delinquency combined with a significant decline in the market value of a higher risk credit. The contingent payment is usually settled in cash and equals the total market value loss incurred by the protection buyer on the reference asset. An alternative to cash settlement (though not often used) is the physical delivery of the reference asset for its par value. This solution is not always viable, however, as banks typically prefer not to sell their loans but rather seek to maintain the customer relationship by managing the underlying credits.

A CDS is a contract where one party pays the other a periodic fee, in return for a variable payment contingent on the default of a third party credit.

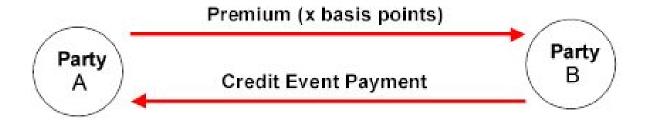
The variable payment will typically be equal to the loss that would have been suffered by owning the underlying security. The payout is defined as Par minus the post-default value. Normally, but not always, the party paying the periodic fee (the buyer of "protection") owns the underlying credit reference and is therefore entering into the CDS as a means of protecting credit exposure to that reference, thus avoiding any need to sell the underlying position.

Credit Default Swaps: example

Party A owns a security that pays an annual coupon of 8%. Party A does not want to (or cannot) sell the security, but no longer wishes to be exposed to the Default Risk of this security for a given period of time (which may or may not be until the maturity of the underlying instrument). Party A therefore buys a credit guarantee from Party B for the given period of time desired, in exchange for a regular payment.

A pays B: x basis points;

B pays A: If default: par minus the post-default value.



Using the CDS, Party A has transferred the risk of default of the underlying security to Party B for the period of the swap. In the event of default of the underlying security, Party B will be responsible for making good the effective reduction in value of the

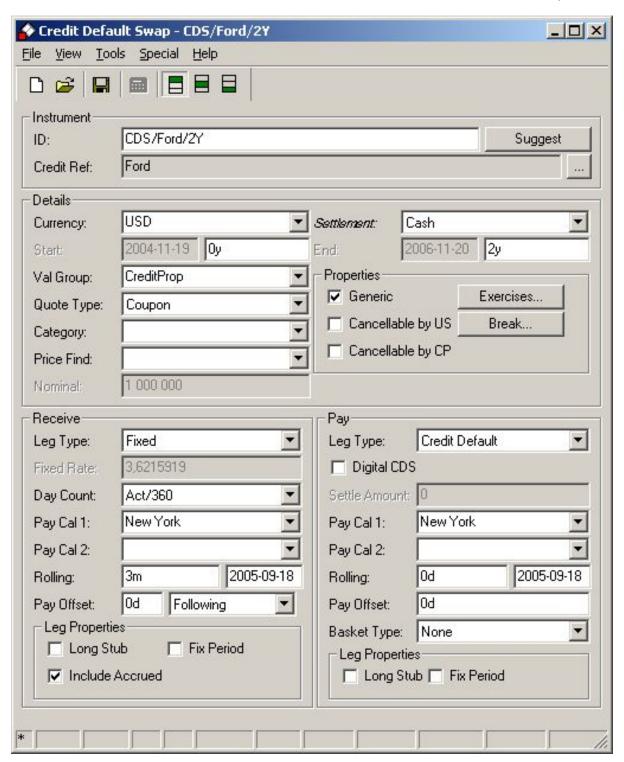
underlying security arising as a result of the credit default event, should such an event occur during the period of the swap.

Credit Default Swaps: representation

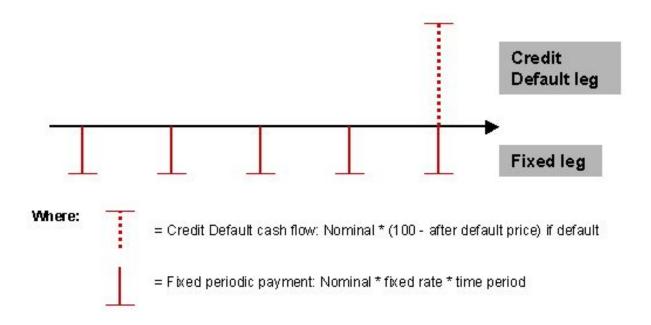
CDS are defined in the Credit Default Swap application. Two characteristics differentiate this application from others.

- The leg type **Credit Default** for defining the credit default leg.
- A field unique to Credit Derivatives instruments: the **Credit Ref** field where the Reference Security is entered. Clicking the "Credit Ref" button selects the desired instrument from the selection subwindow. This reference can be to a non-generic or generic instrument if desired. A generic underlying is used if the contract does not refer to a specific issue, but to an issuer.

Note also that it is possible to define if the settlement, in the event of default, will be cash or physical. To do this, make a selection in the **Settlement** field.



Cash flow representation



The first leg is a Credit Default leg. The cash flow will only take place in the case of default of the credit reference. The value of this cash flow will be calculated using the yield curve mapped to the credit reference, and the yield curve mapped to the currency. In the case of default, the payment will be settled on a settlement date not known in advance. This settlement date does not have anything to do with the end date of the contract as is depicted in the figure.

The second leg is a normal Fixed Leg. The valuation of this leg is done using the yield curve

mapped to the credit reference. This incorporates the fact that these fixed payments will only be paid if no credit default has taken place.

Default events and probabilities

The superscript M stands for marginal probability (unconditional), A for absolute probability and C for conditional probability. Absolute probabilities are also called cumulative probabilities. Default events are denoted as

 $\begin{array}{ll} D_i^M & \text{default occurs in period } [i,i+1] \\ D_i^A & \text{default occurs in period } [0,i] \\ D_i^C & \text{default occurs in period } [i,i+1] \text{ given no} \end{array}$ default up to time *i*.

Probabilities are denoted as

$$p_i^M = P\{D_i^M\}, \qquad p_i^A = P\{D_i^A\}, \qquad p_i^C = P\{D_i^C\}.$$

Credit curves

The term "Credit Curve" is a collective term used for the following three curves.

- Credit Spread Curve. Depending on which type of instrument is being priced this curve is either the Credit Default Swap (CDS) basis-point spreads expressed in par terms or the Eurobond Curve.
- Probability Curve. This curve shows the absolute (cumulative) default probability curve.
- Hazard Rate Curve. This curve shows the default intensity curve.

In addition to these three Credit Curves a **Risk Free Curve** is required to price credit instruments.

The probability curve and hazard rate curve are connected by the following formula:

$$p_i^A = 1 - \frac{1}{1 + h_i},$$

where

t_i time between today and day *i* measured as fraction of year, using Act/365

 h_i hazard rate, annual compounding, Act/365, between today and day i.

To find the Par CDS from the Probability Curve the following formula is used:

$$S_n = (1 - R) \frac{\sum_{i=1}^{n} (p_i^A - p_{i-1}^A) df_i}{\sum_{i=1}^{n} (1 - p_i^A) df_i},$$

where

 S_n par CDS spread at period n

 t_i years between day i-1 and day i with respect to the day count method

 df_i risk-free discount factor between today and day i

R recovery rate.

To find the Eurobond prices from the Probability

Curve the following formula is used:

$$P = N \cdot df_n (1 - p_n^A) + RN \sum_{i=1}^n df_i (p_i^A - p_{i-1}^A)$$
$$+ N \sum_{i=1}^n c_i t_i (1 - p_i^A).$$

where

P price of the Eurobond

N nominal amount

 c_i coupon rate as percentage of nominal for period i.

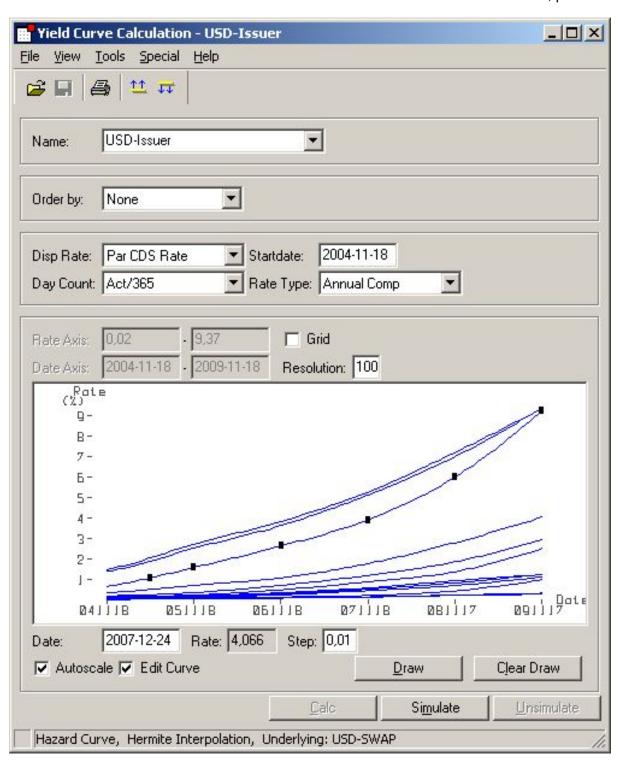
Valuation of a CDS

Example: Hazard Rate CDS

To be able to use this method it is important to first define the relevant spread curves and recovery rate infrastructures. A CDS has a credit reference, which is mapped to a credit curve, where the spread represents the credit quality of the reference. To price a CDS using the Hazard Rate model the Credit Curve and Risk-free curve must first be defined.

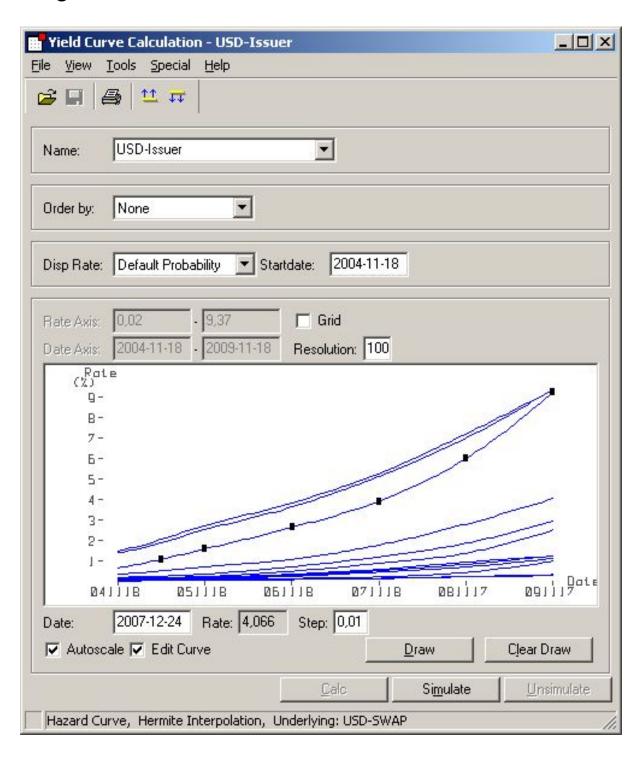
The credit curve or spread curve has to be defined as a yield curve of type "Attribute Spread". The storage type should be set to "Par CDS Rate". The day count method and rate type should be set according to the input spread information. For example, if the input CDS spreads are based on annual credit default swap payments, then the storage rate type should be set to "Annual Comp".

If the storage type is set to "Par CDS Rate", then a hazard rate curve will be calibrated that matches the input par CDS spreads. A Newton–Raphson multi dimensional numerical solver makes the conversion. These settings are illustrated in the following diagram:



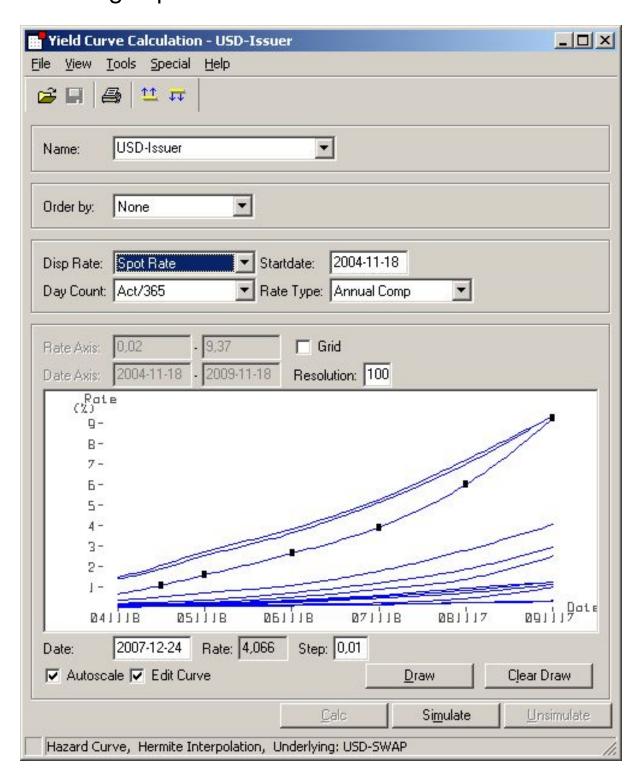
In the Yield Curve Calculation application the default probabilities can be displayed by selecting

"Default Probability", as illustrated in the following diagram:

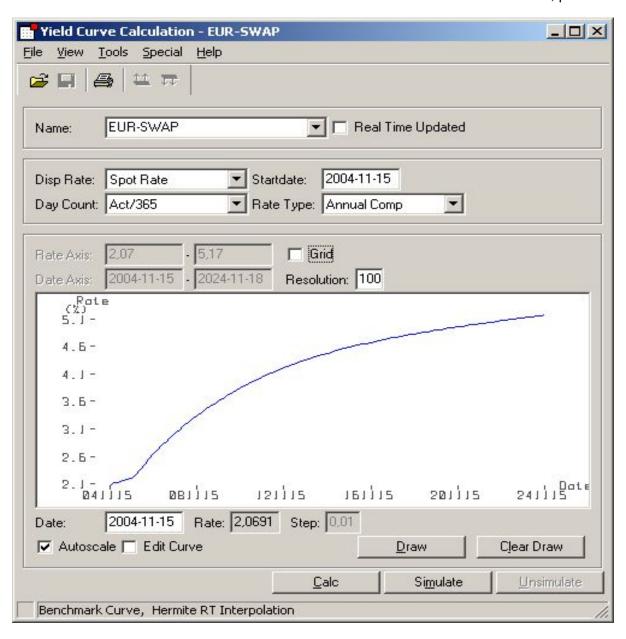


The default probabilities are expressed in

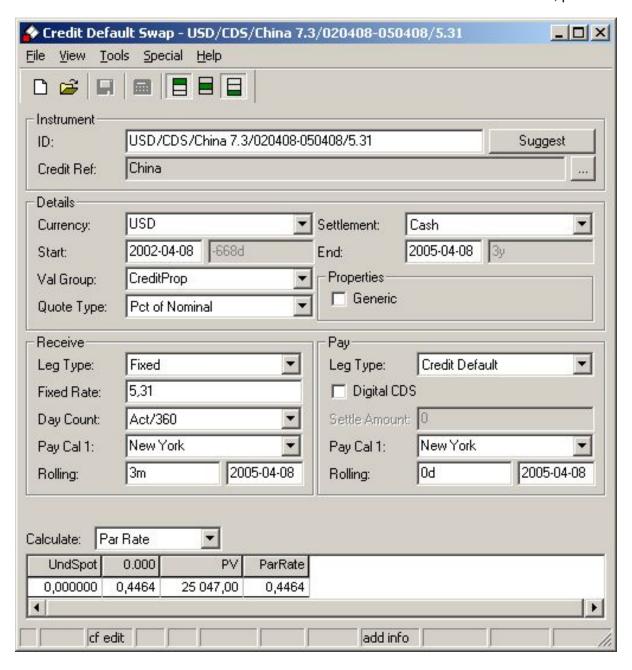
percentage terms and are absolute (cumulative) probabilities. The hazard rates can be displayed by selecting "Spot Rate":



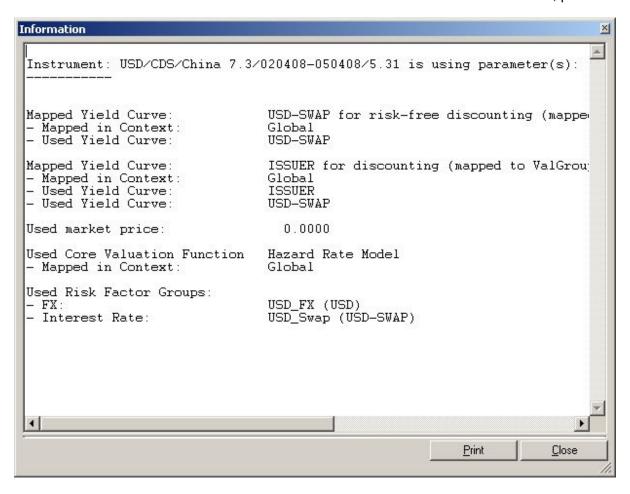
The risk-free curve is used both as a reference curve and for discounting purposes (to find the risk-free discount factors). The risk-free curve is normally represented by a Swap curve. A Swap curve should be represented as a "Benchmark" curve, where the benchmark instruments are used to calibrate the curve. The following example is of a typical USD-SWAP curve:



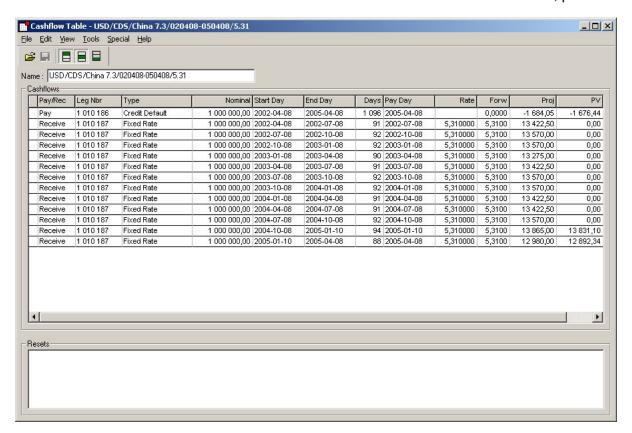
Consider the following CDS:



Details of the CDS are displayed in the Information window as illustrated in the following diagram:



The following diagram is of the Cash Flow Table for the CDS:



The CDS in this example is priced with a CDS spread of 531 basis points. The par rate of this Swap is exactly the same as the spread, which means that the CDS prices back to market. The present value of the Swap will then be zero.

The present value of a Credit Default payment is calculated as a series of contingent cash flows. The contingency is based on whether or not the issuer of the underlying bond defaults.

At the end of each time period (the Default Probability dates), default is assumed to occur and

the default payment is made (the pay off). This payment is then weighted with the probability of default actually occurring during this time period (that is, weighted with the Default Probability referring to this period). Hence we get the expected value of the Credit Default payment at this date.

The expected value is then discounted back to today to derive the present value. Since the default probabilities have been used to model the credit risk the Risk-free rate should be used when determining the discount factor. The present value is calculated in accordance with the following PV formula:

$$PV = \sum_{i=1}^{n} df(i)p_i^M(1-R)N,$$

where

df(i) risk free discount factor between today and day i

R recovery rate

N nominal amount.

The projected cash flow is then calculated by capitalising the present value with the risk free curve.

The present value of a fixed rate payment leg is the sum of all individual cash flows' present value. To calculate the present value of specific cash flow, discount the projected cash flow with the risk-free discount factor and multiply that with the hazard factor.

The projected cash flow, pcf, for a single payment is:

$$pcf = N \frac{c}{100}$$
 · fraction of year,

where

N nominal amount

c coupon payment in percentage terms of the nominal amount.

Then the present value, PV, of this cash flow is calculated by:

$$PV = pcf \cdot df(1 - p_A),$$

where

df risk free discount factor between today and the pay day

- p_A the absolute (cumulative) default probability between today and the pay day
- $1 p_A$ the hazard factor, that is, the absolute (cumulative) *survival* factor between today and the payday.

The present value of the payment leg is then calculated as the sum of the present values of the individual payments.

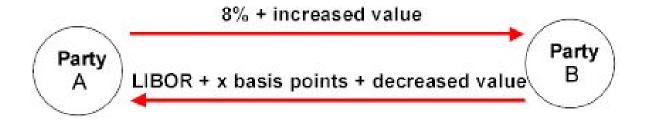
Total Return Swaps (TRS): definition

bilateral Total Return Swaps are financial transactions where the total return (which equals the coupon plus price change) of a fixed income security is exchanged for a funding cash flow, usually LIBOR plus a basis spread. Unlike Credit Default Swaps, payments to balance the underlying credit's price depreciation or appreciation are always exchanged without requiring the occurrence of a specific credit event. This swap structure is beneficial to investors as it involves a leveraged participation in a fixed income instrument without the origination cost. Total Return Swaps are particularly attractive to investment firms that want to diversify their portfolio credit exposure.

A TRS is a contract where one party pays the other the total return on a credit, receiving in exchange a LIBOR-based payment. The total return payment comprises the periodic interest payments on the underlying security plus or minus changes in the underlying value on the underlying security. Price settlement takes place at maturity of the swap.

Total Return Swaps: example

Party A owns a security, which pays an annual coupon of 8%. Party A does not want to (or cannot) sell the security, but no longer wishes to be exposed to the risks/returns of this security for a given period of time (which may or may not be until the maturity of the underlying instrument). Party A therefore transfers the complete risks and returns to Party B for the given period of time desired.



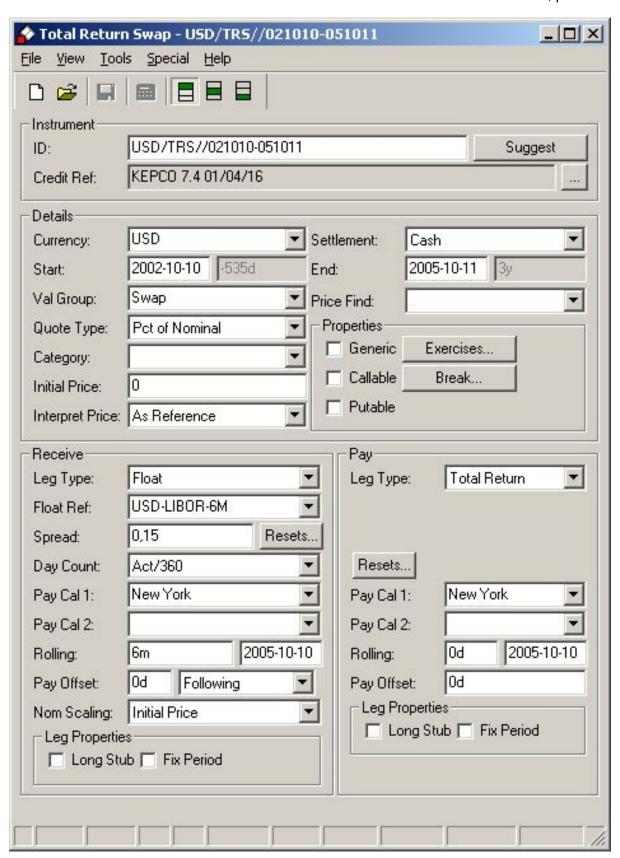
Using the TRS, Party A has transferred all market and credit risk of the security to party B for the period of the swap.

Total Return Swaps: representation

Total Return Swaps (TRS) have their own instrument window. The application has a specific leg type (the "Total return leg") and two fields unique to Credit Derivatives instruments:

Credit Ref. This is the reference security and is stored in the "leg" database table in the field credit_ref. Clicking the Credit Ref button can enter the security and then selecting the desired instrument from the selection subwindow. This reference can be to a notional or generic instrument if desired.

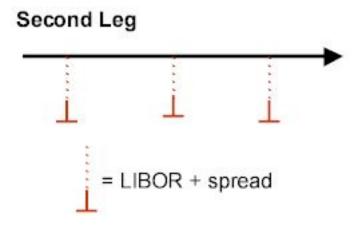
Initial Price. This refers to the initial clean price of the underlying Credit Reference.



FRONT ARENA facilitates both European style and U.S style TRS:

- European style TRS has one total return payment on maturity.
- U.S style TRS divides the total return payment into several payments until maturity.

To represent a Total Return Swap (European style), the leg type **Total Return** is used.



This leg is often referred to as the financing side of the transaction. It is a normal Float Leg, with normal floating cash flows.

U.S style TRS can be replicated by a series of European style TRS that are successively forward starting with no overlaps.

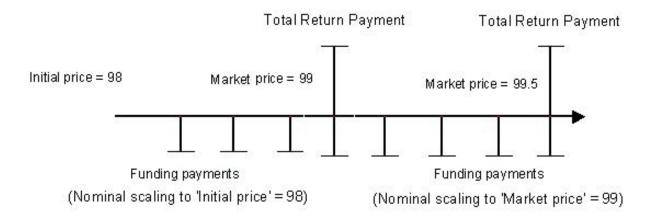
For example, consider two European style TRS with maturity of 0Y-6M and 6M-1Y, the later being forward starting. Holding these two European style TRS is equivalent to holding one U.S style TRS of maturity 0Y-1Y with a rolling period of 6M. This similarity holds for both actual reset fixings and estimated reset values.

To set up a U.S style TRS in FRONT ARENA construct a European style TRS and:

- Set Nominal Scaling to "Price" on the financing side.
- Set the Rolling Period on the total return side to the desired frequency.

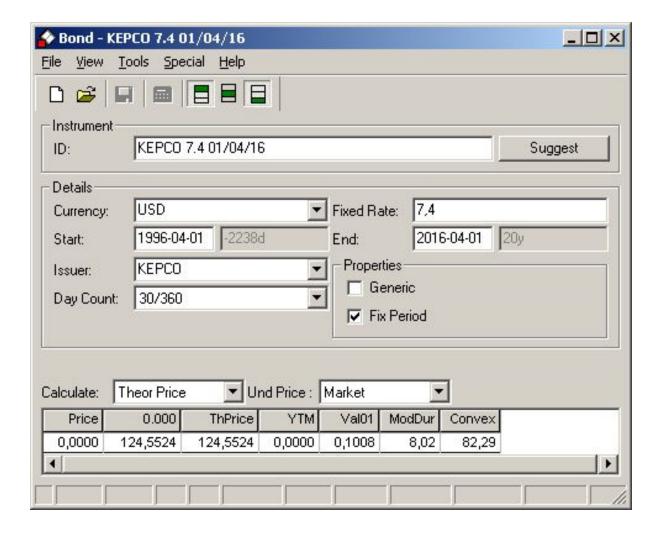
The Nominal Scaling option "Price" is used for U.S style TRS with several total return payments. The funding payments will be scaled with different Nominal Scalings. The first reset will be the reset set in the Initial Price field in the TRS definition. All funding payments will be scaled with the same Nominal Scaling (= Initial Price of the bond reference in the TRS definition).

When a Total Return payment is made and the "Market price" of the reference security does not equal the "Initial price" the funding payments in the next period will be scaled with a new Nominal Scaling set to the "Market price". The Nominal Scaling will change with each new "Market price". This is illustrated in the following diagram:

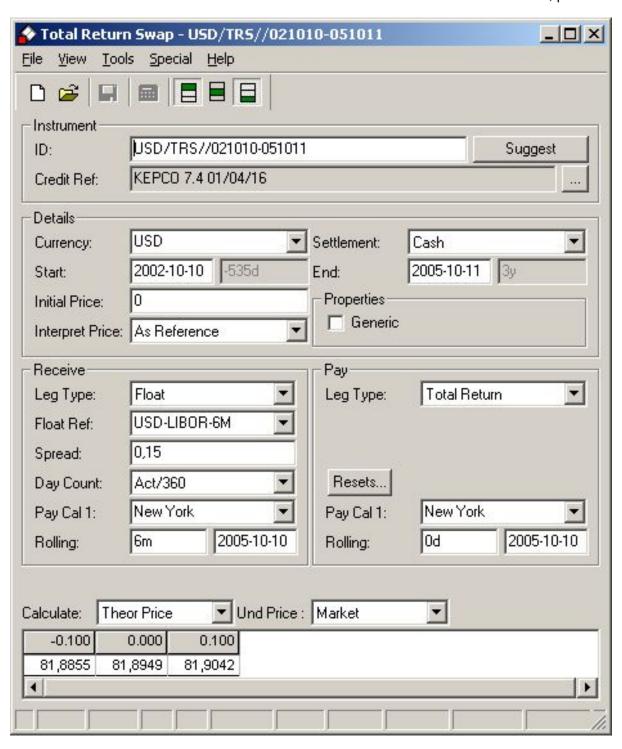


Example

We have a bond.

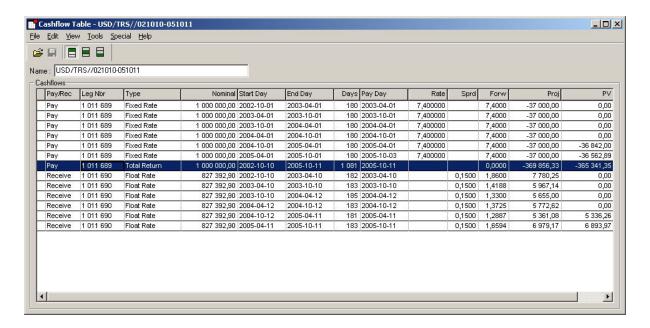


And we buy the TRS protection on the same bond.



We are buying protection for the same nominal amount which we will swap the coupon for so that we receive against the USD-LIBOR-6M rate until 11-

Oct-05.



From the Cash flow table above we can see clearly that we pay away the coupons we receive 7.4% semi-annually from the bond and receive USD-LIBOR-6M. There is also a Total Return payment that will be determined at the end of the contract after fixing the value of the bond. (See fixing table.) At any point before the maturity of this contract the value of Total return Cash flow will be the value of the difference between the initial price and the end expected value of the bond.

Options on Credit

There are mainly two different types of options to long/short credit: Credit Default Swaptions or Credit Spread Options. These option types can be embedded in Callable/Putable structures and can be either default or non-default contingent.

The main difference between Credit Default Swaptions and Credit Spread Options is the modeling of the underlying forward spread.

Credit default swaptions

An option on a Credit Default Swap (CDS) is by definition a credit derivative. It is an option to buy or sell a Credit Default Swap at a pre-defined par spread.

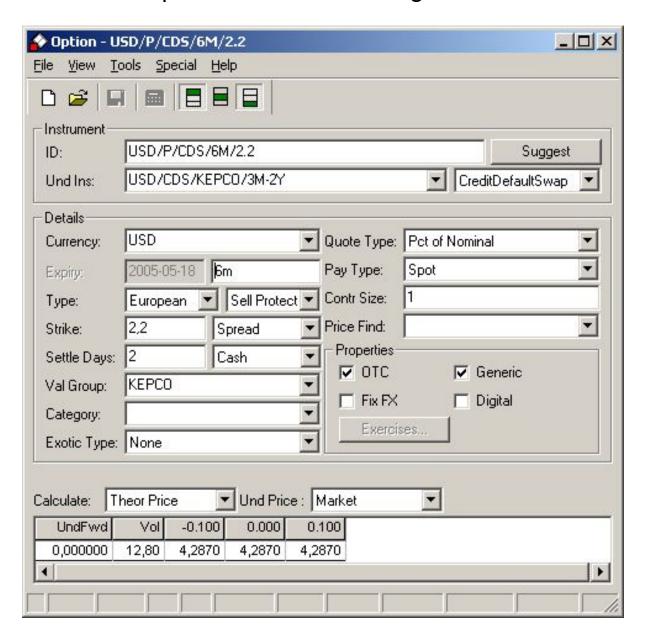
An option on a Credit Default Swap is a right, but not an obligation, to enter into a CDS contract.

Options on Credit Default Swaps are of the type "European" and are contingent on default in that they will be canceled in the event of default of the underlying occurring prior to the expiration of the option.

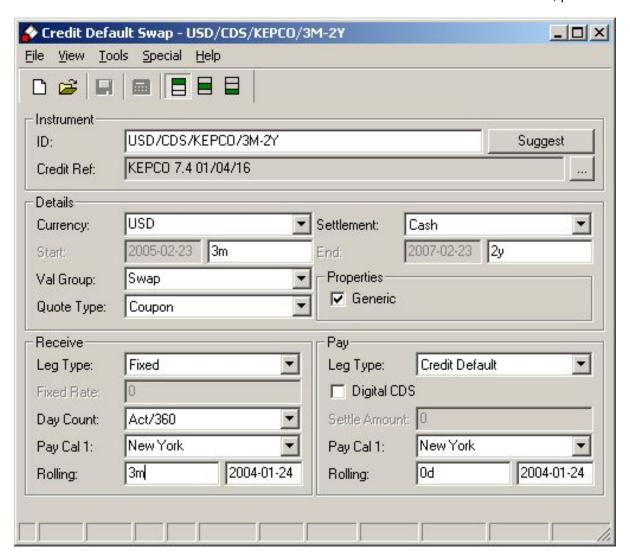
The strike of these options is expressed as the spread of the underlying Credit Default Swap.

Example

Calculating the theoretical price of a Credit Default Swaption with the following instrument data:



The underlying CDS has the following details:



The theoretical price is 4.2870.

Credit Spread Option

A Credit Spread Option is a credit derivative. It is an option to buy or sell a security at a specific spread above another security or the swap curve. The strike for these options is therefore expressed in spread, that is the number of basis points above the reference at which the security can be bought or sold. A Credit Spread option is not the same as an option on a Credit Default Swap.

If the credit quality (ratings) of the reference security deteriorates, the spread will widen.

A Credit Spread Option can be either default contingent or non-default contingent. A non-default contingent option implies that if the underlying security defaults prior to the expiry date of the option, the option will be worthless.

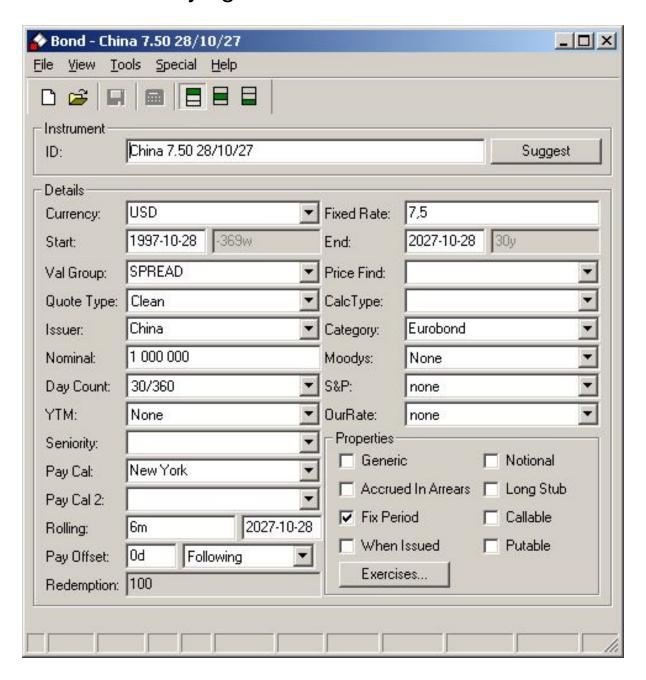
A call Credit Spread Option (also referred to as being a put on spread) is an option to go long on the credit. This is equivalent to a call on asset swap (that is, right to buy the bond). A call credit spread is defined as an option on

a bond/FRN with strike type set to "Spread" for Bonds as underlying instruments and to "Absolute" for FRN's as underlying instruments. This means that an option with strike 0.50 gives the owner of the option the right to buy the underlying bond to a clean price corresponding to an asset swap spread of 50 basis points. Hence the option is in the money if the underlying is traded at an asset swap spread below 0.50 at expiry. If the underlying is an FRN, then the strike (e.g. 100) will be in absolute terms, meaning that the option holder will have the right to buy the underlying at a clean price of 100.

For example, a 1 year option to buy the Eurobond USD/Yokohama/7.625%/2004 at 40 basis points spread above USD swap curve. Currently the bond is now trading at a price corresponding to a 45 basis points spread above the swap curve. If the Yokohama credit improves and the bond trades below 40 basis points one year from now, the option will be exercised at expiration.

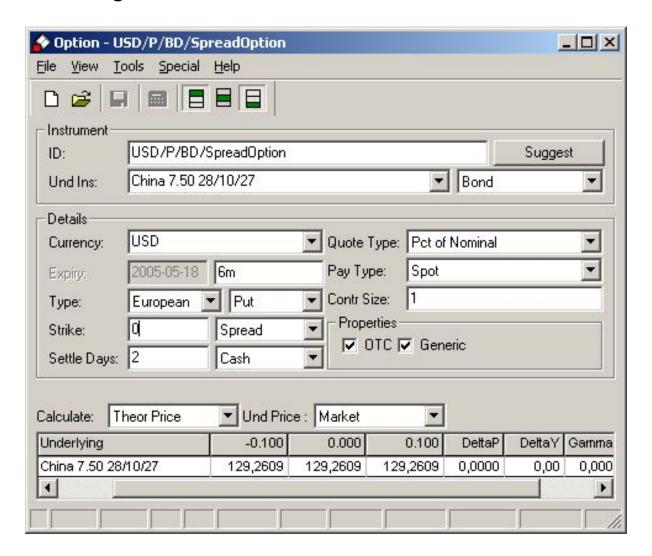
Example

The underlying bond is as follows.



The corresponding Credit Spread Option has the

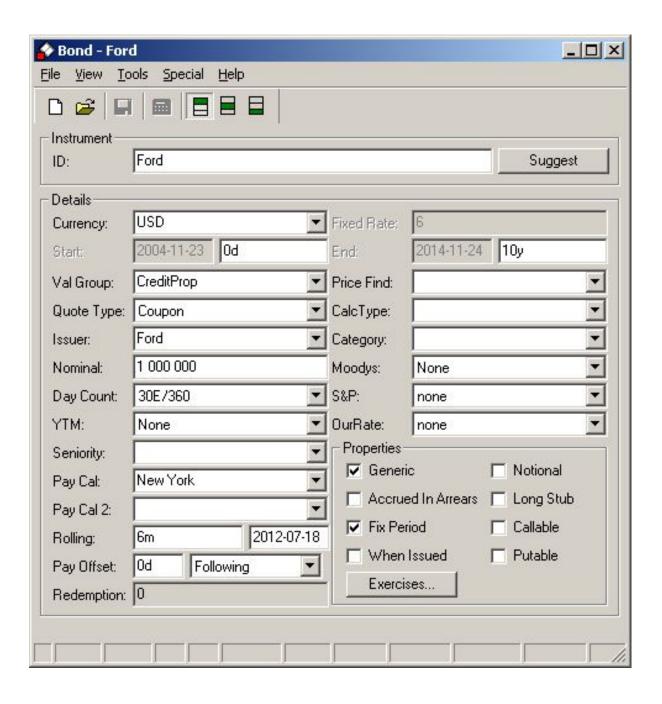
following details.



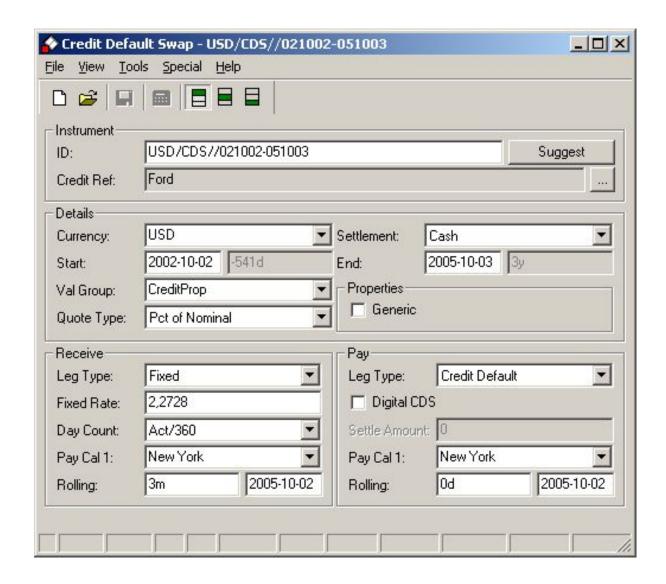
The theoretical price of the option is 129.2609.

Exercises

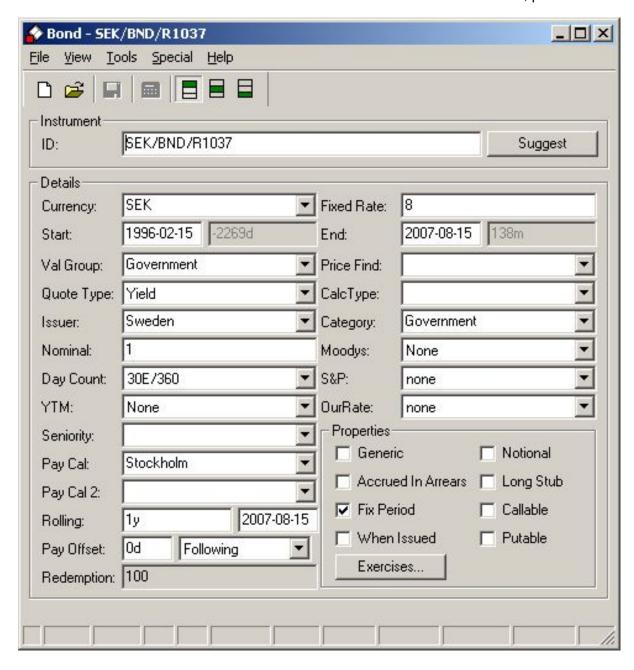
① The underlying bond has the following description.



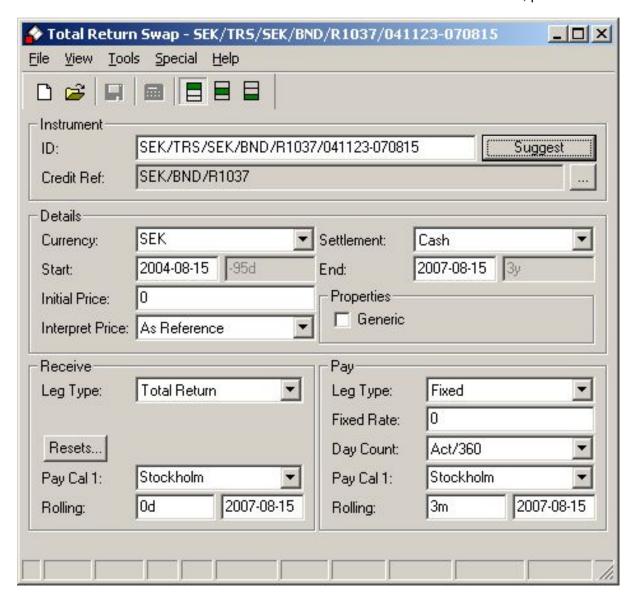
Calculate the par rate of a Credit Default Swap with the following data.



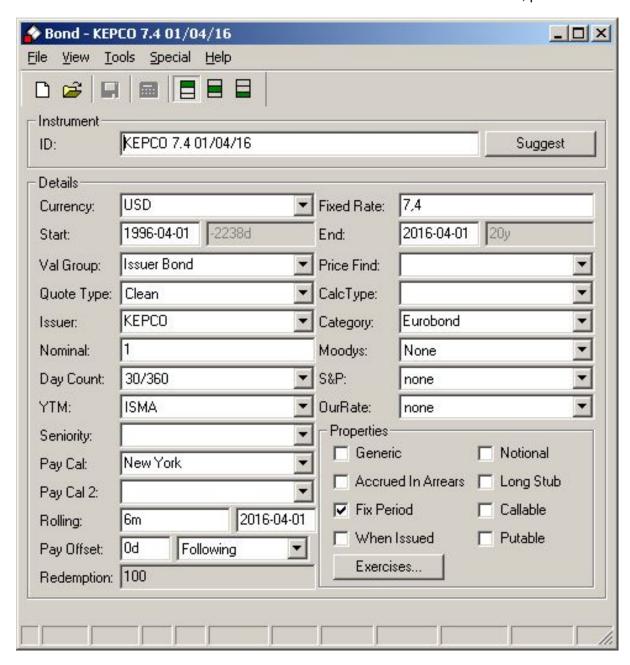
② The underlying bond has the following description.



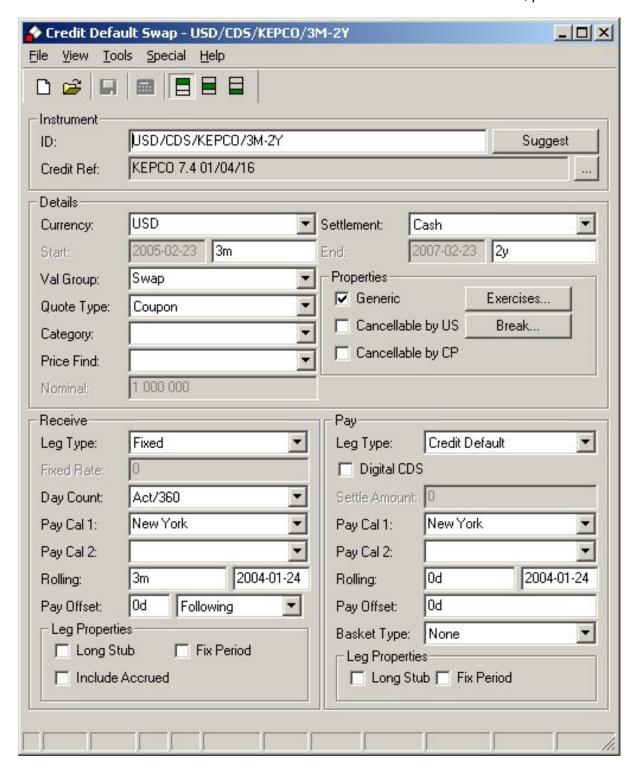
Calculate the theoretical prices of a Total Return Swap with the following characteristics.



3 The underlying bond has the following description.



The corresponding Credit Default Swap has the following characteristics.



Calculate the theoretical price of the following Credit Default Swaption.

