The Option Adjusted Spread model

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Abstract

Contents of the lecture.

☞ The Option Adjusted Spread model.

About the OAS model

The Option Adjusted Spread (OAS) model uses a binomial tree approach and adjusts for the cost of the embedded option and the difference between model price and market price due to other risks, for example credit and liquidity risks.

To adjust the theoretical price on the binomial tree to the actual price, a spread (called option-adjusted spread since the context of OAS started with trying to correct for mis-pricing in option embedded securities) is added to all short rates on the binomial tree **such that the new model price after adding this spread makes the model price equal the market price** (this is the defining purpose of OAS).

The value of option adjusted spread is that it enables investors to directly compare fixed income instruments, which have similar characteristics, but trade at significantly different yields because of embedded options. For example, an investor might be comparing a callable bond to a mortgage-backed security. If the two had comparable credit risk and liquidity, the investor might purchase whichever one had the higher option-adjusted spread (provided > 0) it would offer higher compensation for the risks being taken (due to mis-pricing).

The OAS model in has three dependent variables:

☞ Option Adjusted Spread;

☞ Bond Price;

☞ Volatility.

Definitions

A **bullet bond** is a conventional bond paying a fixed periodic coupon and having no embedded optionality. Such bonds are non amortising, i.e., the principal remains the same throughout the life of the bond and is repaid in its entirety at maturity. Bullet bonds are also called *straight bonds*. In the United States such bonds usually pay a semi-annual coupon. The coupon rate (CR) is stated as an annual rate (usually with semi-annual compounding) and paid on the bond's par value (Par). Thus, a single coupon payment is equal to $1/2 \times \text{CR} \times \text{Par}$.

A **benchmark bullet bond** is a bullet bond issued by the sovereign government and assumed to have no credit risk (e.g., Treasury bond).

A **non-benchmark bullet bond** is a bullet bond issued by an entity other than the sovereign and which, therefore, has some credit risk.

A **callable bond** is a bond where the issuer has the right, but not the obligation, to call back/repurchase the bond at one or more specified points over the bond’s life. If called, the issuer pays the investor the pre-specified call price. The call price is usually higher than the bond’s par value. The difference between the call price and par value is called the *call premium*.

A **putable bond** is a bond where the holder has the right, but not the obligation, to put back the bond at one or more specified points over the bond’s life. If putted, the investor pays the issuer pre-specified put price. The difference between the put price and par value is called the *put premium*.

The bullet bond can be used to find the “value” of the embedded option. For a callable bond the option value is given by the price difference between the bullet bond and the callable bond.

OAS analysis

There are six steps associated with FRONT ARENA OAS analysis. The assumption below is that the method is being applied to a callable bond.

① For every payday, find the benchmark forward rate.

② Build a binomial tree with equal probabilities ($= 1/2$).

③ Calibrate the model by adjusting the nodes in the tree until the model's predicted price matches the prices using the forward rates for all Bond Event Days.

④ Calibrate the model by adding the same number of basis points (the spread factor) to all rates in the tree until the model's predicted price matches the actual market price of the callable bond. The result is the bond's OAS.

⑤ Apply the same OAS to value a bullet bond with terms identical to the callable bond (except that the bullet bond is not callable).

⑥ Take the difference between the value obtained for the callable bond and the value obtained for the non-callable bullet bond. This difference is the value of the embedded call option.

Step 1: benchmark forward rates

A **payday** is any date when the Bond pays a coupon and/or the nominal value. The callable Bond has two statuses, callable and non-callable. The Bond can be callable at certain dates and/or in time-periods and the callable strike price might change on such day. The forward rates are found via interpolation from the yield curve given as **Underlying Yield Curve** (Und_YC) in the Yield Curve Definition application.

Step 2: building a binomial tree

The tree is built using the annual volatility, σ, of the forward rates. The process can be illustrated using the following four forward short rates (all expressed with semi-annual compounding): $f_1 = 6.000\%$, $f_1 = 7.200\%$, $f_3 = 8.150\%$, $f_4 = 8.836\%$.

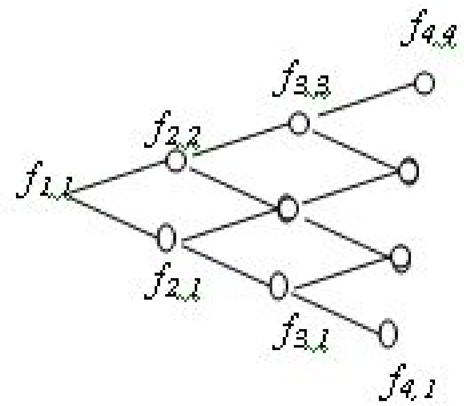
Assume that annual volatility of the forward short rates is 15%. The volatility spread factor Z_i is then defined as:

$$Z_i + e^{2\sigma\sqrt{t_i - t_{i-1}}},$$

and the tree is built with the following relation between the nodes:

$$f_{i,j} = Z_i^{j-1} f_{i,1}, \tag{1}$$

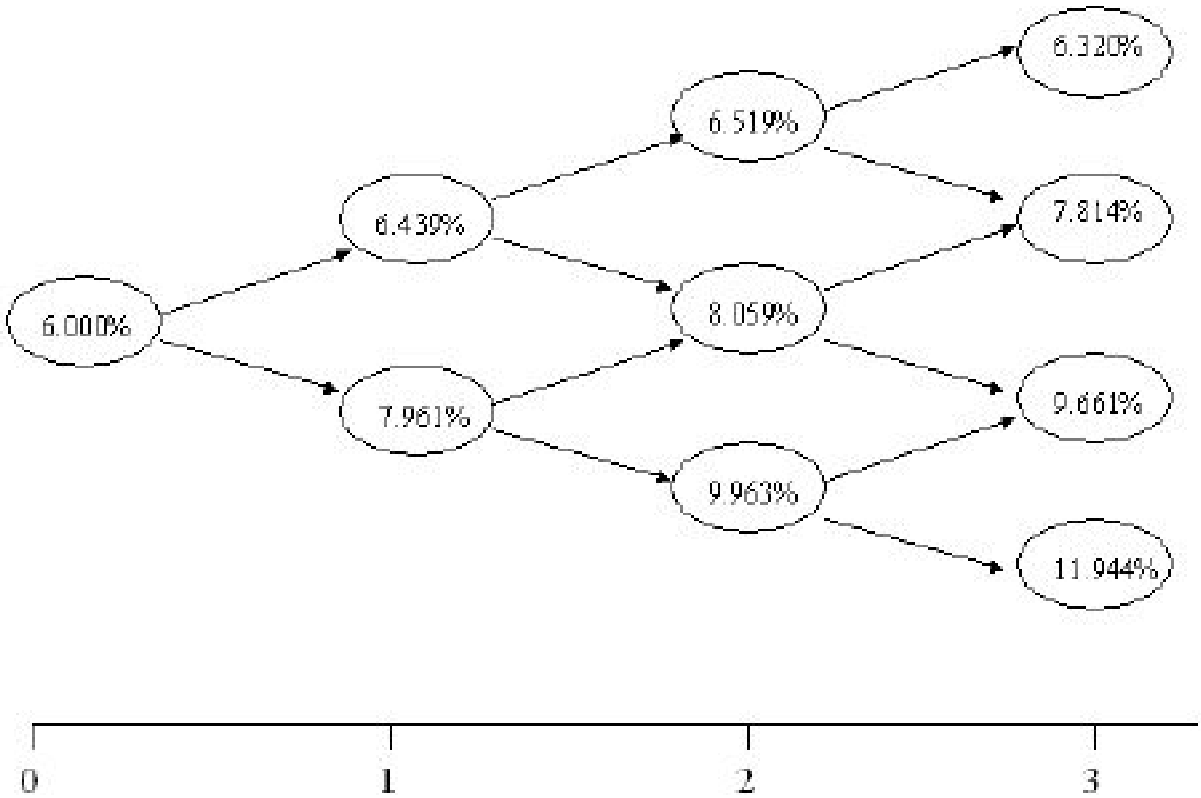
where $f_{1,1} = f_1$, i.e., the forward rate between time 0 and time 1, e.g. the spot rate at time 0. This results in the following tree:



where the rates in the tree is given by:

$$f_{i,1} = \frac{2^{i-1} f_i}{\sum_{j=0}^{i-1} \binom{i-1}{j} Z_i^j}$$

and (1). This results in a tree with the following values:


6.000%
6.439%
7.961%
6.519%
8.059%
9.963%
6.320%
7.814%
9.661%
11.944%
0
1
2
3

Step 3: calibrating the binomial tree

The calibration process involves raising the estimates of the rates in the tree by an amount just sufficient so that the value for all cash flows given by the tree exactly equals the values given by the forward rates. As this is done, the relationship (equation (1) above) between the different nodes must be simultaneously preserved. This is the most critical part in the OAS model where the calibration process is an iterative sequential process. First, the nodes are calibrated at time 1. Once this is finished, the nodes at time 2 are calibrated, and so on. At time 1 we have:

$$\left(\frac{\mathrm{cf}/2}{1+f_{2,1}(t_2-t_1)}+\frac{\mathrm{cf}/2}{1+Z_2 f_{2,1}(t_2-t_1)}\right)\cdot\frac{1}{1+f_{1,1}(t_1-t_0)}$$
$$=\frac{\mathrm{cf}}{(1+f_1(t_1-t_0))(1+f_2(t_2-t_1))}.$$

The first part of the equation is the price of the cash flow cf given by the tree, and the second is the price of the same cash flow given by the forward rates. This equation is solved with respect to the unknown $f_{2,1}$ by a Van Winjgaarden–Decker–Brent method. Then we use the relationship

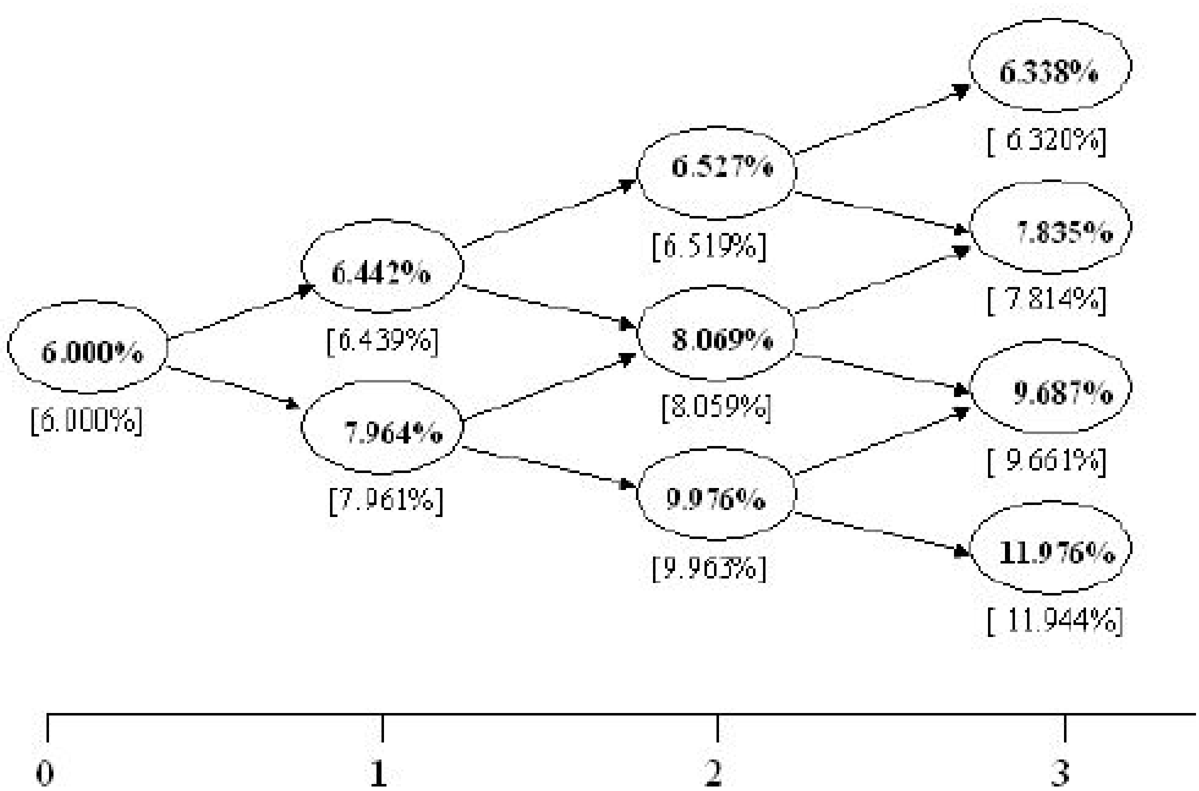
$$f_{2,2} = Z_2 f_{2,1}$$

to find $f_{2,2}$.

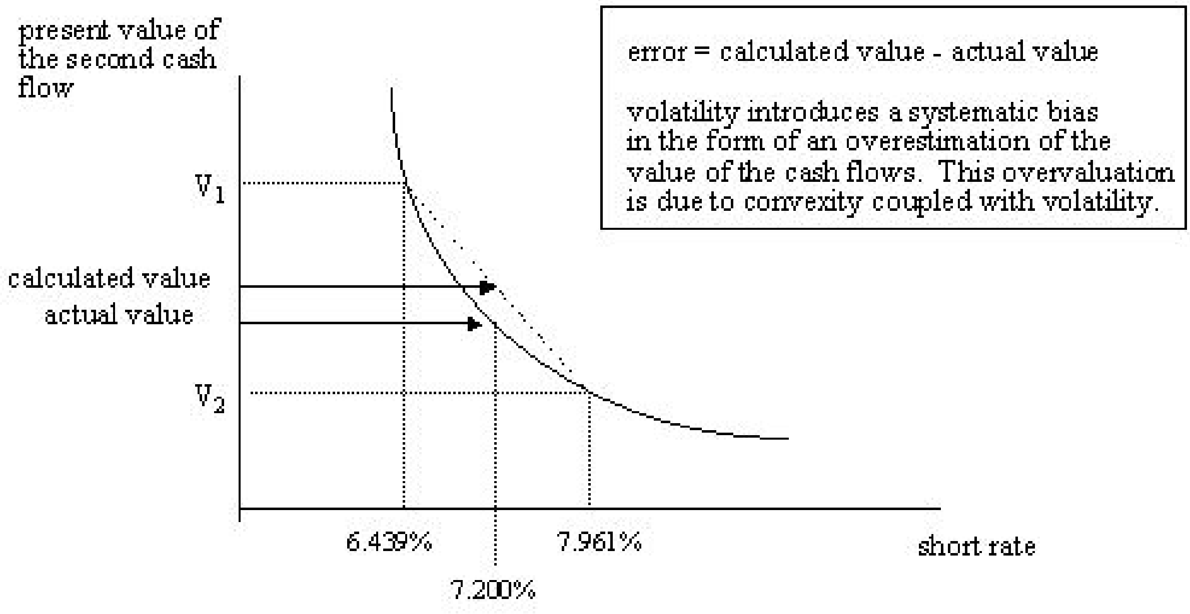
At this point we know the calibrated nodes up to time 1. At the next level the following equation needs to be solved (note, it is not necessary to know the size of the cash flow).

$$\begin{aligned}
&\frac{1}{2}\left\{\frac{1}{2}\left(\frac{1}{1+Z_3^2 f_{3,1}(t_3-t_2)}+\frac{1}{1+Z_3 f_{3,1}(t_3-t_2)}\right)\cdot\frac{1}{1+f_{2,2}(t_2-t_1)}\right.\\
&\left.+\frac{1}{2}\left(\frac{1}{1+Z_3 f_{3,1}(t_3-t_2)}+\frac{1}{1+f_{3,1}(t_3-t_2)}\right)\cdot\frac{1}{1+f_{1,2}(t_2-t_1)}\right\}\cdot\frac{1}{1+f_{1,1}(t_1-t_0)}\\
&=\frac{1}{(1+f_1(t_1-t_0))(1+f_2(t_2-t_1))(1+f_3(t_3-t_2))}.
\end{aligned}$$

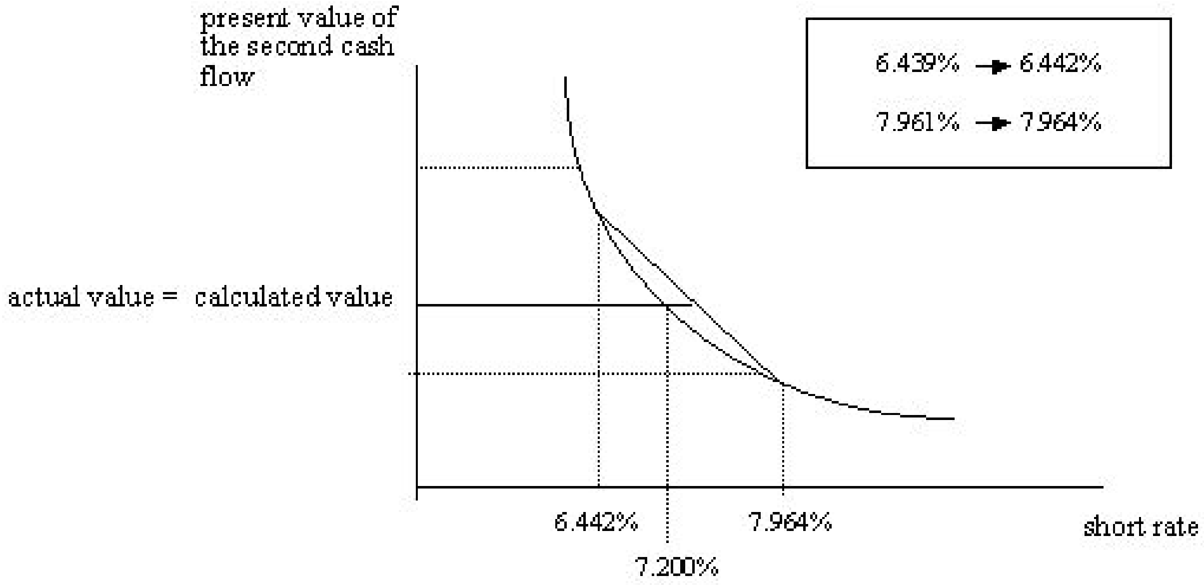
Solving this equation for $f_{3,1}$ also gives $f_{3,2}$ and $f_{3,3}$ from the relations $f_{3,2} = Z_3 f_{3,1}$ and $f_{3,3} = Z_3 f_{3,2}$. If we use the same method for cash flows at all times in the tree, the tree will be fully calibrated to produce the same value as the forward rates for all cash flows. The new calibrated tree is now:



The rates in the calibrated tree are compared with the rates from the un-calibrated. The reason for the previous calibration is shown in the figure below where the error is caused by the bond's convexity.



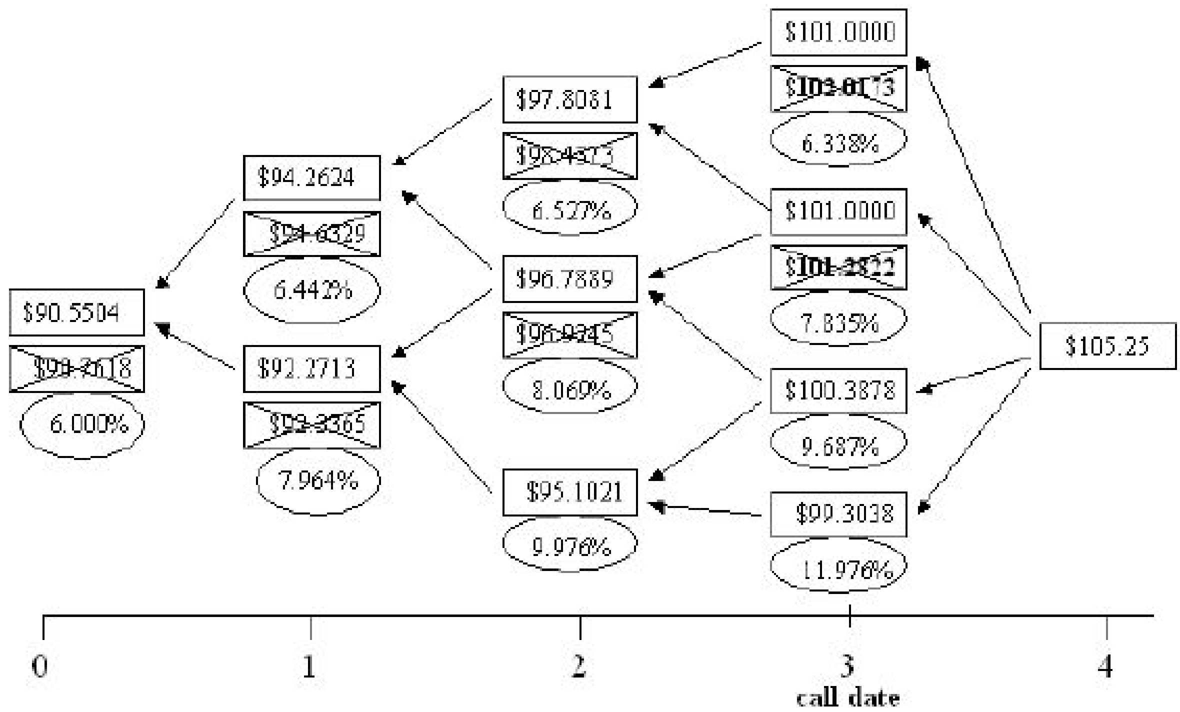
Notice that the present value curve is not linear. The curvature represents convexity. The value of the cash flow, labeled the "calculated value" above, is an average of the two values V_1 and V_2. Note that this average is higher than the actual value. With the calibration we have shifted the rates to have the following situation:


present value of the second cash flow
6.439% → 6.442%
7.961% → 7.964%
actual value = calculated value
6.442%
7.964%
7.200%
short rate

Step 4: calibrating the binomial tree with an OAS

The calibrated binomial tree just derived is applicable to valuing a benchmark bullet bond. Now we consider how this same calibrated tree could be adapted to value a non-benchmark (corporate) callable bond. To simplify the analysis, it is assumed that a corporation incurs no transaction costs either when it calls a bond or when it issues a new bond, and that it will always call a bond if it is rational to do so.

Consider a 24-month corporate bond paying an annual coupon of 10.50% in two semi-annual instalments (each coupon is therefore $\$5.25$). The bond is callable in 18 months (period 3) at $\$101.00$. Suppose that the bond's offer price is $\$103.75$ — this is the price at which you could buy this bond. The goal is to derive this same value with the model. To get this value a constant spread is added to all of the rates in the tree until the value of the bond cash flows equal the price of the callable corporate bond. In the calibration procedure we replace the values of the bond with the call value if the bond can be called back at this time, and the value at this point exceeds the call value — this is shown for the final cash flow in the figure below.



The same is done for all cash flows, and the sum of these is taken. Then the tree is adjusted to find a new shifted tree. The correct value for the callable corporate bond gives a spread of 90.465 basis points. This spread is called the **bond's option adjusted spread** or OAS. Essentially, interpret the OAS is interpreted as the number of basis points that must be added to each and every rate in the calibrated binomial tree of risk-free short rates to obtain a model predicted price that precisely equals the observed market value of the bond. These basis points represent the risk premium for bearing the credit risk associated with the bond. The same sort of analysis could have been performed if the bond had contained an embedded put option.

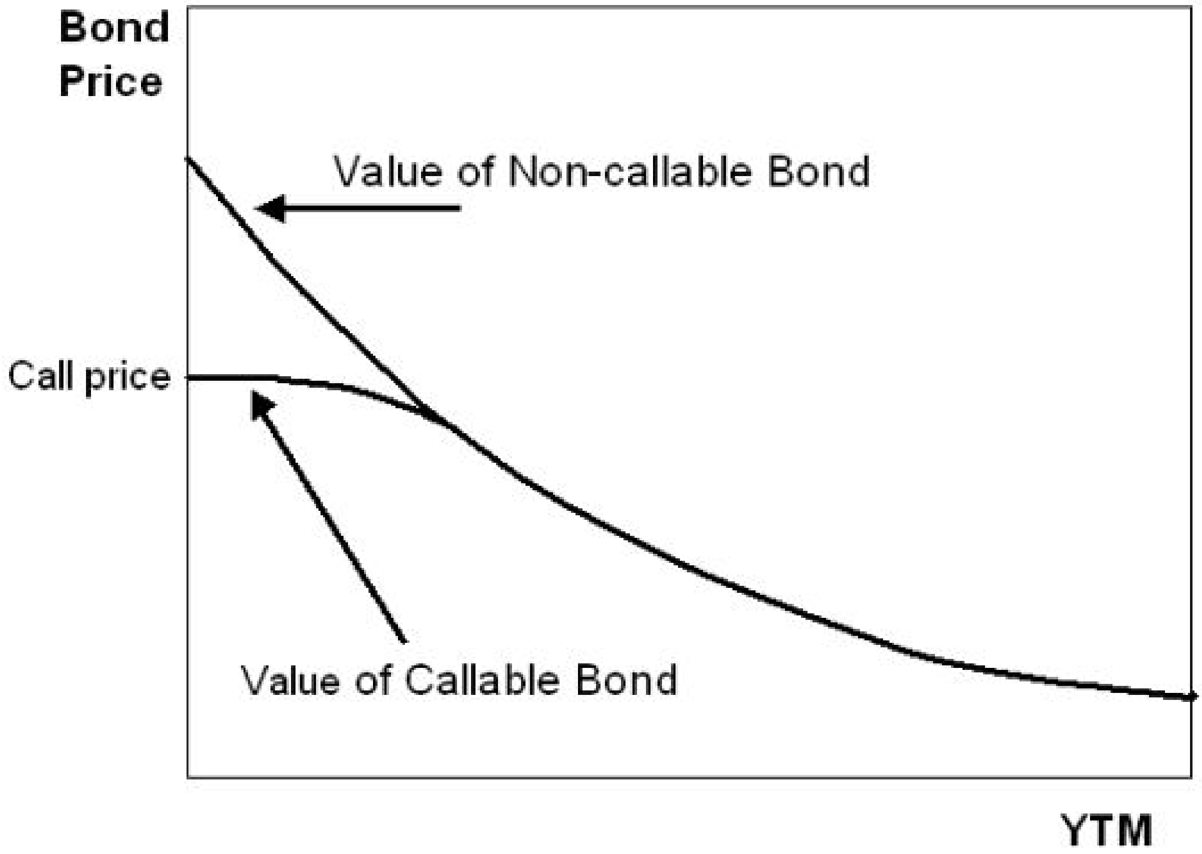
Step 5 and 6: using analysis to value the embedded option

Now, the OAS can be used to determine the value of the option that is embedded in a callable bond. To accomplish this task we ask “what would the value of the bond be at the same OAS if the bond had not been callable”. In this case, the answer is $\$103.8143$.

A callable bond may be viewed as a portfolio consisting of a long position in a bullet bond and a short position in a call option on a bullet bond that begins on the option’s call date. Therefore,

$$B_{\text{callable}} = B_{\text{bullet}} - C_{\text{bullet}},$$

This implies that $C_{\text{bullet}} = 0.0643$. Therefore, the option is worth $\$0.0643$ for every $\$100$ of par. Because of the embedded option in a callable bond, the curve, Bond Price as function of YTM, will differ from the curve for a non-callable (bullet) bond. This is shown in the figure below.


Bond
Price
Value of Non-callable Bond
Call price
Value of Callable Bond
YTM

Impact of volatility

Note that higher volatility increases the value of the option. Since the bondholder is short the option, this lowers the “adjusted yield” and the Option Adjusted Spread. For the same reason a lower volatility assumption increases the value of the bond and leads to a higher Option Adjusted Spread. Remember, this is only the case for a callable bond.

Effective duration and convexity

Since a callable (or putable) bond has cash flows that differ under different interest rate scenarios, it follows that Macauley duration is an inappropriate measure for these bonds. In other words, when a bond contains embedded options, the modified duration is a poor indicator of the interest rate risk associated with holding the bond. The OAS approach makes it possible to derive a better measure of interest rate risk. This measure is called the **bond's effective duration** or **option-adjusted duration**.

In order to calculate a modified duration for any bond it is necessary to assume that the cash flows of the bond do not change with interest rates (callable bonds violate this necessary assumption). The most intuitive way to calculate an effective duration is to first calculate the callable bond's fair value using the OAS approach (as done above). Next, it is assumed that the benchmark yield curve shifts upward by exactly one basis point. The benchmark forward rates are then re-derived, as is the calibrated binomial tree of interest rates. With the new binomial tree the upward shifted value of the callable bond is calculated. Similarly, it is then assumed that the benchmark yield curve shifts downward by exactly one basis point, and the same values are recalculated as above. With this tree we calculate the downward shifted value of the bond.

The effective Macauley duration D and convexity C is then given by:

$$D = \frac{P_- - P_+}{2P_0 \Delta y}$$

and

$$C = \frac{P_+ + P_- - 2P_0}{P_0 (\Delta y)^2},$$

where

P_-	the down shifted price
P_+	the up shifted price
P_0	the unshifted price
Δy	the shift in the yield curve.

If this technique is used for the corporate bond for which we calculated an OAS of 90.465 basis points, the effective duration will be 1.745 and the effective convexity 4.045. Without the embedded option the values are 1.782 and 4.166 respectively. In this case the differences are small, but for bonds with long maturity the difference between Modified and Effective Duration can be significant.

Example