The LIBOR market model

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Abstract

Contents of the lecture.

- ☞ The LIBOR market model: theory.
- ☞ Context mapping
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The LIBOR market model

An interest rate derivative is a contract whose value depends on an underlying interest rate. The valuation of such a derivative may be dependent on the value of the underlying rate at several different future dates. Whilst models such as the Black model assume log-normality for a single forward rate or swap rate when pricing such derivatives as, caplets and swaptions respectively, many interest rate derivatives have future payoffs dependent on several underlying rates.

The LIBOR Market model also referred to, as the "BGM/J model" is a multi-factor term structure model that allows for future volatility patterns to be considered. The LIBOR Market framework uses LIBOR rates for modeling, which are market observable and has therefore become very popular among traders. Existing interest rate models are limited by the fact that, first, they model non-market observable quantities and, secondly, often use one factor.

Black model

The standard model for valuing interest rate options, caps/floors and European swaptions, is the Black model. The Black model is used by traders in the market to price these derivatives and as will be seen later on, the analytical Black formulas will play a key role when calibrating the LIBOR Market model.

Before attempting to calculate values (prices) for these interest rate derivatives it is necessary to make certain assumptions about the underlying rates.

The basic assumptions under the Black model are:

- ☞ the underlying forward rate or swap rate is a log normally distributed random variable;
- ☞ the volatility of the underlying is constant;
- ☞ prices are arbitrage free;
- ☞ there is continuous trading in all instruments.

Forward rates

Before defining the underlying variables, the rates, it is important to introduce the concept of discount bonds. Discount bonds are traded in the market but more commonly are so called coupon-bearing bonds. These have several guaranteed future cash flows instead of just one. The coupon-bearing bonds can be reduced to a set of ordinary discount bonds. In fact, to get the price of a discount bond one often imputes this price from (suitable) coupon-bearing bonds.

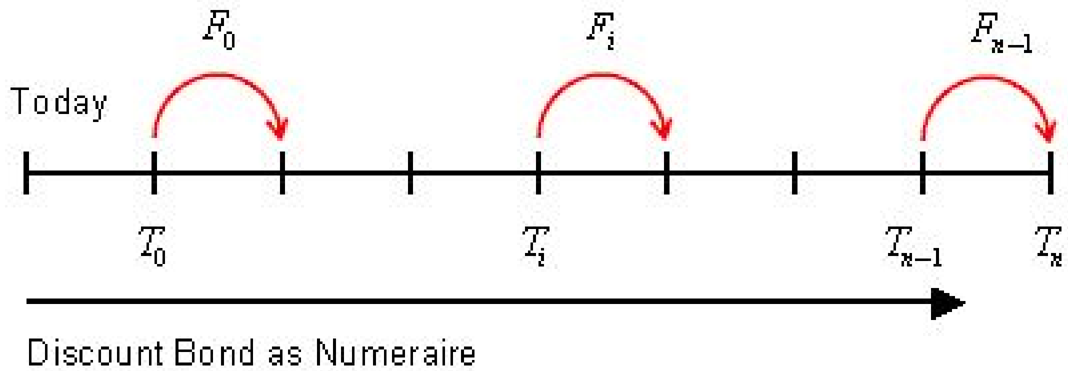
A contract, which gives the holder an amount 1 at some future date T, is referred to as **discount bond**. 1 is called the **notional** or **face value** and T is referred to as the **maturity date**. The price at time t of a discount bond with maturity T and face value 1 is denoted by $P(t, T)$.

To deal with this assumption of a continuum of prices (also known as the discount function), in practice one selects a set of *discount bond prices* (the benchmark instruments) and then simply interpolates between them. For theoretical purposes it is (almost) always assumed that there is a discount bond available for every maturity date T.

The **simply compounded forward rate** at time t spanning the future period $[T_1, T_2]$, $F(t, T_1, T_2)$ is defined by

$$\frac{P(t,T_2)}{P(t,T_1)} = \frac{1}{1 + F(t,T_1,T_2)(T_2 - T_1)}.$$

The following diagram illustrates a set of forward rates spanning the set of dates $\{T_i\}$:



These simply compounded forward rates also define the LIBOR rates (London Inter Bank Offer Rates). There are equivalents to LIBOR in other countries for example the Swedish version referred to as STIBOR (Stockholm Inter Bank Offer Rate).

Because the LIBOR rates are expressed in terms of discount bonds it is easy to see that these could be an alternative to the discount bonds when building the discount function in certain circumstances. More explicitly, where one was receiving money at a discrete set of known future dates $T_1, \ldots, T_n$, and wanted to calculate todays value of these future cash flows, then it would be sufficient to know the value of the discount function at these dates given by a discrete set of LIBOR rates.

Caps and floors

A **caplet** connected with the forward rate $F(T_i, T_i, T_i + \tau_i)$ and with strike k, is a contract, which at time $T_i + \tau_i$ gives the holder the cash flow:

$$\tau_i \max\{F(T_i, T_i, T_i + \tau_i) - k, 0\}.$$

A **cap** is a set of caplets. The price of a caplet using the Black model is given by:

$$V_i(0) = P(0, T_{i+1}) N_i \tau_i (F_i(0)\Phi(a_{1i}) - k\Phi(a_{2i})),$$

where

k	the strike of an interest rate cap
V_i	value of ith caplet
τ_i	time to maturity of the ith caplet
T_i	reset dates of forward rate
P	discount factor
N_i	notional principal of each caplet
F	forward rate
τ_i	tenor or $T_{i+1} - T_i$.

a_{1i} and a_{2i} are given by

$$a_{1i} = \frac{\ln(F_i(0)/k) + \sigma_i^2 T_i/2}{\sigma_i \sqrt{T_i}},$$
$$a_{2i} = a_{1i} - \sigma_i \sqrt{T_i},$$

and σ_i denotes the volatility for the ith caplet.

A **floorlet** connected with the forward rate $F(T_i, T_i, T_i + \tau_i)$ and with strike k, is a contract, which at time $T_i + \tau_i$ gives the holder the cash flow:

$$\tau_i \max\{k - F(T_i, T_i, T_i + \tau_i), 0\}.$$

A **floor** is a set of floorlets. The price of a floorlet using the Black model is given by:

$$V_i(0) = P(0, T_{i+1}) N_i \tau_i (k\Phi(-a_{2i}) - F_i(0)\Phi(-a_{1i})),$$

LIBOR Market Model

Let the tenor structure be $0 = T_0 < T_1 < \cdots < T_n$ and i an integer ranging over the resets of the rates, e.g. $1 \leq i \leq n$.

We define $\eta(t)$ to be the unique index such that $T_{\eta(t)}$ is the next tenor date after t.

The (one factor) model is given by the following stochastic differential equation (SDE) for the underlying rates (swap or forward):

$$\frac{df_i}{f_i} = \mu_i(f_i(t), t)\, dt + \sigma_i(t)\, dz(t),$$

where

f_i	forward/swap rate at time i
μ_i	drift term
σ_i	volatility of rate i
$z(t)$	Wiener process.

The solution to this SDE is:

$$f_i(t) = f_i(0) \exp\left(\int_0^T \left(\mu_i(s) - \frac{1}{2}\sigma_i^2(s) \right) ds + \int_0^T \sigma_i(s)\, dz(s) \right).$$

The drift terms μ_i depend on the choice of numeraire and can be determined by applying the assumption of no arbitrage. Suppose we have forward rates as the underlying rates and choose $P(T_0, T_{i+1})$ as the numeraire. Then the drift terms become

$$\mu_i(t) = \sigma_i(t) \sum_{k=\eta(t)}^{i} \frac{\tau_i f_i(t) \sigma_k(t)}{1 + \tau_i f_i(t)}.$$

A one-factor model means that all the rates are perfectly instantaneously correlated. In this case, a single Wiener process is sufficient to evolve the rates. This is not often a reasonable assumption, and eliminates one of the advantages of employing the LIBOR Market model. A m-factor model is one where m independent Wiener processes are used to evolve the rates. In this case the equation becomes

$$\frac{df_i}{f_i} = \mu_i(f_i(t), t)\, dt + \sum_{k=1}^{m} \sigma_{i,k}(t)\, dz_k(t).$$

where $\sigma_{i,k}(t)$ is the component of the volatility of $f_i(t)$

attributable to the kth factor. They must satisfy

$$\sum_{k=1}^{m} \sigma_{i,k}^2(t) = \sigma_i^2(t).$$

The solution of the SDE is

$$f_i(t) = f_i(0)\exp\left(\int_0^T \left(\mu_i(s) - \frac{1}{2}\sigma_i^2(s)\right) ds + \sum_{k=1}^{m} \int_0^T \sigma_{i,k}(s)\, dz_k(s)\right).$$

Cap volatility calibration

Assume that each underlying rate $f_i(t)$ has a lognormal distribution with variance equal to $\sigma_B^2 t$, where σ_B^2 is the implied Black volatility, which can be read from the market. Then the instantaneous volatility at reset for each rate is related to the above expression in the following way:

$$\int_0^{T_i} \sigma_i^2(t)\, dt = \sigma_B^2 T_i. \tag{1}$$

There are (infinitely) many solutions to these equations, and our goal is to pick one that fits our needs. Let

$$\sigma(t) = (a + bt)e^{-ct} + d$$

and

$$\sigma_i(t) = k_i \sigma(T_i - t).$$

The calibration proceeds as follows.

① Find values on the constants a, b, c, and d such that equation (1) fit as close as possible.

② Set values of the k_i as

$$k_i = \sqrt{\frac{\sigma_B^2 T_i}{\int_0^{T_i} \sigma_i^2(t)\, dt}}.$$

The second step ensures equality for the equations in (1), that is, the instantaneous volatility and the implied Black volatility is equal at each reset. This completes the volatility calibration for caps.

Context mapping

In order to use the LIBOR Market Model when valuing instruments the following context mappings must be performed:

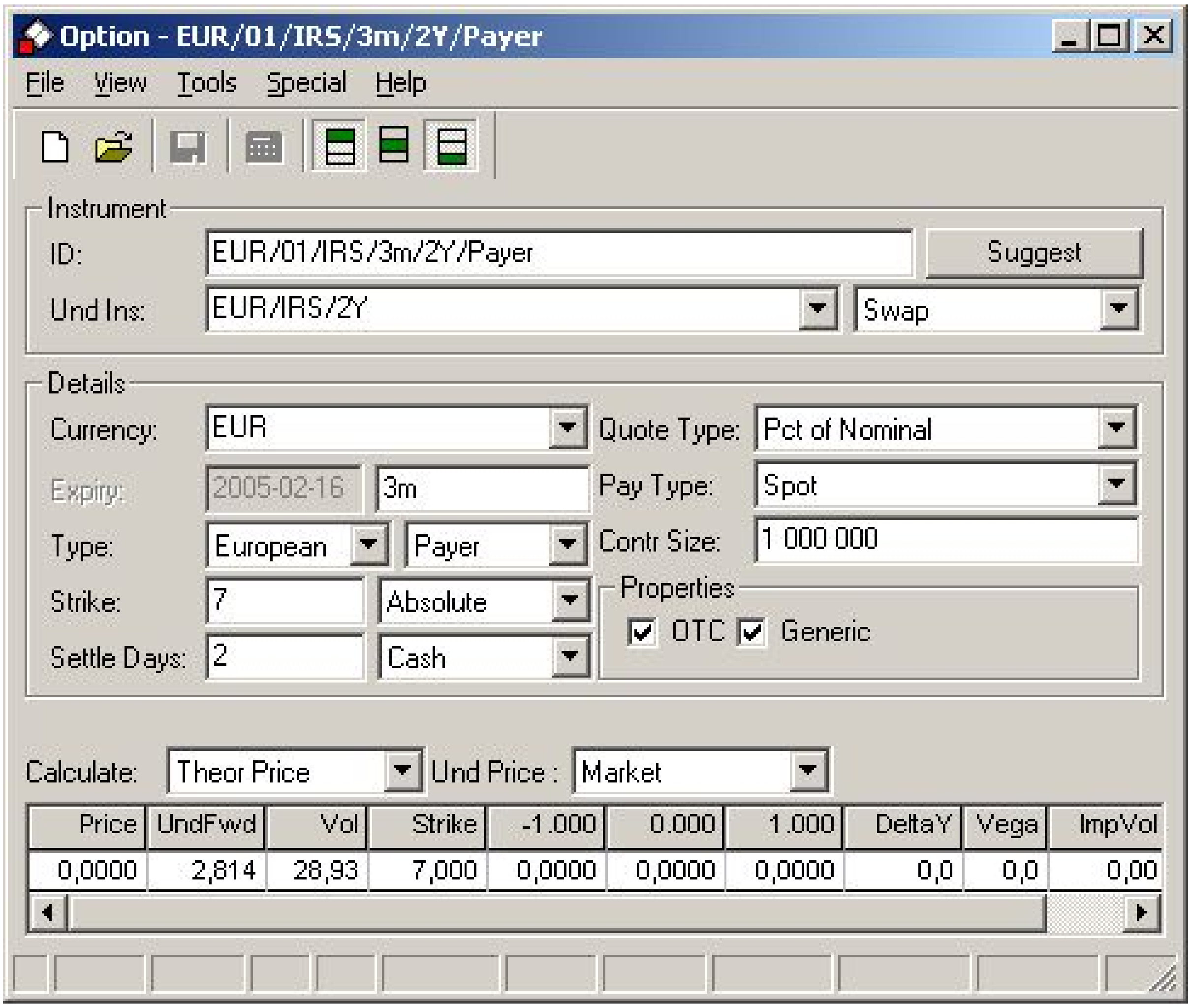
① Map the instrument to the **Core Valuation Function** > **LIBOR Market Model**. This mapping tells FRONT ARENA to value the instrument with the LIBOR Market Model.

② Map the instrument to an appropriate correlation matrix. The LIBOR Market Model requires a correlation matrix as input, and this mapping makes sure it gets one.

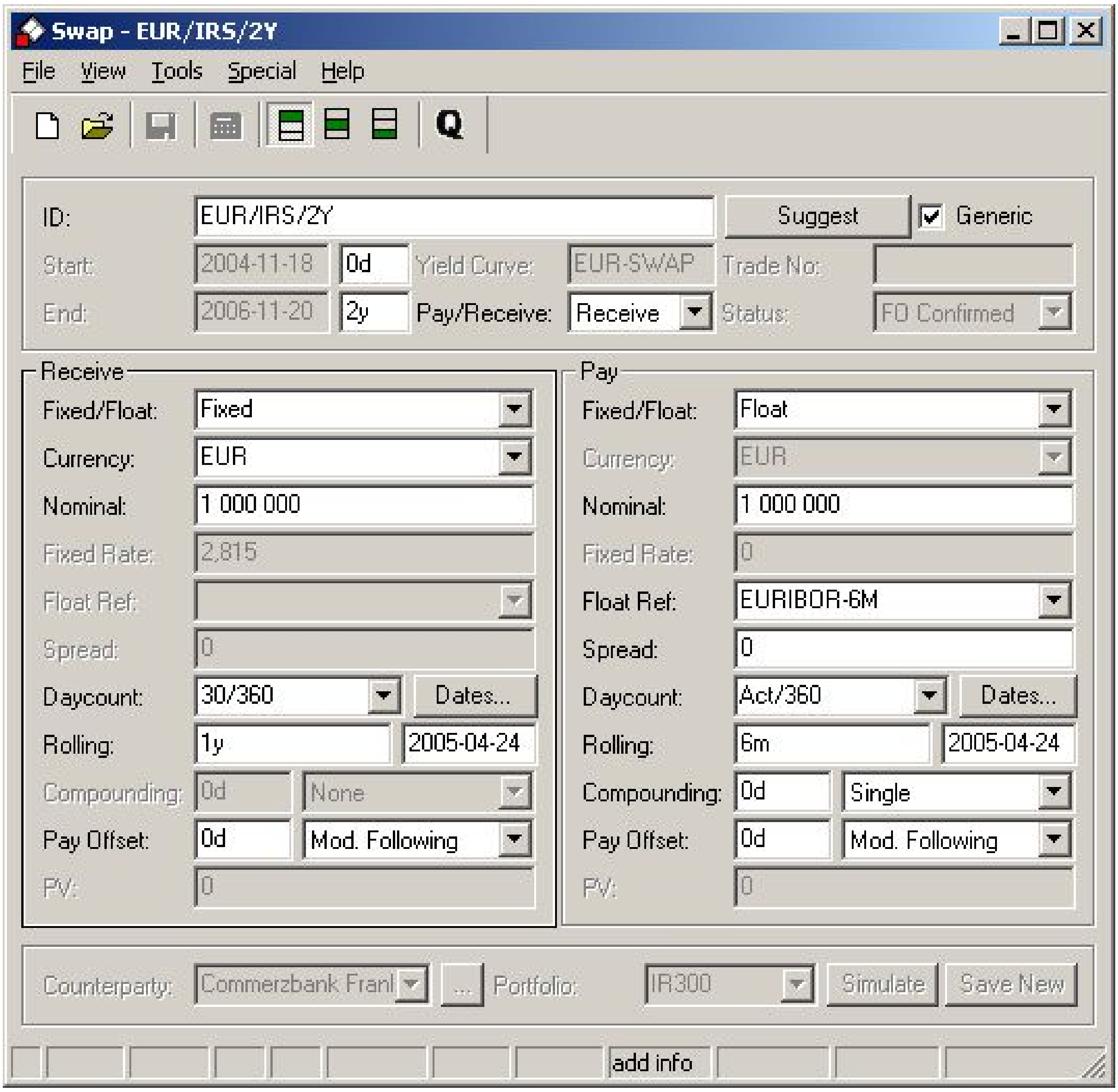
③ Map the instrument to an appropriate volatility Landscape. If the instrument is a Cap/Floor it suffices to map a volatility Landscape to the rate index. If the instrument is a Swaption, we must, in addition, map a volatility Landscape to the instrument itself. The LIBOR Market Model then uses these volatilities (or this volatility if the instrument is a Cap/Floor) in its calibration step.

These mappings are specific for the LIBOR Market Model. There are also some general valuation mappings that must be done.

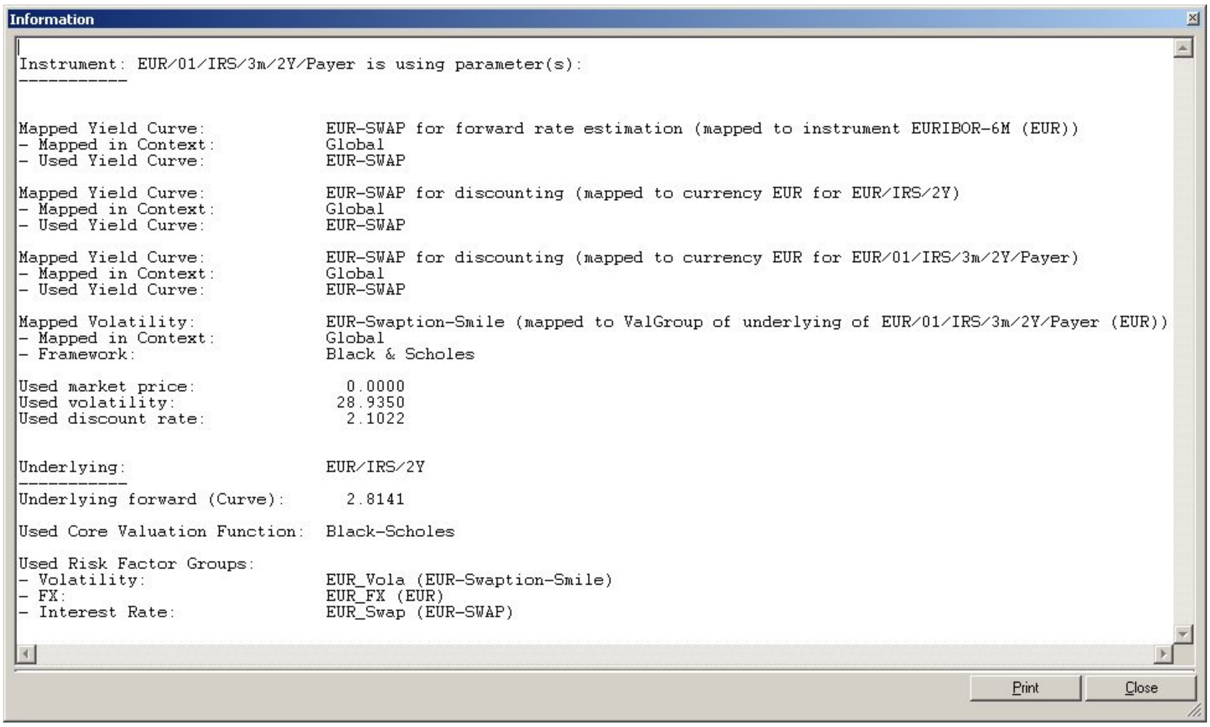
To illustrate this mapping process lets consider the following Swaption:



It has the following underlying swap:

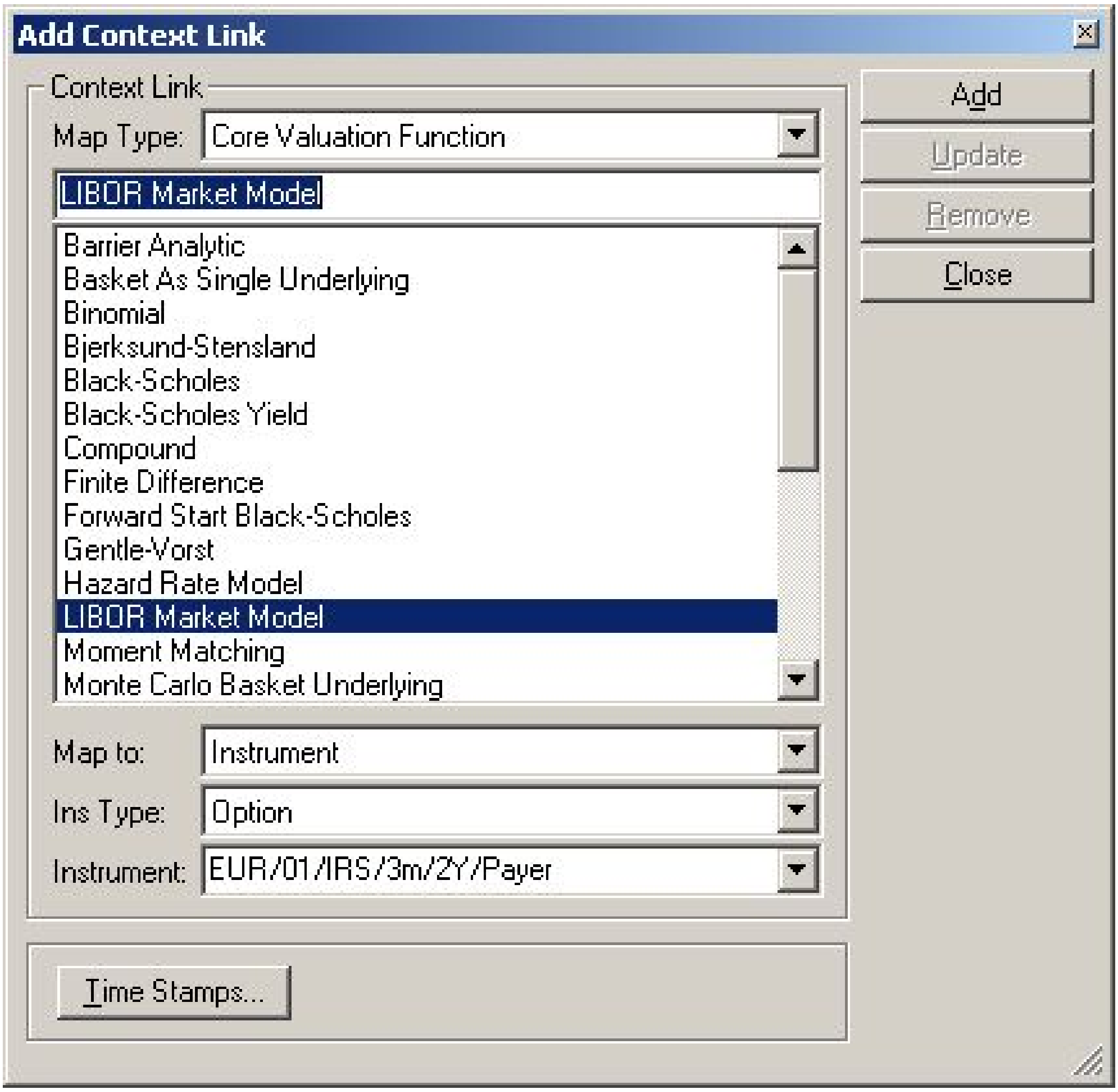


In the **Special** > **Information** window of the Swaption we can see the mappings that apply. They are as follows:

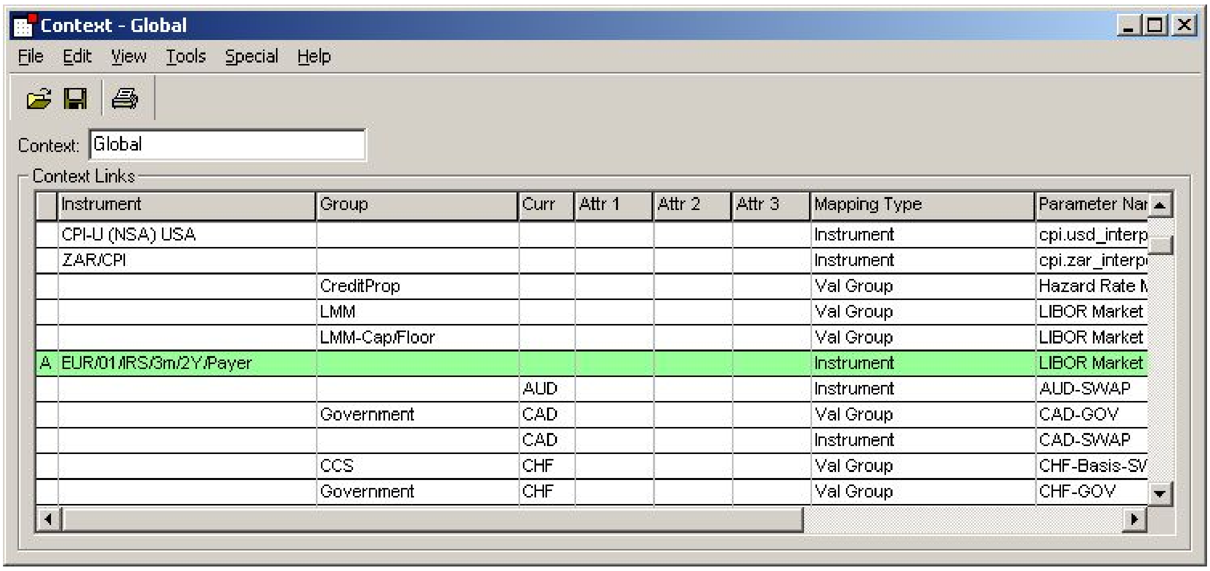


In order to value the Swaption with the LIBOR Market Model, it must first be mapped to the **Core Valuation Function** LIBOR Market Model in the **Context** application.

First, open an appropriate context in **File** > **Open**. In this example it is 'Global'. To add a context link select **Edit** > **Add ContextLink** as follows:

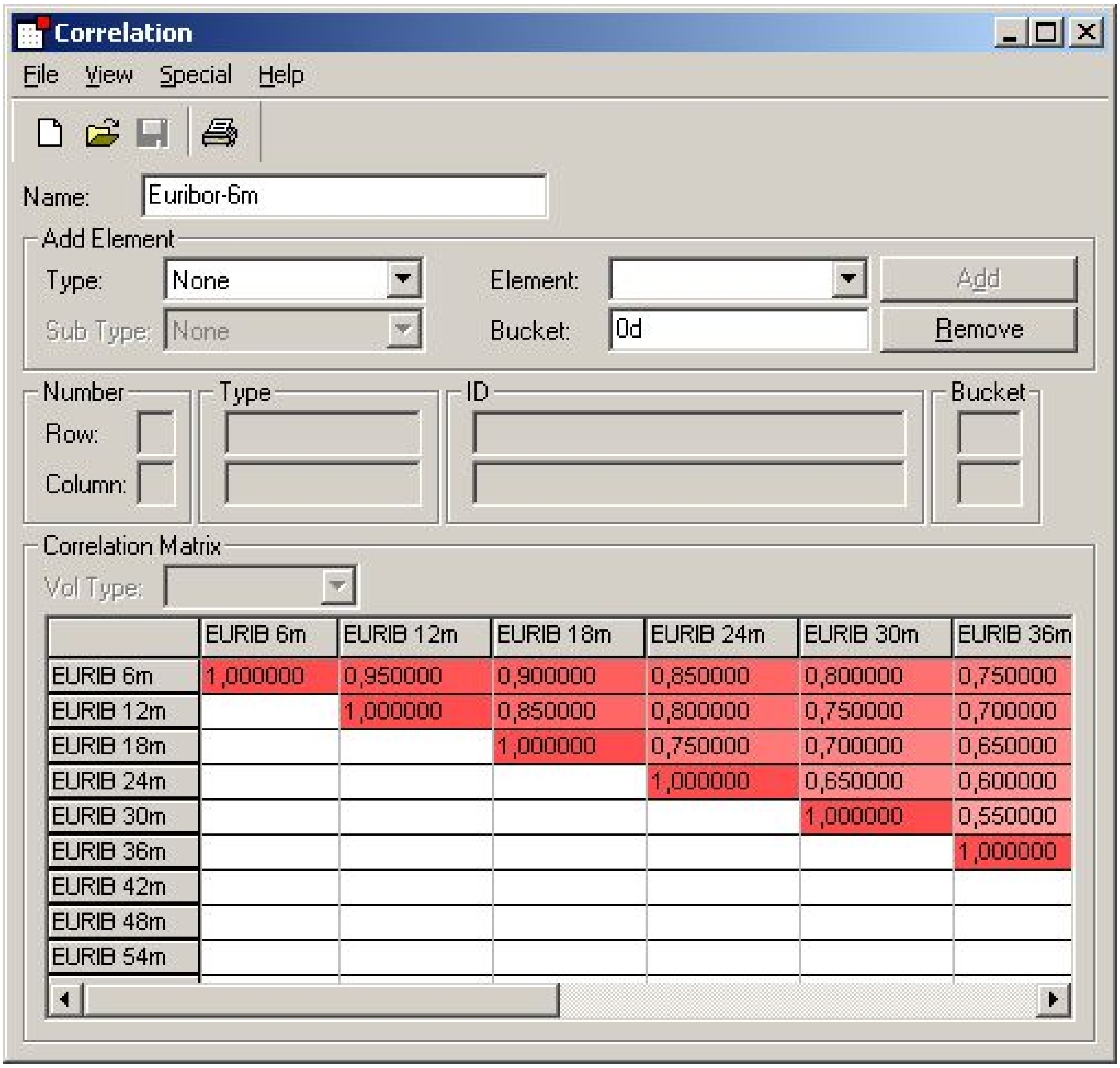


Once the ContextLink has been added it will appear in the **Context** application window as displayed in the following figure:

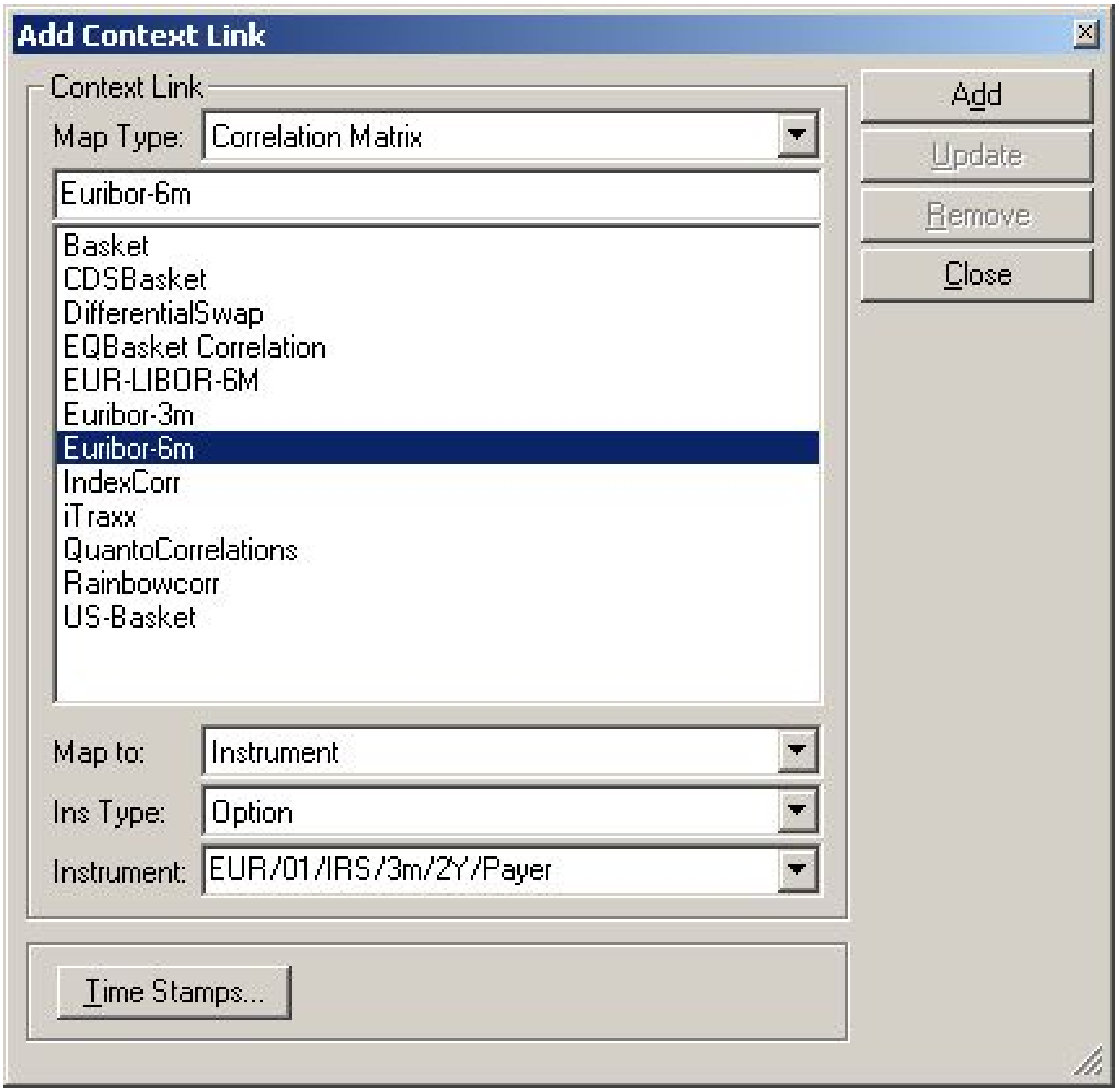


In the above illustration it can be seen that the “EUR/01/IRS/3m/2Y/Payer” instrument is associated with the “LIBOR Market Model” Core Valuation Function. Therefore, this Core Valuation Function will be used in the valuation.

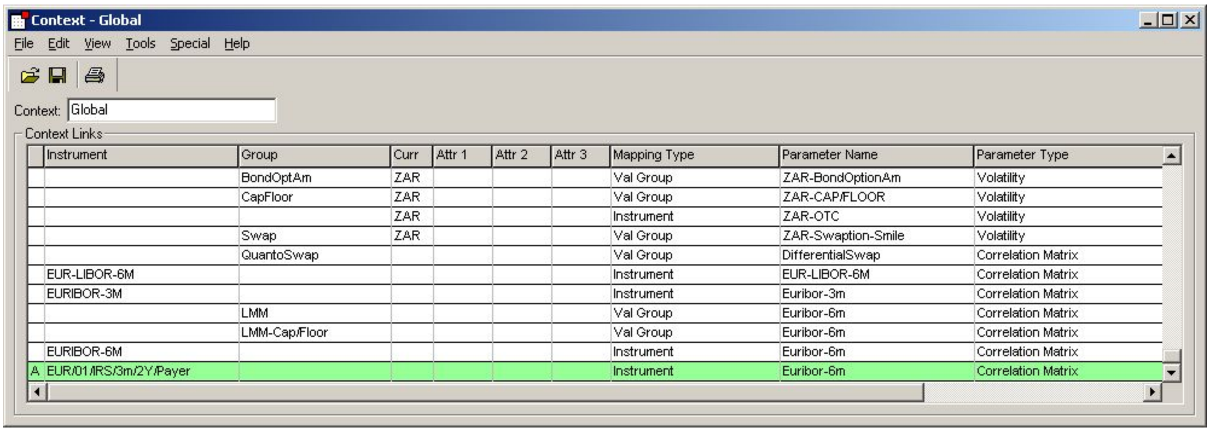
An integral part of the LIBOR Market valuation process is setting the correlations between the rates, which are user defined. Correlations matrices are set up using the **Correlation** application, which is accessed by selecting **Data** > **Correlation** from the PRIME. The correlation matrix used by “EUR/01/IRS/3m/2Y/Payer” is illustrated in the figure below:



To add this context link select **Edit** > **Add ContextLink** as the following figure shows:

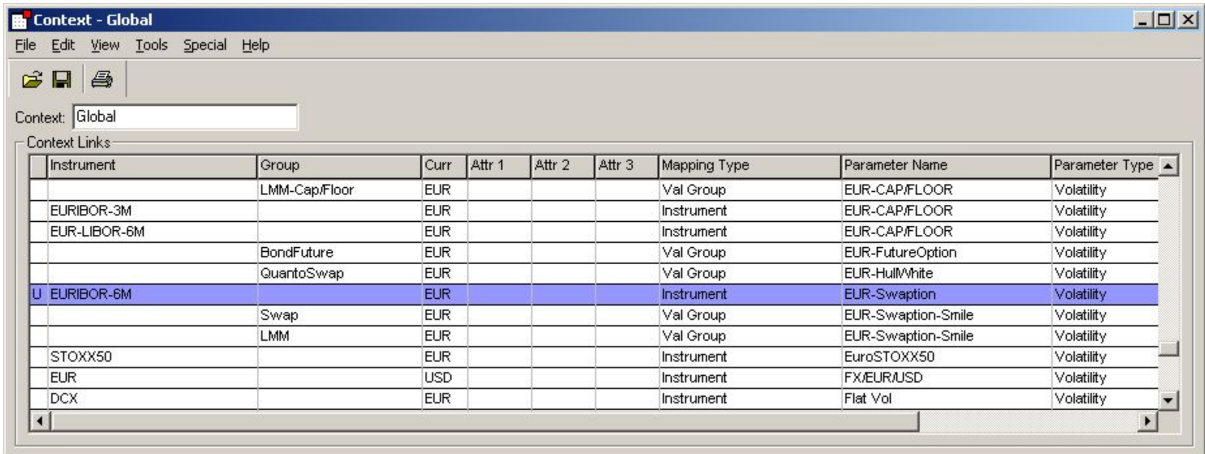


Once the ContextLink has been added it will appear in the **Context** application window as displayed in the following figure:



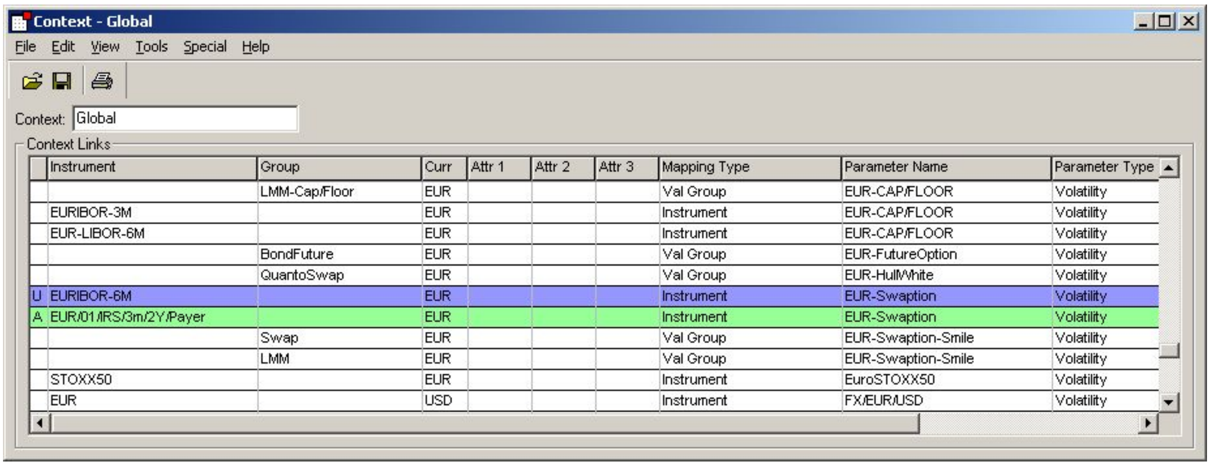
	Instrument	Group	Curr	Attr 1	Attr 2	Attr 3	Mapping Type	Parameter Name	Parameter Type
		BondOptAm	ZAR				Val Group	ZAR-BondOptionAm	Volatility
		CapFloor	ZAR				Val Group	ZAR-CAP/FLOOR	Volatility
			ZAR				Instrument	ZAR-OTC	Volatility
		Swap	ZAR				Val Group	ZAR-Swaption-Smile	Volatility
		QuantoSwap					Val Group	DifferentialSwap	Correlation Matrix
	EUR-LIBOR-6M						Instrument	EUR-LIBOR-6M	Correlation Matrix
	EURIBOR-3M						Instrument	Euribor-3m	Correlation Matrix
		LMM					Val Group	Euribor-6m	Correlation Matrix
		LMM-Cap/Floor					Val Group	Euribor-6m	Correlation Matrix
	EURIBOR-6M						Instrument	Euribor-6m	Correlation Matrix
A	EUR/01/IRS/3m/2Y/Payer						Instrument	Euribor-6m	Correlation Matrix

To enable the calibration process to take place it is necessary to map a volatility Landscape to the underlying rate index used by “EUR/01/IRS/3m/2Y/Payer”, in our case “Euribor-6m”. The mapping procedure is performed in the same way as for the correlation matrix mapping and the result is pictured in the following figure:



	Instrument	Group	Curr	Attr 1	Attr 2	Attr 3	Mapping Type	Parameter Name	Parameter Type
		LMM-Cap/Floor	EUR				Val Group	EUR-CAP/FLOOR	Volatility
	EURIBOR-3M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
	EUR-LIBOR-6M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
		BondFuture	EUR				Val Group	EUR-FutureOption	Volatility
		QuantoSwap	EUR				Val Group	EUR-HullWhite	Volatility
U	EURIBOR-6M		EUR				Instrument	EUR-Swaption	Volatility
		Swap	EUR				Val Group	EUR-Swaption-Smile	Volatility
		LMM	EUR				Val Group	EUR-Swaption-Smile	Volatility
	STOXX50		EUR				Instrument	EuroSTOXX50	Volatility
	EUR		USD				Instrument	FX/EUR/USD	Volatility
	DCX		EUR				Instrument	Flat Vol	Volatility

Once more, to enable the calibration to take place it is also necessary to map “EUR/01/IRS/3m/2Y/Payer” to an appropriate volatility Landscape. In this example the volatility Landscape 'EUR-Swaption' is chosen as illustrated in the following figure:



	Instrument	Group	Curr	Attr 1	Attr 2	Attr 3	Mapping Type	Parameter Name	Parameter Type
		LMM-Cap/Floor	EUR				Val Group	EUR-CAP/FLOOR	Volatility
	EURIBOR-3M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
	EUR-LIBOR-6M		EUR				Instrument	EUR-CAP/FLOOR	Volatility
		BondFuture	EUR				Val Group	EUR-FutureOption	Volatility
		QuantoSwap	EUR				Val Group	EUR-HullWhite	Volatility
U	EURIBOR-6M		EUR				Instrument	EUR-Swaption	Volatility
A	EUR/01/IRS/3m/2Y/Payer		EUR				Instrument	EUR-Swaption	Volatility
		Swap	EUR				Val Group	EUR-Swaption-Smile	Volatility
		LMM	EUR				Val Group	EUR-Swaption-Smile	Volatility
	STOXX50		EUR				Instrument	EuroSTOXX50	Volatility
	EUR		USD				Instrument	FX/EUR/USD	Volatility

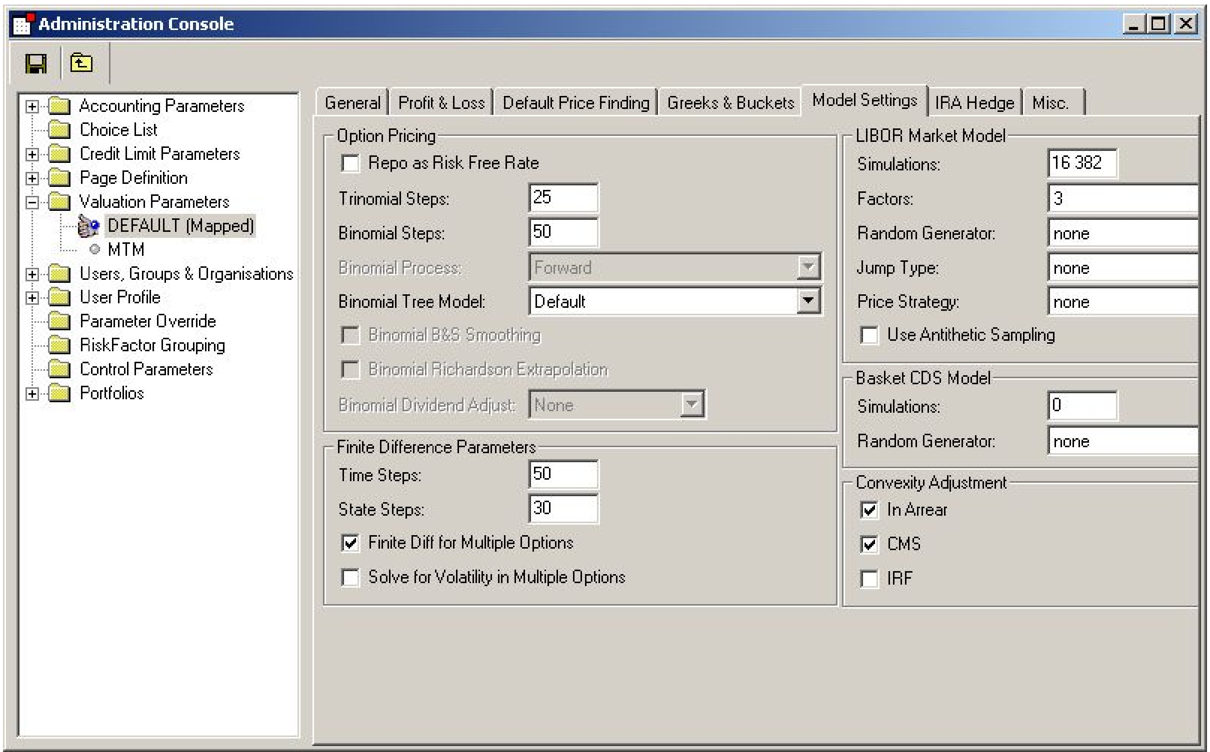
There is one difference between how you map a Swaption and a Cap/Floor. For a Cap/Floor it is sufficient (and necessary) to map a volatility Landscape to the rate index.

A Swaption needs in addition, one volatility Landscape mapped to the instrument itself. The other mappings are necessary for both Swaptions and Caps/Floors.

Valuation parameters

Before running the LIBOR Market Model in FRONT ARENA, it is necessary to specify the number of Monte Carlo simulations and the number of factors to be used by the model. This is done in **Admin** > **Administration Console** application.

In the illustration below the number of simulations used is $16,382$ and the number of factors is 3:



Caps and floors

It is assumed that all the necessary settings, as discussed before, have been taken care of. The different Caps/Floors that can be valued in FRONT ARENA using the LIBOR Market Model are plain vanilla, “Ratchet”, “Sticky”, “Momentum”, “Flexi”, and “Chooser”.

All are described there, with the exception of a plain vanilla. Assuming that the correct settings have been done, entering a plain vanilla Cap/Floor and value it with the LIBOR Market Model is no different had it been valued with Black’s Model.

From hereon we only discuss Caps. Definitions of the Floors can easily be derived from the corresponding Cap definitions.

Notation

$T_1 < T_1 < \cdots < T_n$	the reset dates of the cap
K_i	the strike at the ith reset
L_i	the LIBOR with reset at T_i.

The following table maps the different fields in the Cap definition in FRONT ARENA to a variable name that is more appropriate to use in the mathematical formulas.

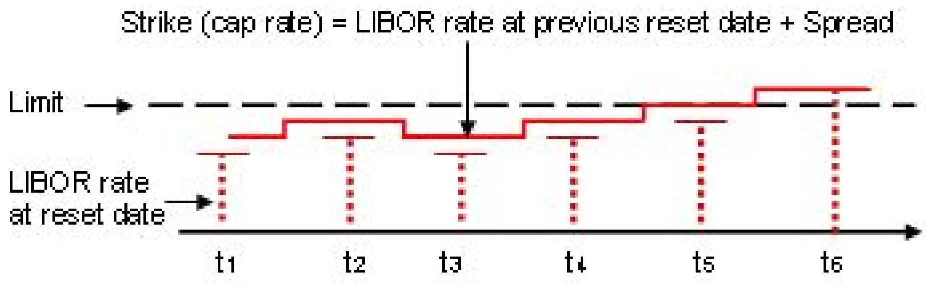
Field	Variable
Spread	X
Limit	m
Strike	K
Barrier	b

Ratchet cap

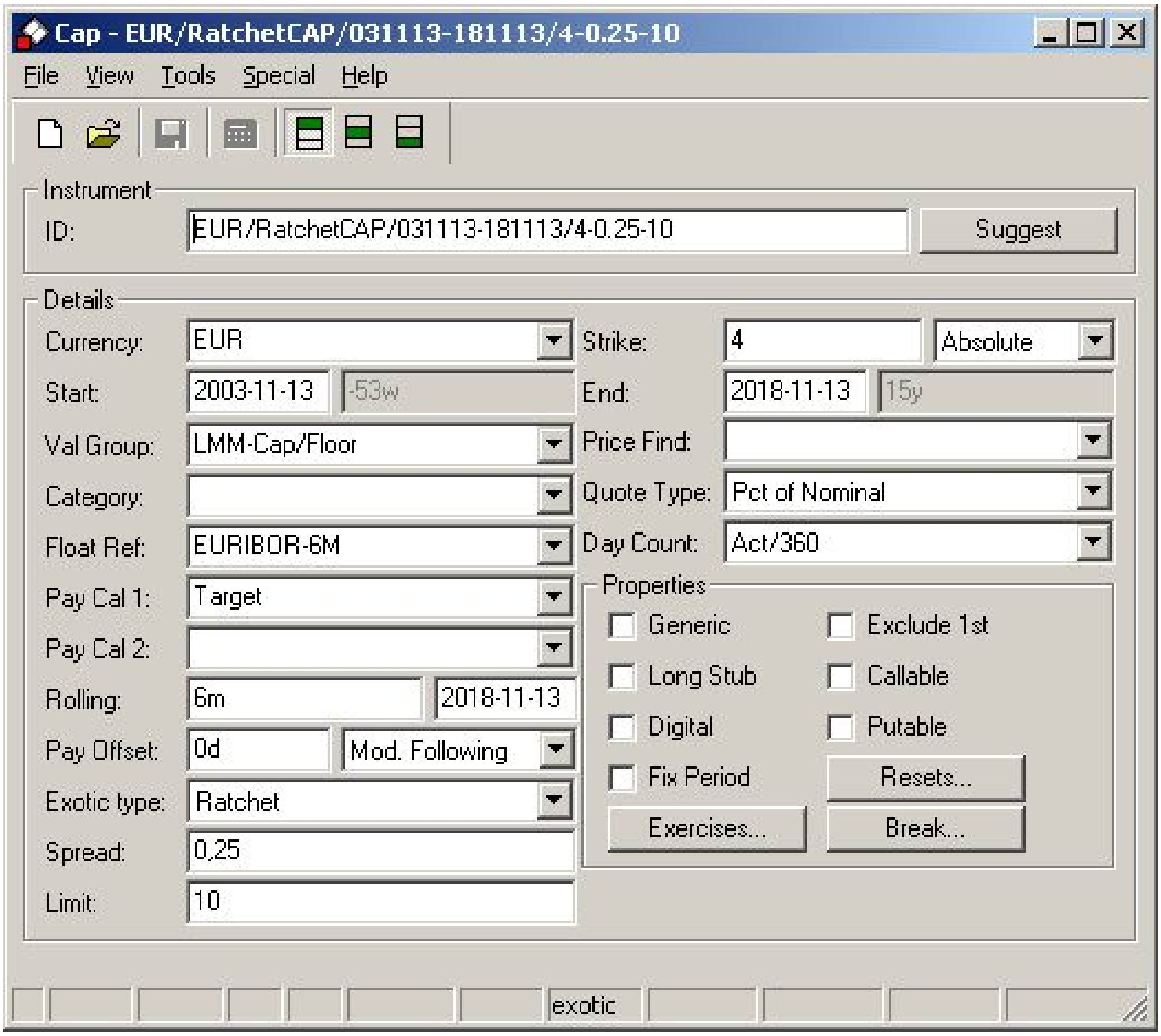
A Ratchet Cap is like a plain vanilla Cap except that the strike is given by:

$$K_i = \begin{cases} \min\{K, m\}, & i = 1 \\ \min\{K_{i-1} + X, m\}, & i > 1. \end{cases}$$

Ratchet Caps incorporate rules for determining how the cap rate for each caplet is set. The cap rate equals the LIBOR rate at the previous reset date plus a spread. A limit is then set on the strike level, above which a strike cannot be set.



Entering a Ratchet Cap involves setting three additional fields in the Cap definition compared to entering a plain vanilla Cap. To start with, we have to set **Exotic Type** > **Ratchet**. This makes the fields **Spread** and **Limit** appear in the Cap definition. Finally, we need to set **Spread** and **Limit**.



Note that the **Strike** field in the Cap definition has a different meaning in a Ratchet Cap than in a plain vanilla Cap. In a Ratchet Cap it decides the initial value on the strike.

Sticky cap

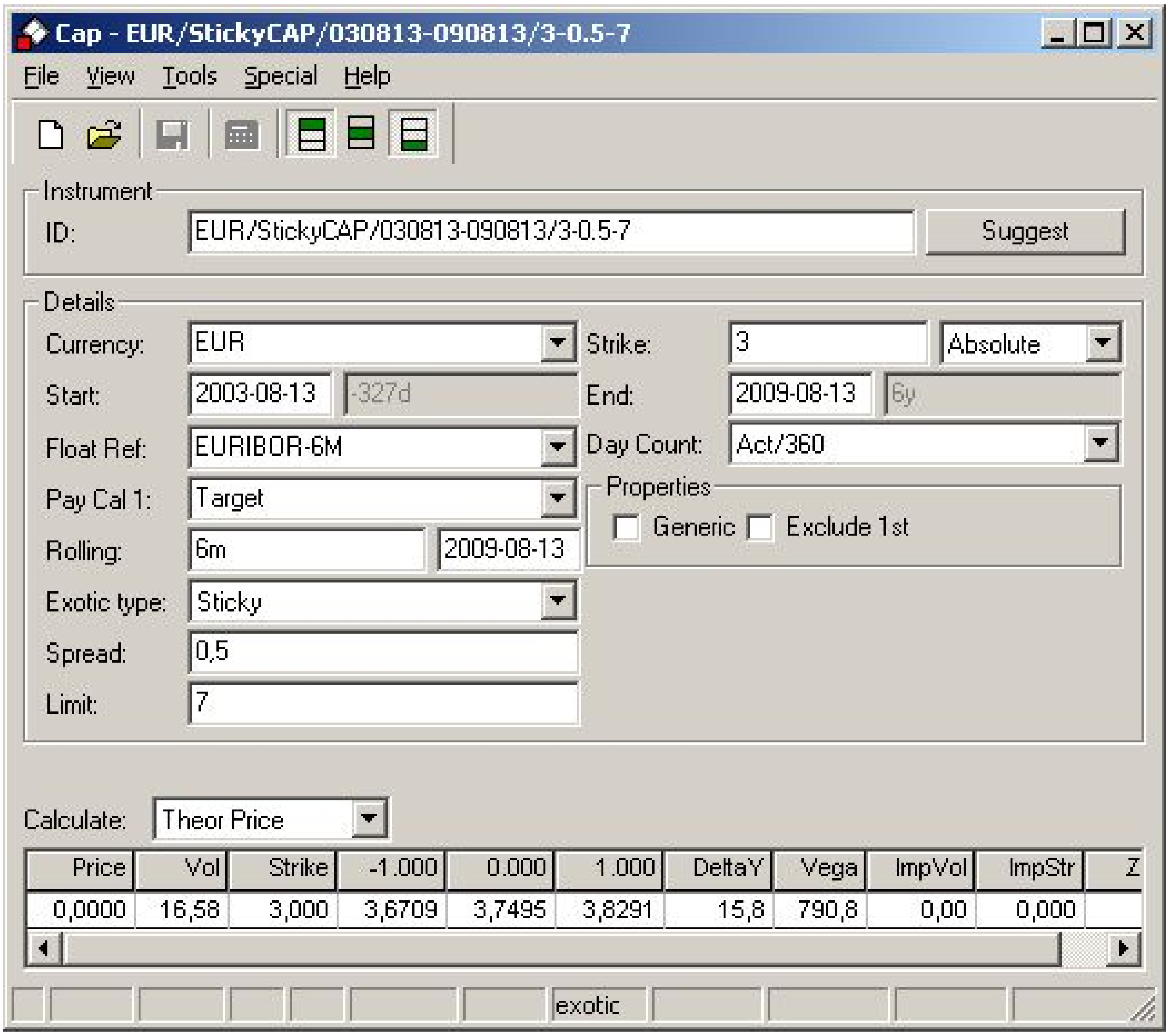
A Sticky Cap is like a plain vanilla Cap except that the strike is given by:

$$K_i = \begin{cases} \min\{K, m\}, & i = 1 \\ \min\{\min\{K_{i-1}, L_{i-1}\} + X, m\}, & i > 1. \end{cases}$$

In a Sticky Cap, the cap rate equals the previous capped rate plus a spread. A limit is then set on the strike level, above which a strike cannot be set.

Entering a Sticky Cap involves setting three additional fields in the Cap definition compared to entering a plain vanilla Cap. To start with, it is necessary to set **Exotic Type** > **Sticky**. This makes the fields **Spread** and **Limit** appear in the Cap definition. Finally, we need to set **Spread** and **Limit**.

Note that the **Strike** field in the Cap definition has a different meaning in a 'Sticky' Cap than in a plain vanilla Cap. In a 'Sticky' Cap it decides the initial value on the strike.


Cap - EUR/StickyCAP/030813-090813/3-0.5-7
File View Tools Special Help
Instrument
ID: EUR/StickyCAP/030813-090813/3-0.5-7
Suggest
Details
Currency: EUR
Strike: 3
Absolute
Start: 2003-08-13
-327d
End: 2009-08-13
6y
Float Ref: EURIBOR-6M
Day Count: Act/360
Pay Cal 1: Target
Properties
Generic
Exclude 1st
Rolling: 6m
2009-08-13
Exotic type: Sticky
Spread: 0,5
Limit: 7
Calculate: Theor Price
Price Vol Strike -1.000 0.000 1.000 DeltaY Vega ImpVol ImpStr Z
0,0000 16,58 3,000 3,6709 3,7495 3,8291 15,8 790,8 0,00 0,000
exotic

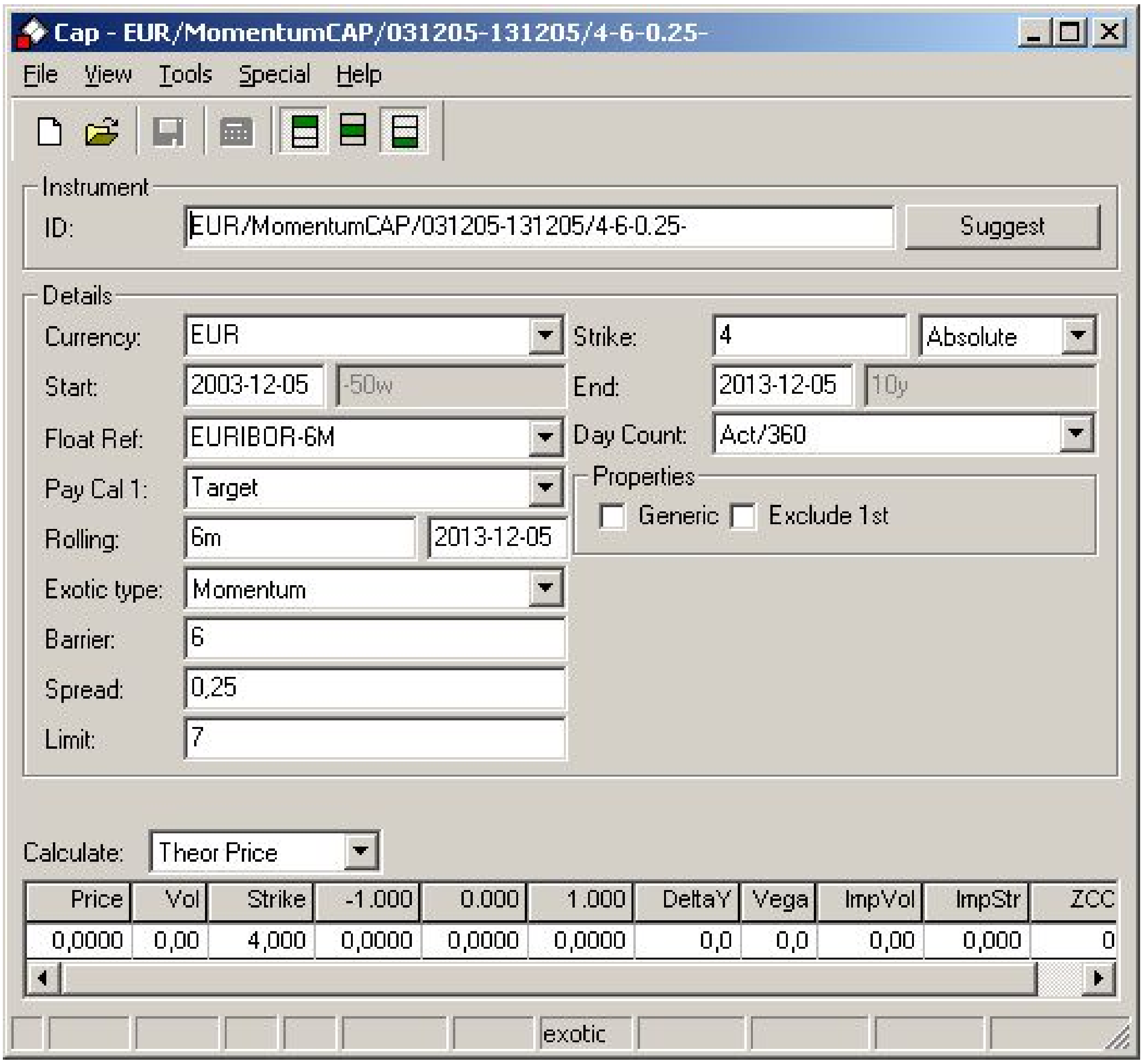
Momentum cap

A Momentum Cap is like a plain vanilla Cap except that the strike is given by:

$$K_i = \begin{cases} \min\{K, m\}, & i = 1 \\ \min\{K_{i-1} + X, m\}, & i > 1, L_i - b > L_{i-1} \\ \min\{K_{i-1}, m\}, & i > 1, L_i - b \le L_{i-1}. \end{cases}$$

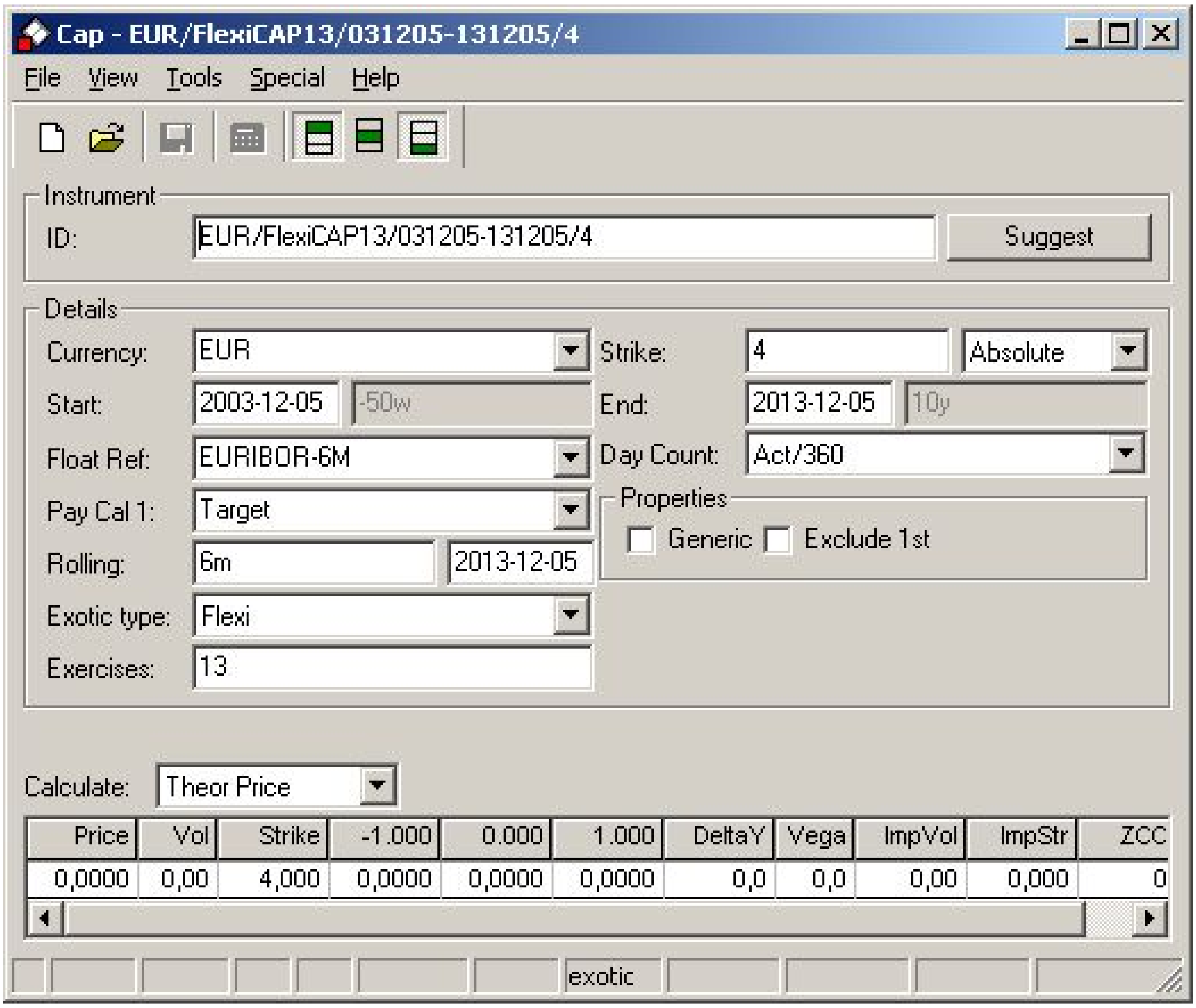
Entering a Momentum Cap involves setting four additional fields in the Cap definition compared to entering a plain vanilla Cap. To start with, we have to set **Exotic Type** > **Momentum**. This makes the fields **Barrier**, **Spread** and **Limit** appear in the Cap definition. Finally, we need to set **Barrier**, **Spread** and **Limit**.

Note that the **Strike** field in the Cap definition has a different meaning in a Momentum Cap than in a plain vanilla Cap. In a Momentum Cap it decides the initial value on the strike.


Cap - EUR/MomentumCAP/031205-131205/4-6-0.25-
File View Tools Special Help
Instrument
ID:
EUR/MomentumCAP/031205-131205/4-6-0.25-
Suggest
Details
Currency:
EUR
Strike:
4
Absolute
Start:
2003-12-05
-50w
End:
2013-12-05
10y
Float Ref:
EURIBOR-6M
Day Count:
Act/360
Pay Cal 1:
Target
Properties
Generic
Exclude 1st
Rolling:
6m
2013-12-05
Exotic type:
Momentum
Barrier:
6
Spread:
0,25
Limit:
7
Calculate:
Theor Price
Price Vol Strike -1.000 0.000 1.000 DeltaY Vega ImpVol ImpStr ZCC
0,0000 0,00 4,000 0,0000 0,0000 0,0000 0,0 0,0 0,00 0,000 0
exotic

Flexi cap

Let a be a positive integer less than or equal to the number of resets, i.e. $0 < a \leq n$. A Flexi Cap is like an ordinary Cap except that only the a first in-the-money Caplets are exercised.



Entering a Flexi Cap involves setting two additional fields in the Cap definition compared to entering a plain vanilla Cap. To start with, we have to set **Exotic Type** $>$ **Flexi**. This makes the field **Exercises** appear in the Cap definition, which corresponds to the integer a seen above. Finally, we need to set this field.

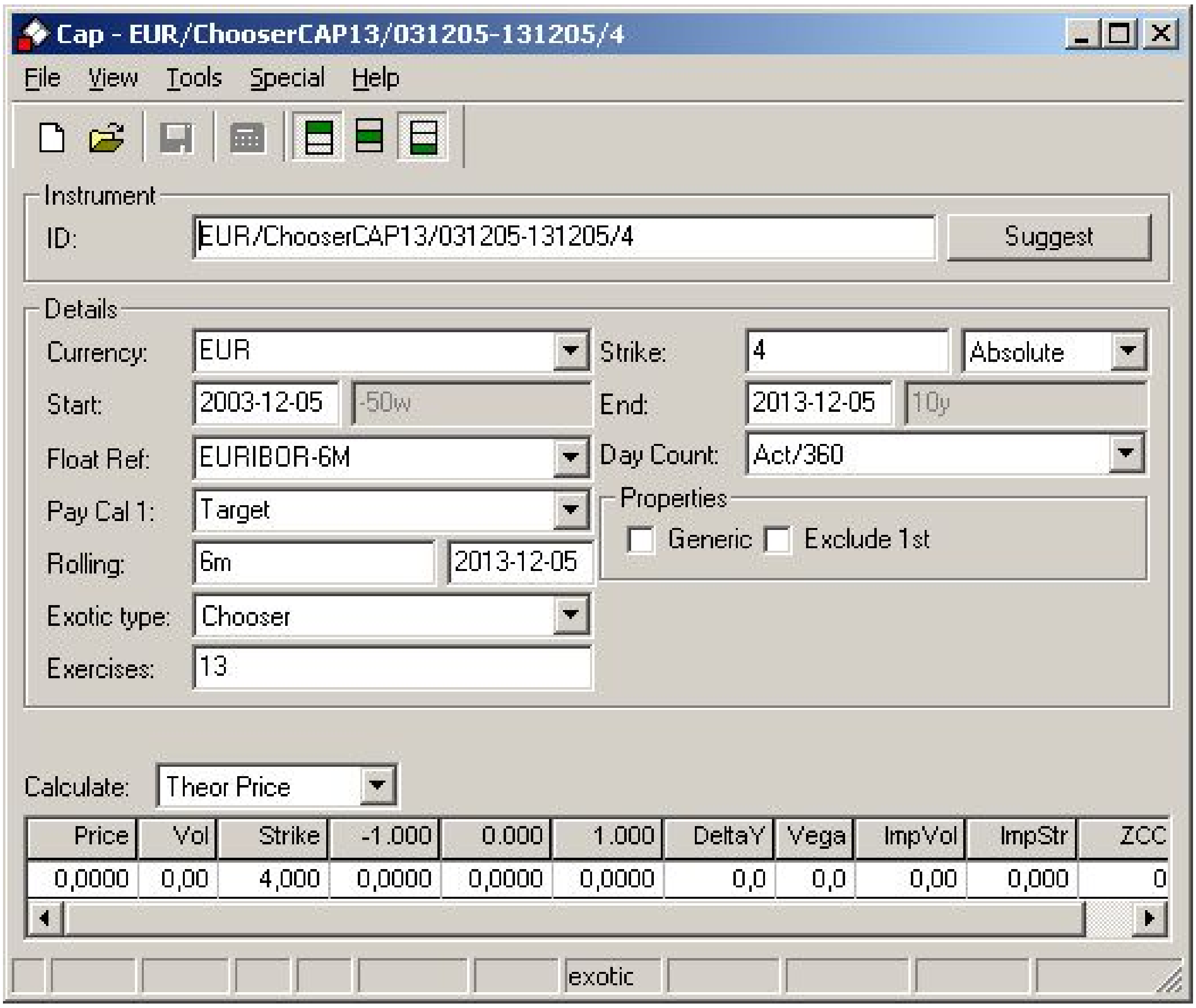
Chooser cap

Let a be a positive integer less than or equal to the number of resets, i.e. $0 < a \leq n$. A chooser Cap is like a Flexi Cap except that the contract holder can choose which a Caplets to exercise. Once the reset of a Caplet has taken place, it can no longer be chosen.

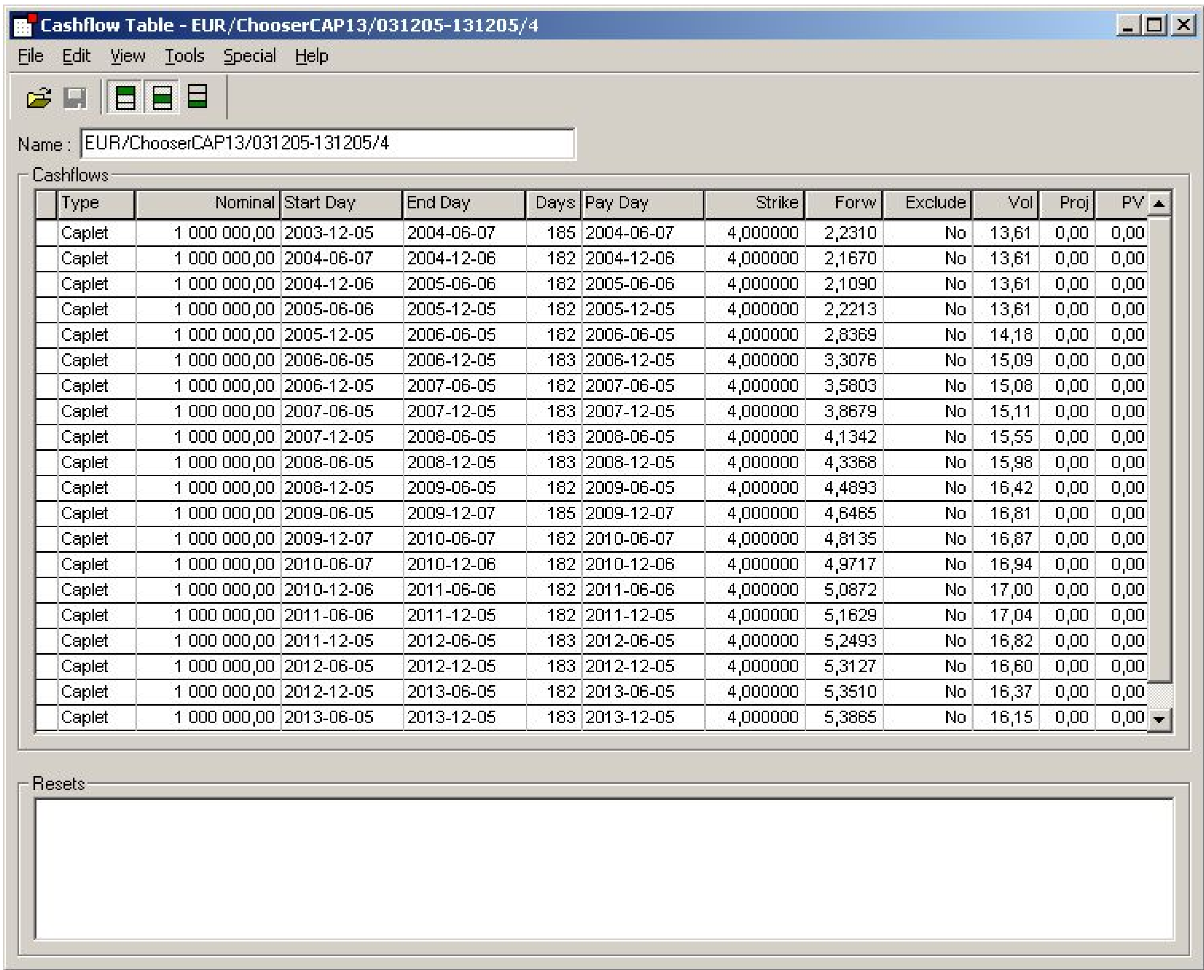
This is how the LIBOR Market Model in FRONT ARENA values a chooser Cap. Of course, it includes all the chosen Caplets. Suppose that c Caplets have been chosen, and that $c < a \leq n$. This means that there are still $a - c$ Caplets that can be chosen. In this case it picks $a - c$, or as many as possible but no more than $a - c$, of the remaining Caplets with highest present value. In other words, it employs an optimal strategy for the remaining of the Cap.

Initially, entering a chooser Cap involves setting two additional fields in the Cap definition compared to entering a plain vanilla Cap. To start with, we have to set **Exotic Type** > **Chooser**. This makes the field **Exercises** appear in the Cap definition, which corresponds to the integer a seen above. Finally, we need to set this field.

During the life of the chooser Cap we must choose which of the a Caplets to exercise. We choose a Caplet by entering a non-zero value in the record **fixed_time** of the database table **CashFlow**.



Now we have defined the chooser Cap. Let us take a look at the **Tools** > **Cash Flow Table** when we have not chosen any Caplet to exercise.

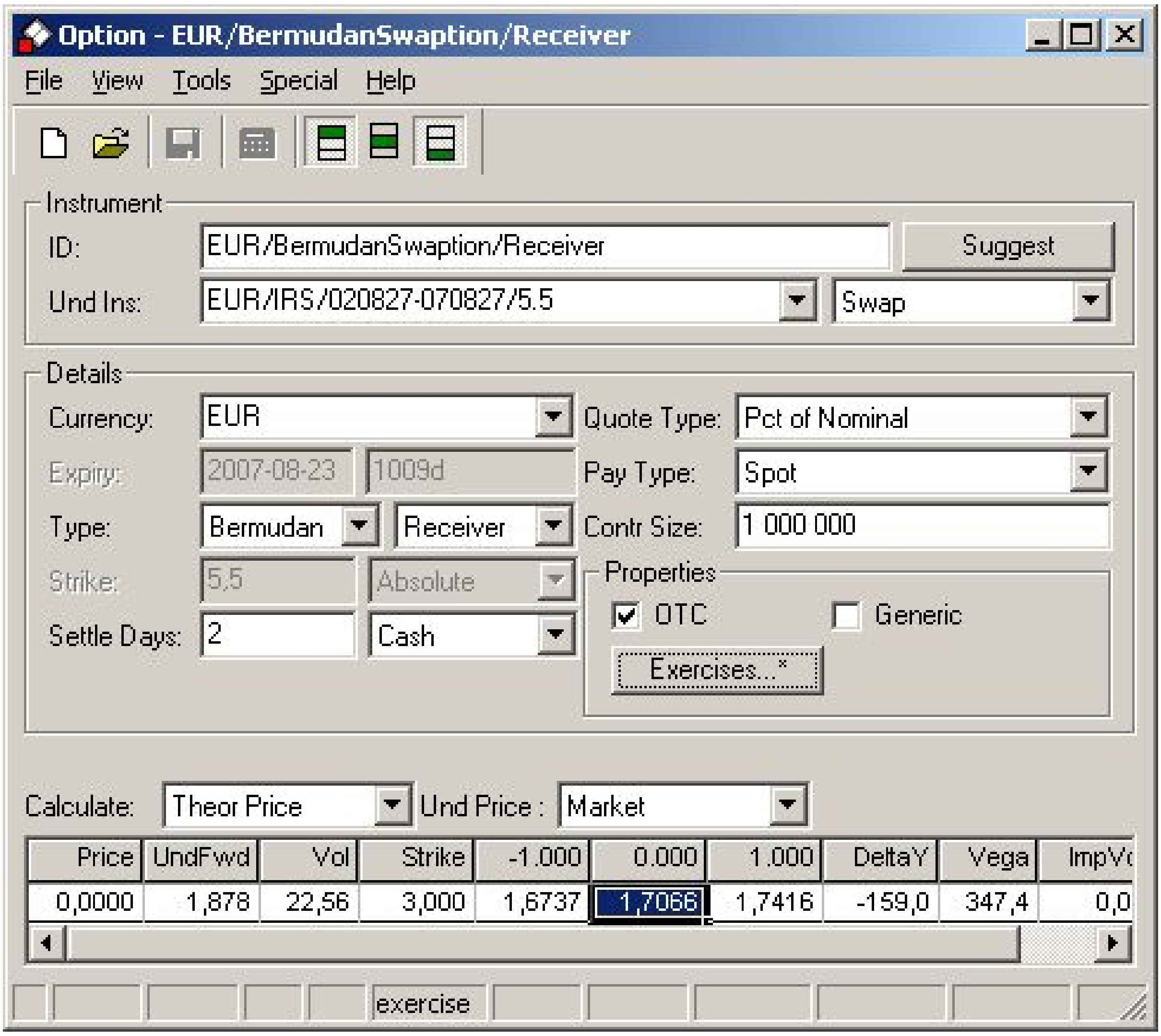


Type	Nominal	Start Day	End Day	Days	Pay Day	Strike	Forw	Exclude	Vol	Proj	PV
Caplet	1 000 000,00	2003-12-05	2004-06-07	185	2004-06-07	4,000000	2,2310	No	13,61	0,00	0,00
Caplet	1 000 000,00	2004-06-07	2004-12-06	182	2004-12-06	4,000000	2,1670	No	13,61	0,00	0,00
Caplet	1 000 000,00	2004-12-06	2005-06-06	182	2005-06-06	4,000000	2,1090	No	13,61	0,00	0,00
Caplet	1 000 000,00	2005-06-06	2005-12-05	182	2005-12-05	4,000000	2,2213	No	13,61	0,00	0,00
Caplet	1 000 000,00	2005-12-05	2006-06-05	182	2006-06-05	4,000000	2,8369	No	14,18	0,00	0,00
Caplet	1 000 000,00	2006-06-05	2006-12-05	183	2006-12-05	4,000000	3,3076	No	15,09	0,00	0,00
Caplet	1 000 000,00	2006-12-05	2007-06-05	182	2007-06-05	4,000000	3,5803	No	15,08	0,00	0,00
Caplet	1 000 000,00	2007-06-05	2007-12-05	183	2007-12-05	4,000000	3,8679	No	15,11	0,00	0,00
Caplet	1 000 000,00	2007-12-05	2008-06-05	183	2008-06-05	4,000000	4,1342	No	15,55	0,00	0,00
Caplet	1 000 000,00	2008-06-05	2008-12-05	183	2008-12-05	4,000000	4,3368	No	15,98	0,00	0,00
Caplet	1 000 000,00	2008-12-05	2009-06-05	182	2009-06-05	4,000000	4,4893	No	16,42	0,00	0,00
Caplet	1 000 000,00	2009-06-05	2009-12-07	185	2009-12-07	4,000000	4,6465	No	16,81	0,00	0,00
Caplet	1 000 000,00	2009-12-07	2010-06-07	182	2010-06-07	4,000000	4,8135	No	16,87	0,00	0,00
Caplet	1 000 000,00	2010-06-07	2010-12-06	182	2010-12-06	4,000000	4,9717	No	16,94	0,00	0,00
Caplet	1 000 000,00	2010-12-06	2011-06-06	182	2011-06-06	4,000000	5,0872	No	17,00	0,00	0,00
Caplet	1 000 000,00	2011-06-06	2011-12-05	182	2011-12-05	4,000000	5,1629	No	17,04	0,00	0,00
Caplet	1 000 000,00	2011-12-05	2012-06-05	183	2012-06-05	4,000000	5,2493	No	16,82	0,00	0,00
Caplet	1 000 000,00	2012-06-05	2012-12-05	183	2012-12-05	4,000000	5,3127	No	16,60	0,00	0,00
Caplet	1 000 000,00	2012-12-05	2013-06-05	182	2013-06-05	4,000000	5,3510	No	16,37	0,00	0,00
Caplet	1 000 000,00	2013-06-05	2013-12-05	183	2013-12-05	4,000000	5,3865	No	16,15	0,00	0,00

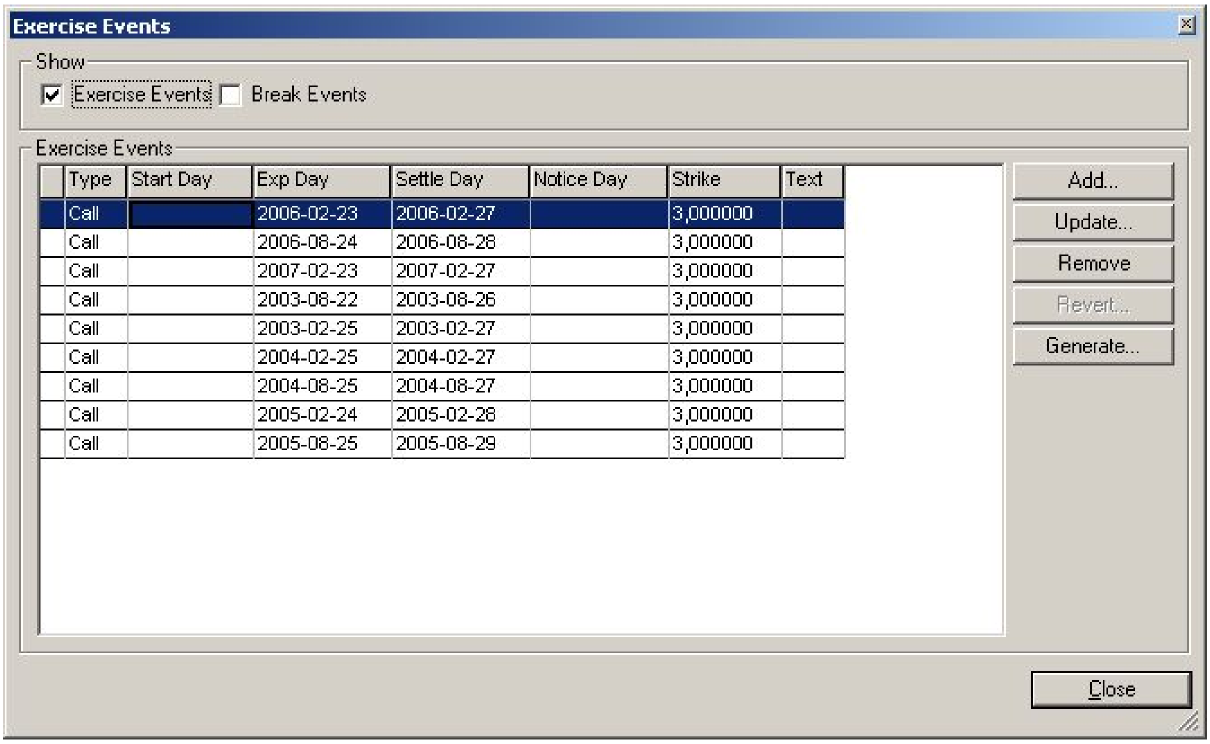
What is interesting is the PV column. We see that the second Caplet is worth 0 although it has a Pay Day in the future. This is because we have not chosen to exercise this Caplet. We will return to this example in the lecture about ARENA SQL.

Example: Bermudan call swaption

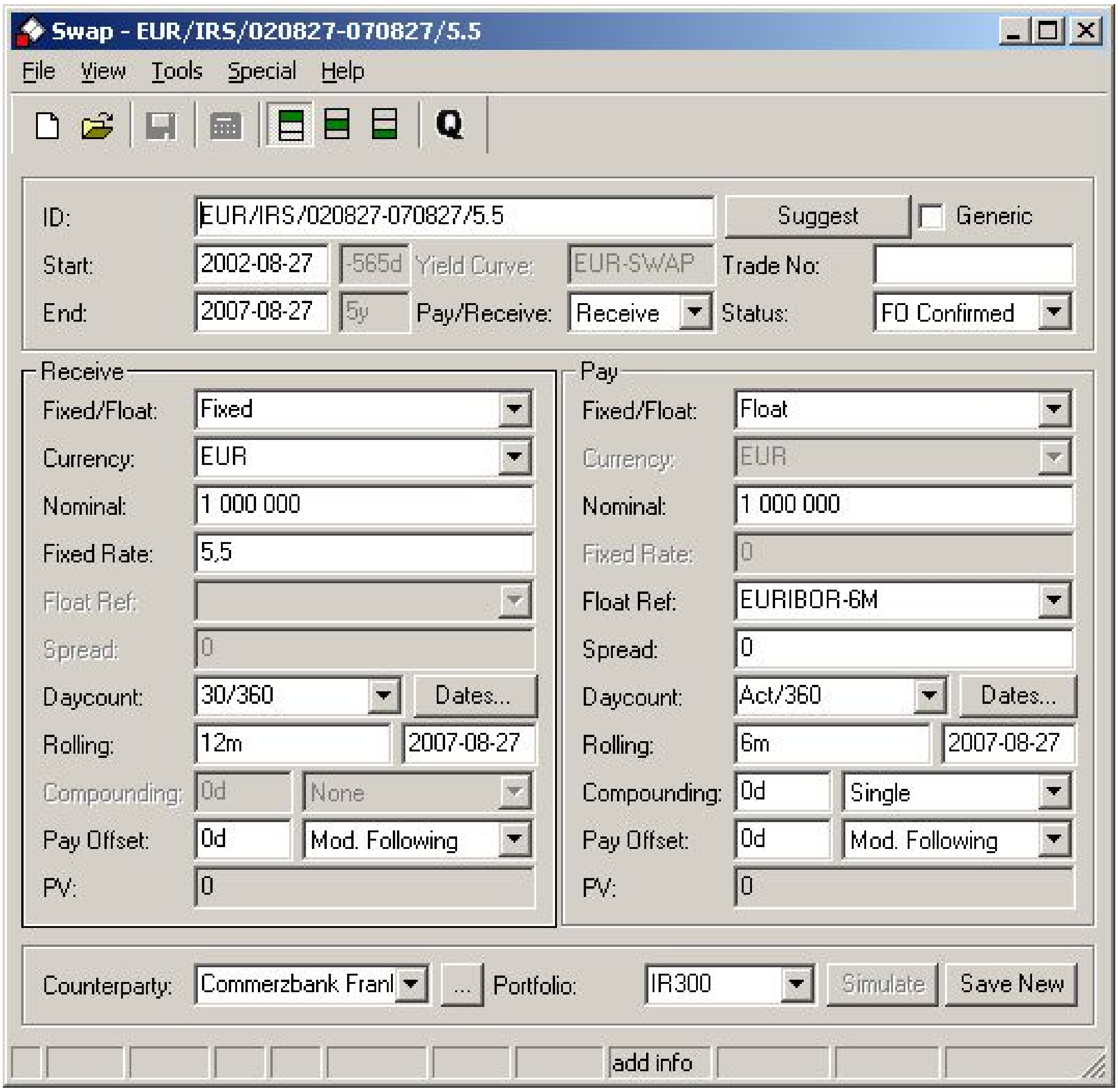
Bermudan Swaptions are defined in the Option application by selecting the **Type** > **Bermudan**. The exercise dates are entered in the **Exercise Events** dialog accessed by clicking the **Exercises...** button. This is illustrated in the following figure:



Before proceeding, the underlying instrument must be selected. In this example the instrument EUR/IRS/020827-070827/5.5 has been chosen. The next step is to enter the relevant exercise dates and strike rates. Click the **Exercises...** button to display the **Exercise Events** window:

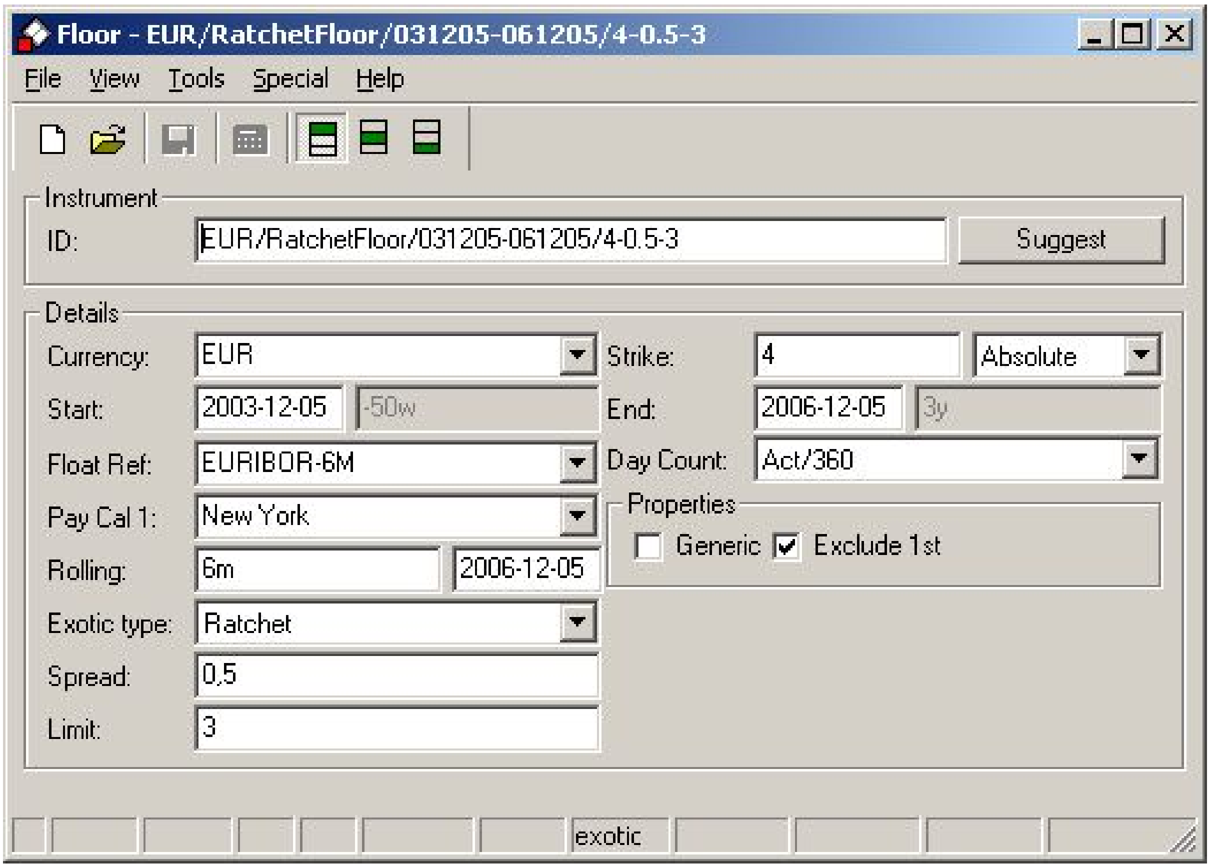


The underlying swap has the following details:


Swap - EUR/IRS/020827-070827/5.5
File View Tools Special Help
ID: EUR/IRS/020827-070827/5.5
Suggest
Generic
Start: 2002-08-27 -565d Yield Curve: EUR-SWAP Trade No:
End: 2007-08-27 5y Pay/Receive: Receive Status: FO Confirmed
Receive
Fixed/Float: Fixed
Currency: EUR
Nominal: 1 000 000
Fixed Rate: 5,5
Float Ref:
Spread: 0
Daycount: 30/360 Dates...
Rolling: 12m 2007-08-27
Compounding: 0d None
Pay Offset: 0d Mod. Following
PV: 0
Pay
Fixed/Float: Float
Currency: EUR
Nominal: 1 000 000
Fixed Rate: 0
Float Ref: EURIBOR-6M
Spread: 0
Daycount: Act/360 Dates...
Rolling: 6m 2007-08-27
Compounding: 0d Single
Pay Offset: 0d Mod. Following
PV: 0
Counterparty: Commerzbank Franl ... Portfolio: IR300 Simulate Save New
add info

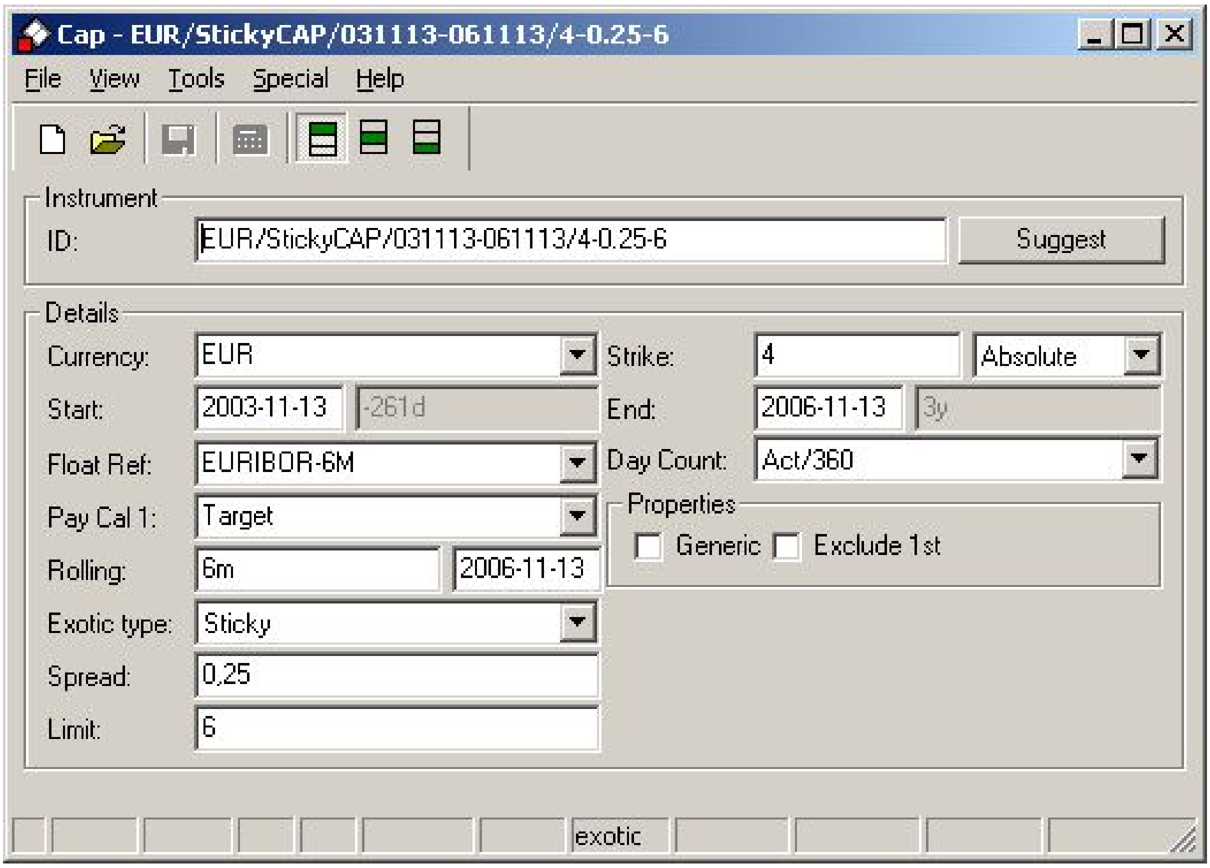
Exercises

① Calculate the theoretical price of the ratchet floor with the following parameters



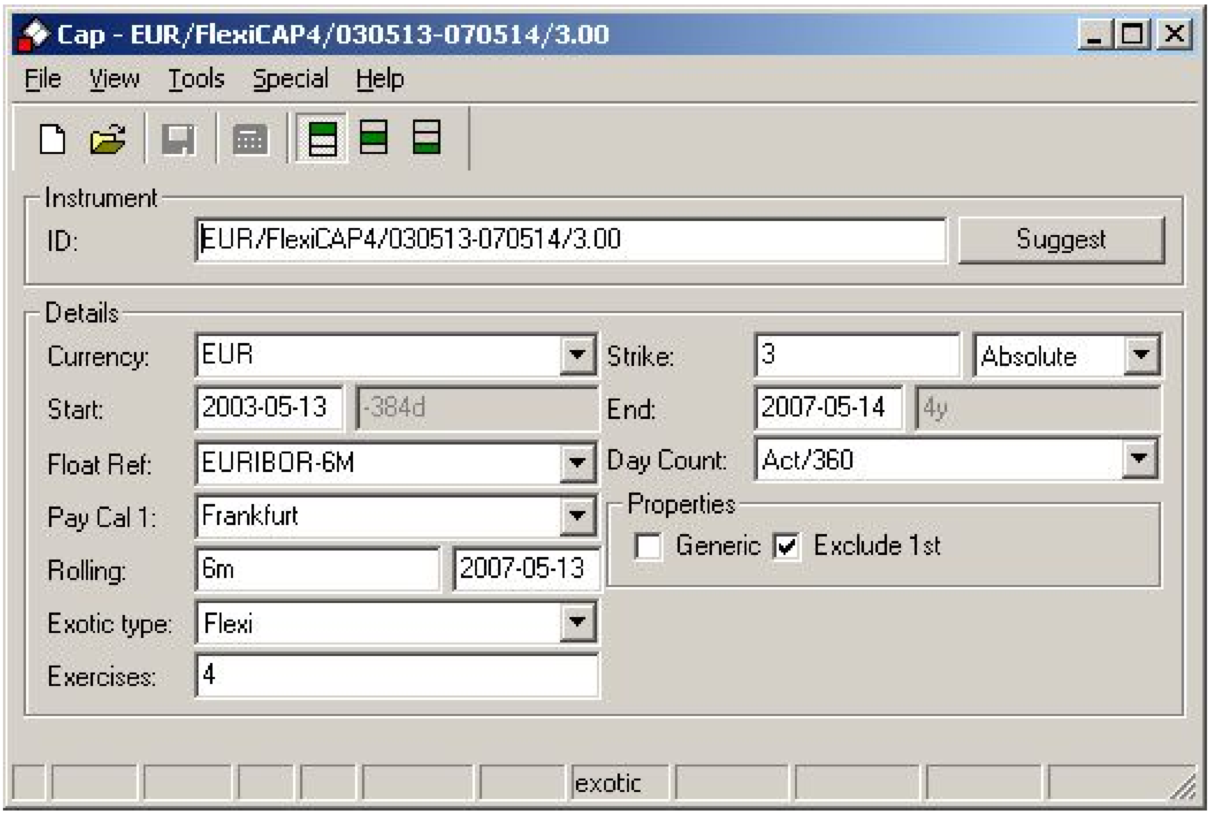
using the LIBOR market model.

② Calculate the theoretical price of the sticky cap with the following parameters



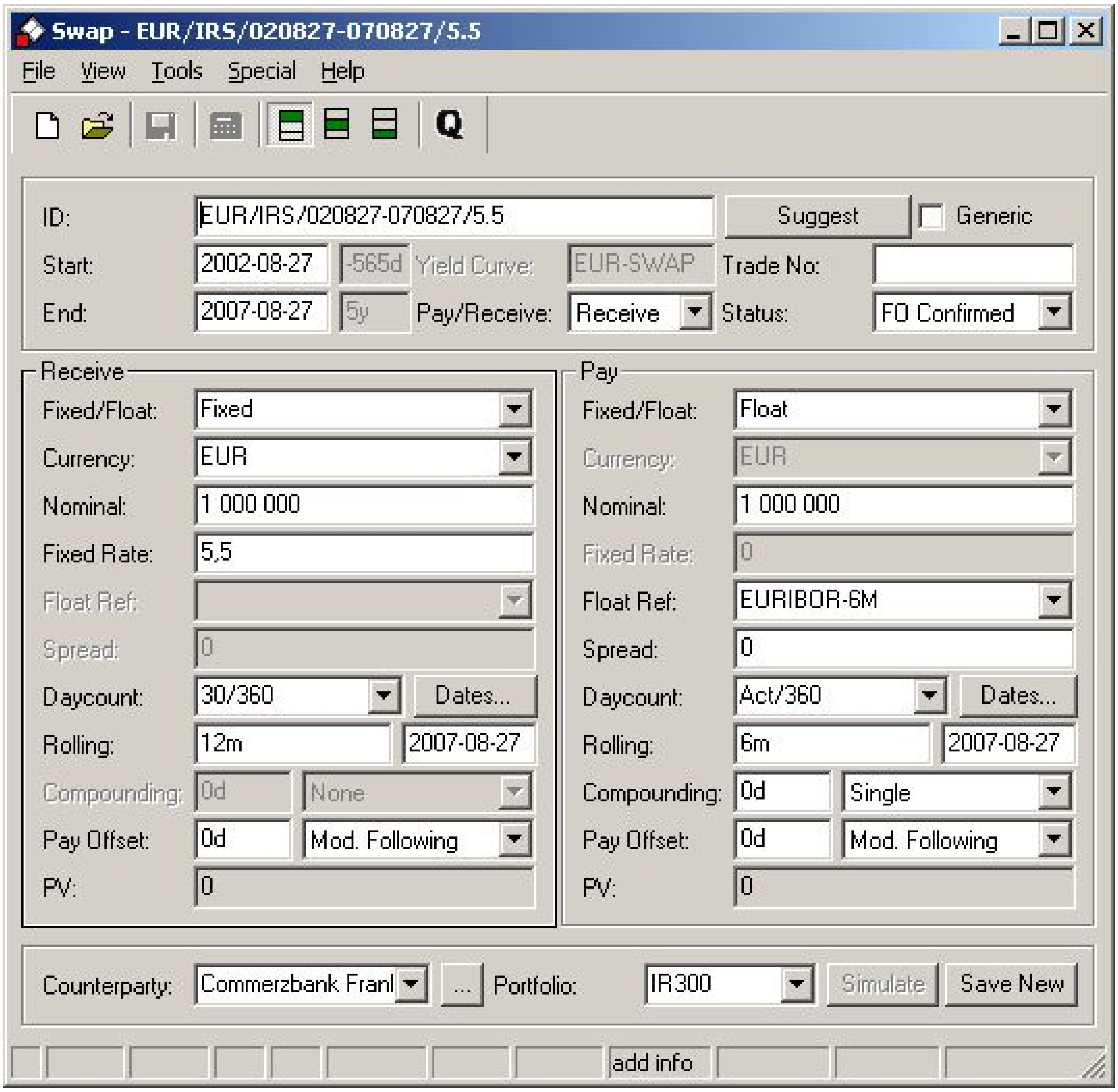
using the LIBOR market model.

③ Calculate the theoretical price of the flexi cap with the following parameters

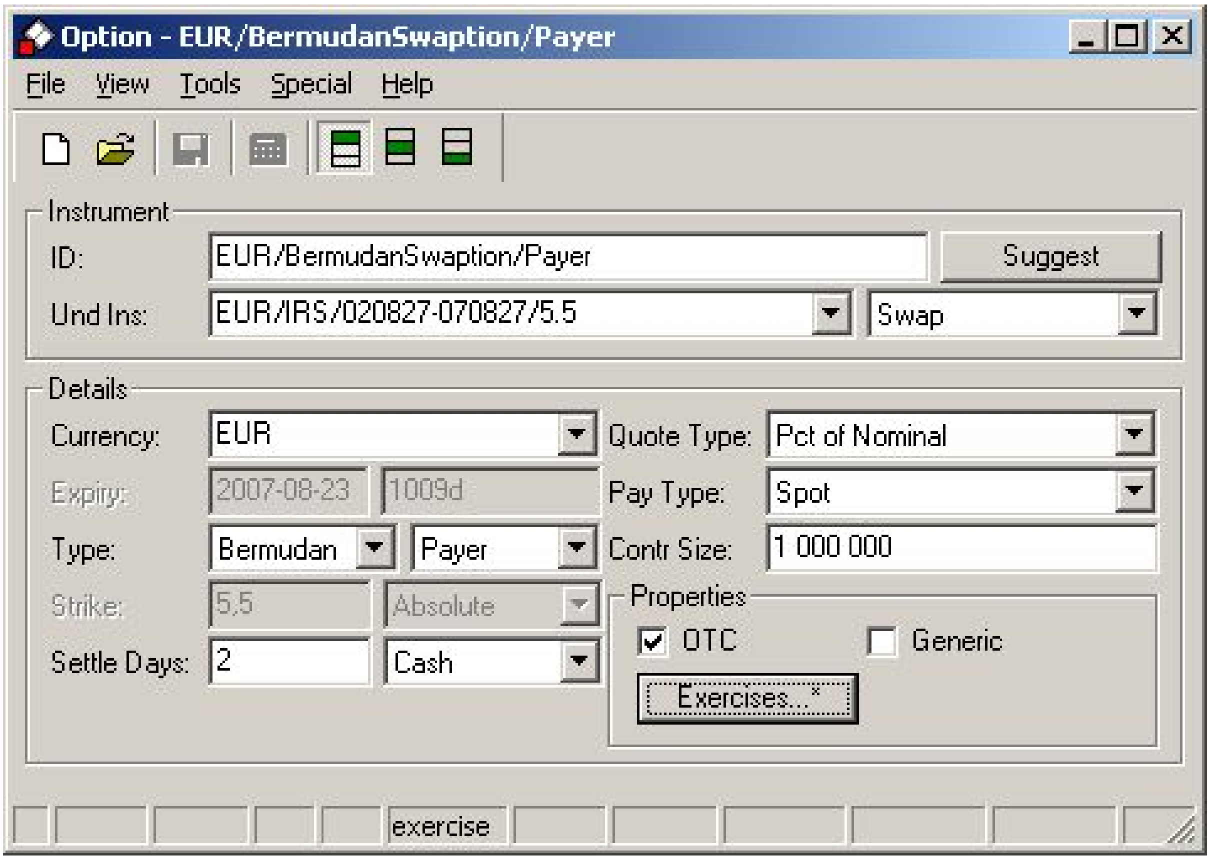


using the LIBOR market model.

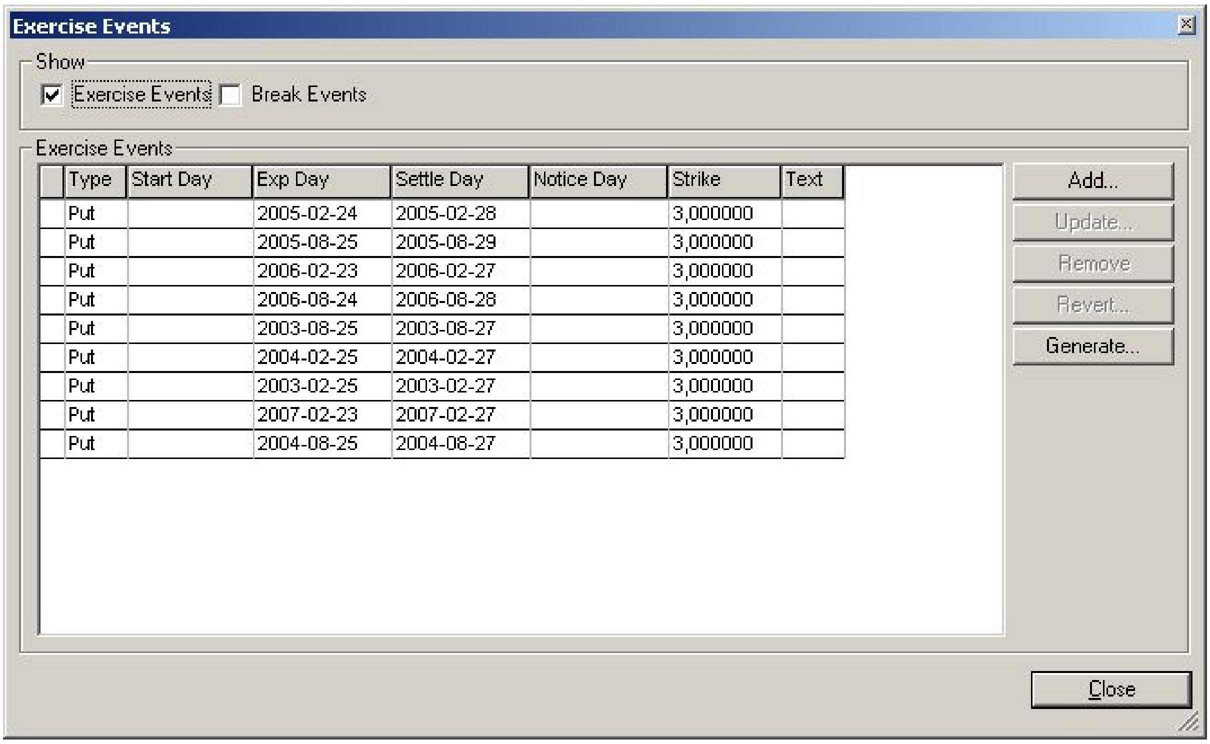
④ The underlying swap has the following details:



The corresponding Bermudan swaption has the following parameters



and the following exercise dates:



Type	Start Day	Exp Day	Settle Day	Notice Day	Strike	Text
Put		2005-02-24	2005-02-28		3,000000	
Put		2005-08-25	2005-08-29		3,000000	
Put		2006-02-23	2006-02-27		3,000000	
Put		2006-08-24	2006-08-28		3,000000	
Put		2003-08-25	2003-08-27		3,000000	
Put		2004-02-25	2004-02-27		3,000000	
Put		2003-02-25	2003-02-27		3,000000	
Put		2007-02-23	2007-02-27		3,000000	
Put		2004-08-25	2004-08-27		3,000000	

Calculate the theoretical price of the swaption, using the LIBOR market model.