Instruments with underlyings

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Abstract

Contents of the lecture.

☞ Futures and forwards.

☞ Options.

Introduction

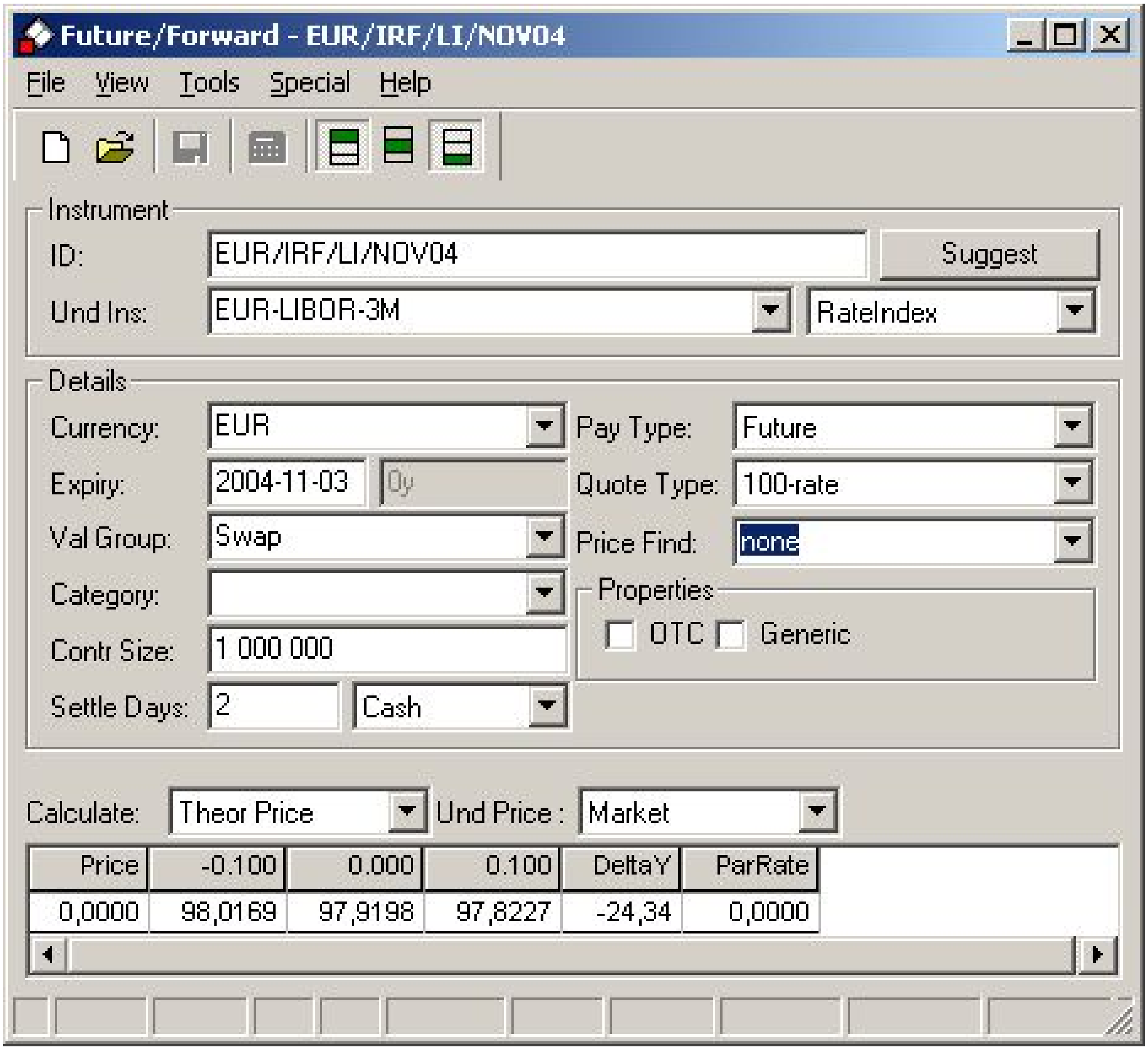
Instruments such as futures, forwards and options are not treated as sets of cash flows in PRIME. These instruments have the common feature that they have varying kinds of underlying instruments.

The underlying instrument and its current value is of course, of great importance when valuing futures and options. In many applications, it is possible to choose whether the underlying market price should be used directly or whether the underlying price should be calculated from the yield curve used by the underlying.

Interest rate futures: theory

Interest rate futures are treated differently from futures on securities. The valuation is based on the implied forward rate of the rate index used as the underlying. The time period used, starts on the settlement date of the future and ends on settlement date + rate index time period. The theoretical futures price is then achieved by taking 100 minus the implied forward rate. If the future is mapped to a term structure volatility, the implied forward rate is adjusted with the convexity effect.

Interest rate futures: example



Consider the EUR-LIBOR-3M interest rate future that expires 2004-11-03. The instrument data is as follows:

Quote Type:	*100-rate*
Settlement	*Cash*
Settlement days:	*2*
Pay Type:	*Future*

The result is shown in figure. The theoretical price is 97.9198.

Futures and forwards on securities: theory

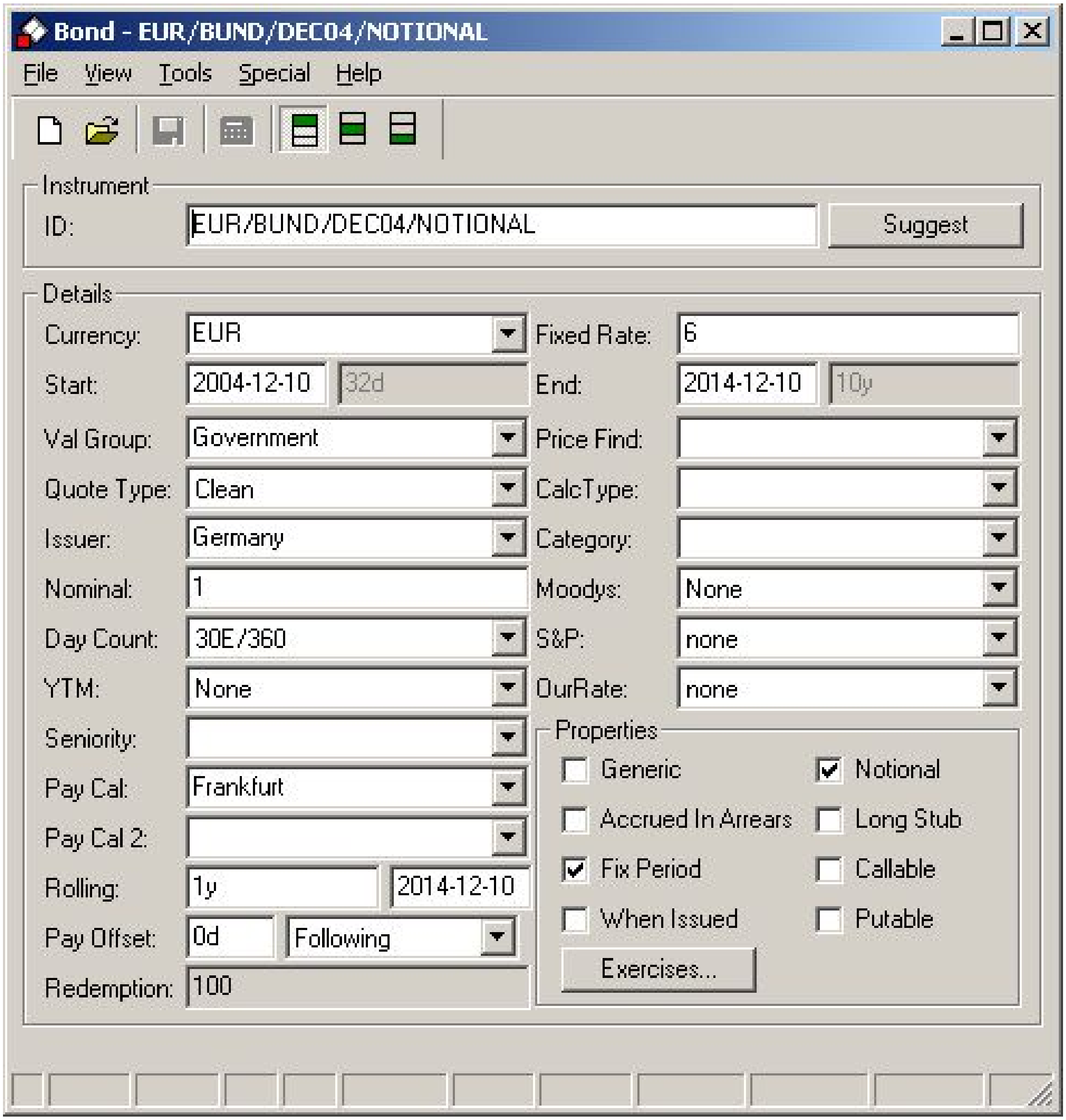
In the valuation of both futures and forwards on securities, the analysis starts with the price of the underlying. Based on an arbitrage argument, the forward price is then derived by comparing the cost of carry of the underlying with the revenues in terms of accrued interest, dividends and coupon payments.

The cost of carry is calculated using the repo curve mapped to the underlying. The dates used in this analysis will be the spot date (normally two banking days from the valuation date) to the settlement date of the future. This will give the theoretical forward price.

For futures on securities, the calculation of underlying values using the yield curve (when the theoretical value of the underlying is used) will be made with the curve mapped to the future and not with the curve mapped to the underlying.

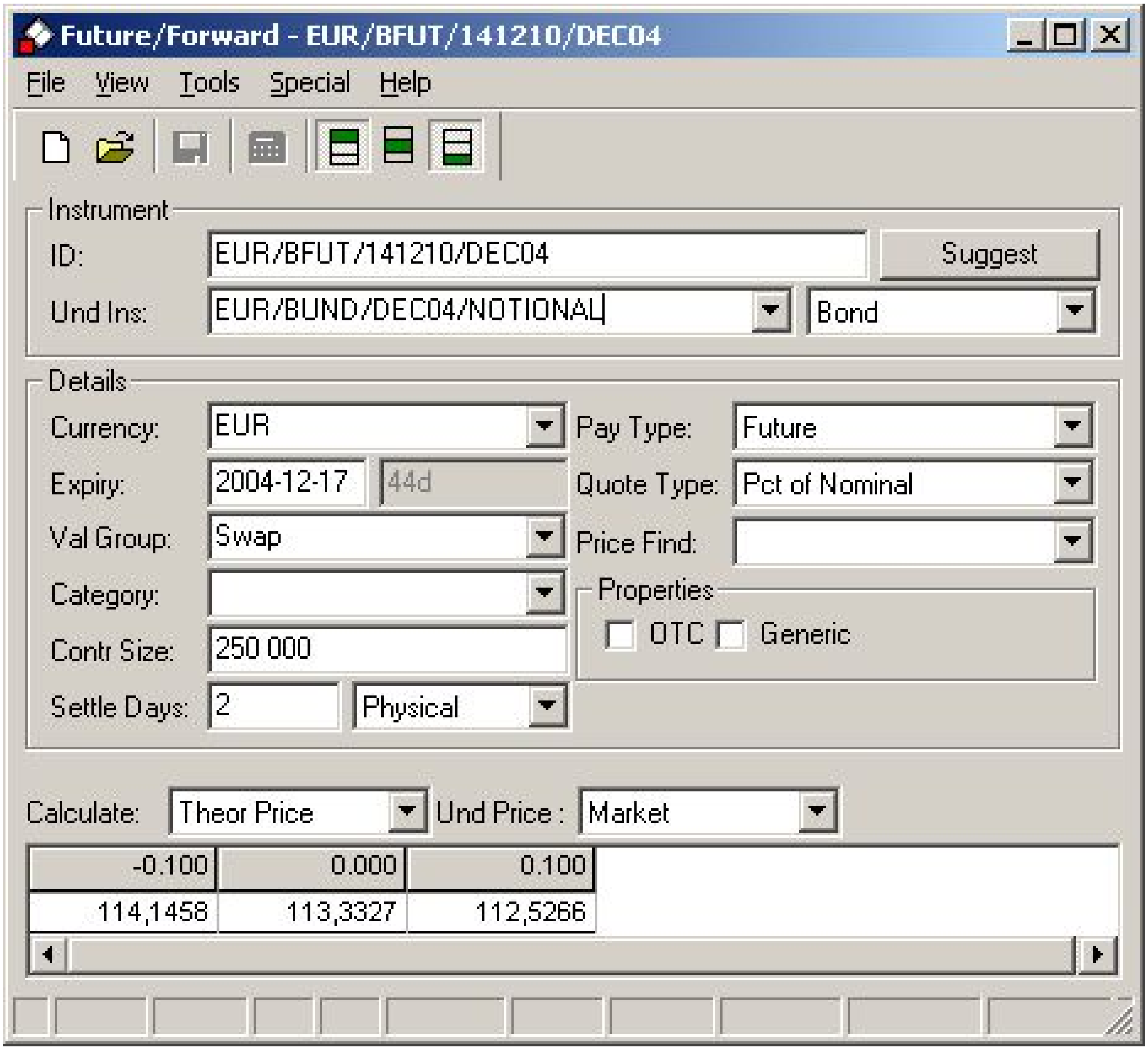
Futures and forwards on securities: example

The underlying bond is shown in the figure.



Consider the bond future that expires 2004-12-17 with

Quote Type:	*Pct of nominal*
Contr Size:	*250 000*
Settlement:	*Physical*
Settlement Days:	*2*



The theoretical price is 113.3327.

Options: theory

By an option, we mean a contract that gives the owner the right to buy (call option) or sell (put option) an underlying instrument.

All options on interest rate underlyings can be valued using either the Black–Scholes model or term structure based models. This is user-defined and depends on the volatility structure that the option is mapped to. Some instruments, like compound options, must be valued using a term structure model.

The Black–Scholes model has many variations in PRIME, depending on the type of option (European/American), pay type of the option and the underlying, etc., but the general framework is the same.

The Black–Scholes model

The Black–Scholes model was first developed for European options on non-dividend paying stocks. The model has subsequently been extended to cope with American options and other underlyings.

The basic assumptions are:

- ☞ The underlying is a log normally distributed random variable.
- ☞ The volatility of the underlying is constant.
- ☞ Interest rates are constant.
- ☞ There are no transaction costs in any capital markets.
- ☞ Borrowing and lending can be done at the constant interest rate.
- ☞ There is continuous trading in all instruments.

A general formulation of the Black–Scholes formula (sometimes called Black76) for European options can be written as

$$c = P(t)[FN(d) - XN(d - \sigma\sqrt{\tau})]S$$

for call options and

$$p = P(t)[XN(\sigma\sqrt{\tau} - d) - FN(-d)]S$$

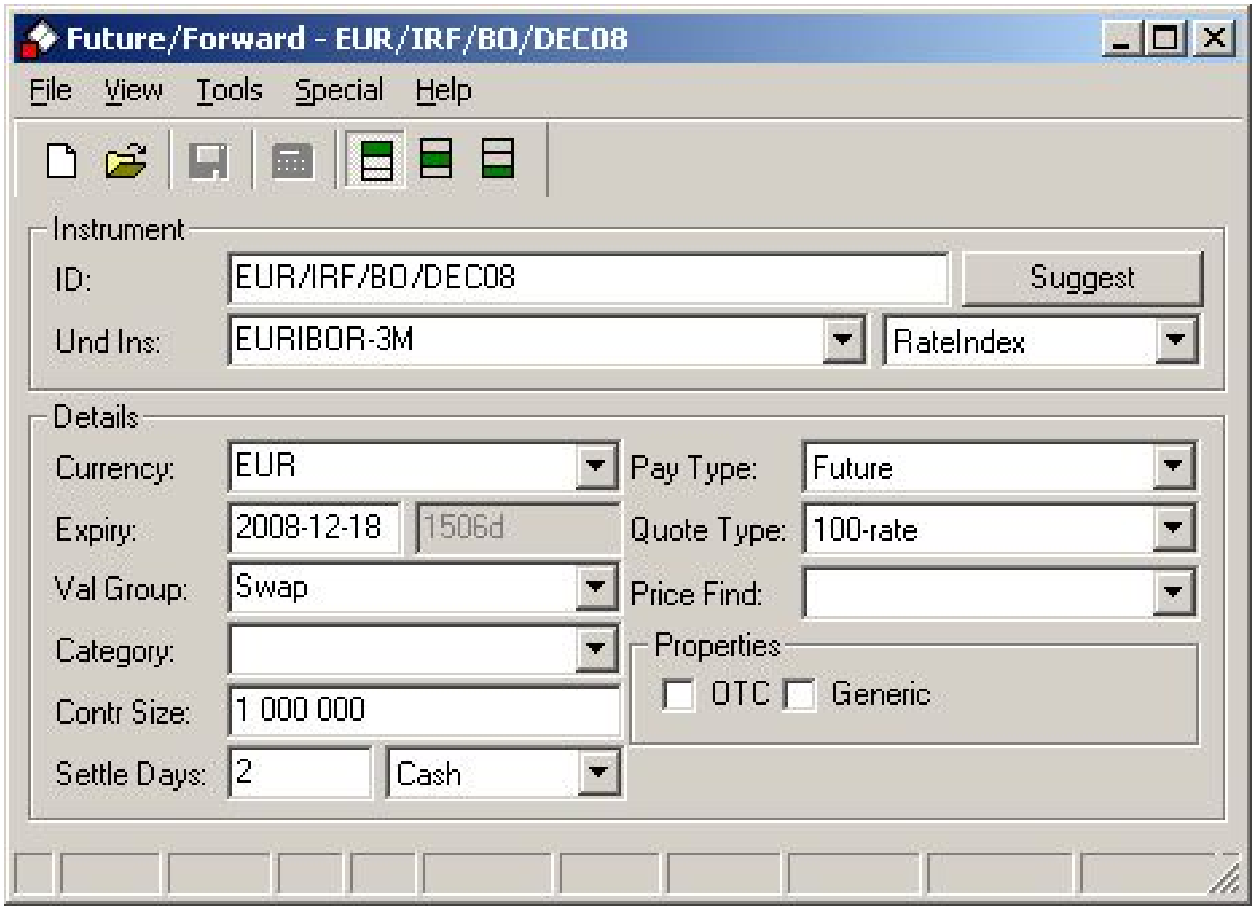
for put options, where:

F	=	forward price/rate of underlying
X	=	strike price/rate of option
τ	=	time to maturity of option
σ	=	volatility
$P(t)$	=	discount factor to maturity of option (pay day) for spot traded options, or 1 for future-style options
S	=	optional scale factor to convert option value in yield terms to monetary terms
$N(d)$	=	normal distribution function
d	=	$\dfrac{\ln(F/X)}{\sigma\sqrt{\tau}} + \dfrac{\sigma}{2}\sqrt{\tau}$

The way this general formula is applied differs somewhat, depending on the underlying of the option. The discount factor is calculated from the yield curve mapped to the option. This rate can therefore be different from the repo rate, which is calculated from the repo curve mapped to the underlying.

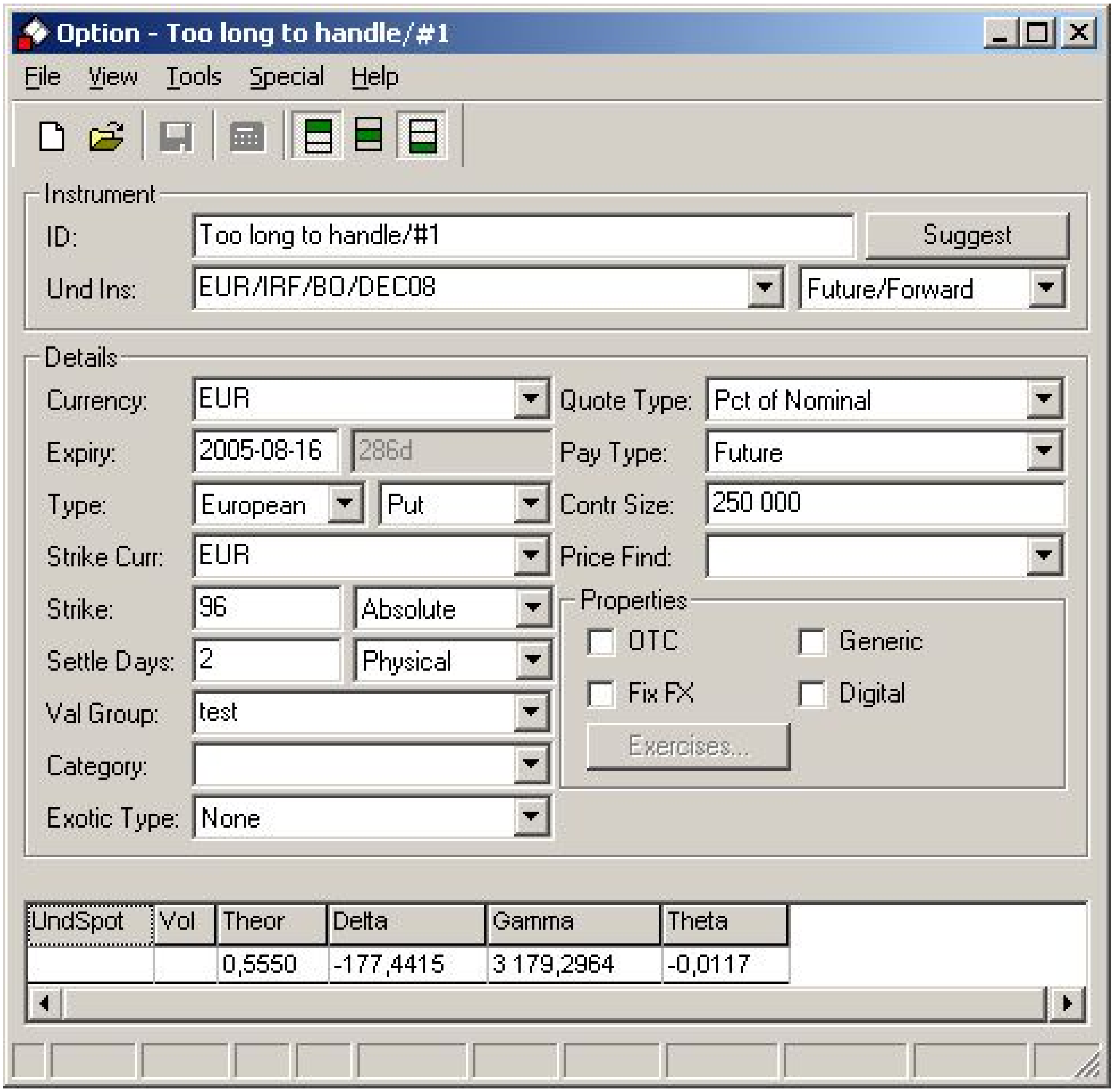
Example: option on futures

The underlying futures is shown in the figure.



Consider the following option data:

Quote type:	*Pct of Nominal*
Option Type	*Put*
Contract Size:	*250 000*
Strike	*96*
Expiry	*2005-08-16*
Settlement	*Physical delivery 2 days*
Exercise Type	*European*
Pay Type:	*Future*



The result is shown in figure. The theoretical price is €0.555.

Bond options: theory

For options on bonds, the clean forward price is calculated according to the standard forward pricing formula given as

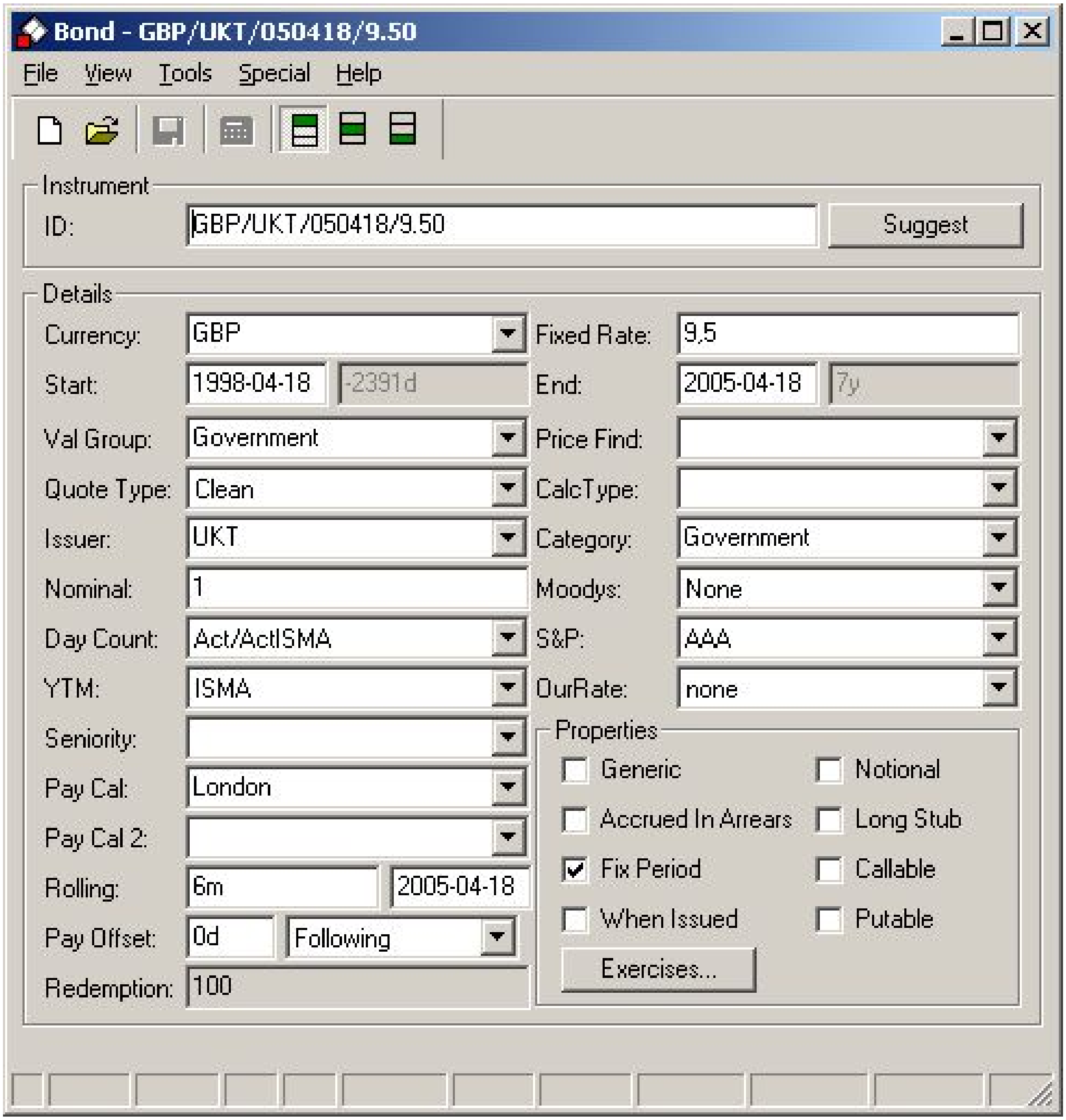
$$\sigma_p = \sigma_y \frac{y}{P} \left| \frac{dP}{dy} \right|,$$

where

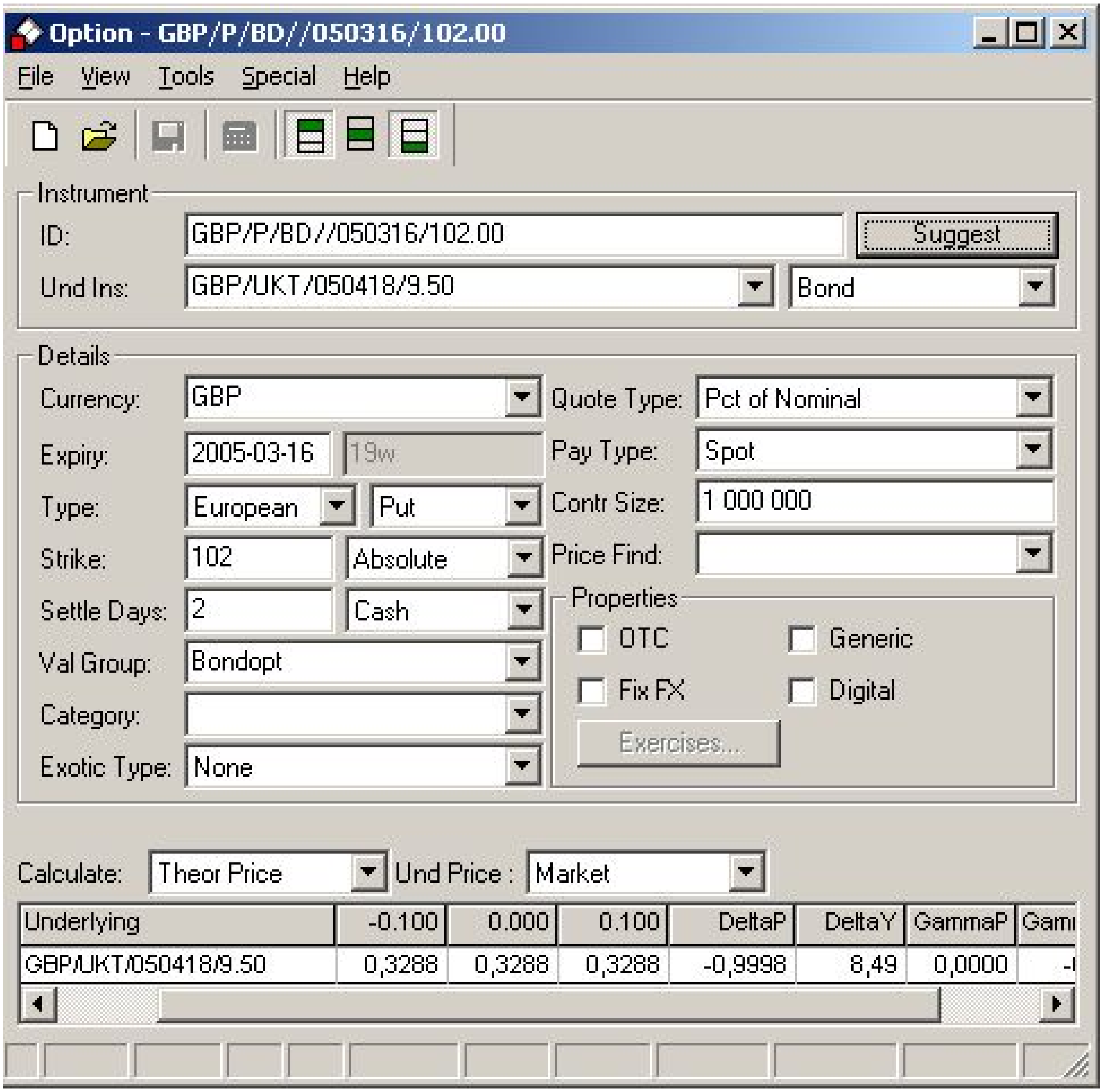
σ_p	=	price volatility
σ_y	=	yield volatility
y	=	yield of underlying bond
P	=	price of underlying bond

Bond options: example

The underlying bond is shown in the figure.



We are going to calculate the theoretical price of the option with the following data


Option - GBP/P/BD//050316/102.00
File View Tools Special Help
Instrument
ID: GBP/P/BD//050316/102.00
Suggest
Und Ins: GBP/UKT/050418/9.50
Bond
Details
Currency: GBP
Quote Type: Pct of Nominal
Expiry: 2005-03-16
19w
Pay Type: Spot
Type: European
Put
Contr Size: 1 000 000
Strike: 102
Absolute
Price Find:
Settle Days: 2
Cash
Properties
OTC
Generic
Val Group: Bondopt
Fix FX
Digital
Category:
Exercises...
Exotic Type: None
Calculate: Theor Price
Und Price : Market
Underlying -0.100 0.000 0.100 DeltaP DeltaY GammaP Gam
GBP/UKT/050418/9.50 0,3288 0,3288 0,3288 -0,9998 8,49 0,0000

Quote type:	*Pct of Nominal*
Option Type	*Put*
Contract Size:	*1 000 000*
Strike	*102*
Expiry	*2005-03-16*
Settlement	*Cash 2 days*
Exercise Type	*European*
Pay Type:	*Spot*

The theoretical price is GBP 0.3288.

Swaptions: theory

For swaps, the forward rate used is the forward par rate of the underlying swap. The par rate calculation is exact, in the sense that it takes account of all the exact cash flows of the swap.

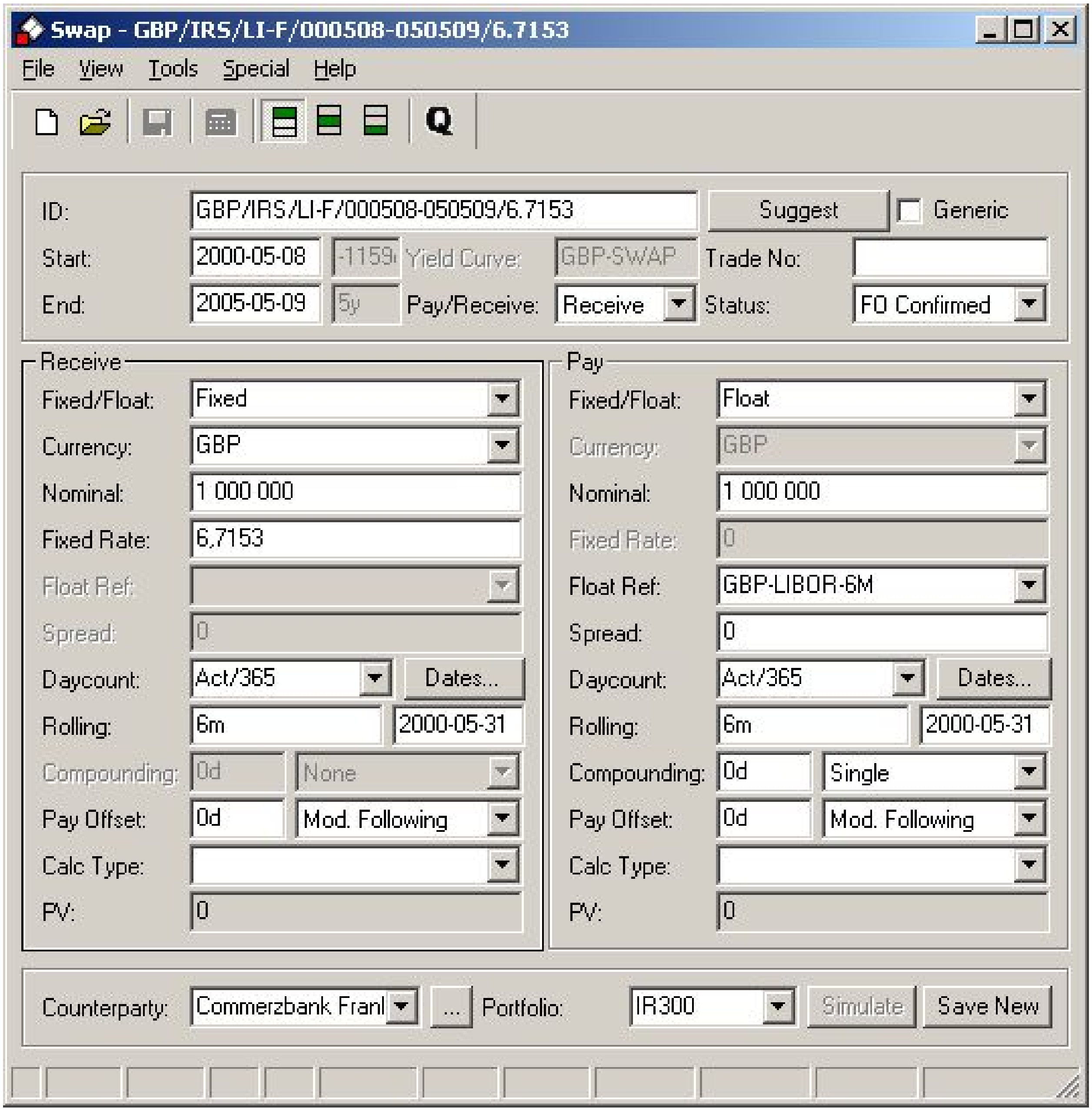
This implies that amortising swaps, reset in arrears swaps, or swaps with a spread can be used as underlyings. If a generic swap is selected as the underlying, the start date of that swap will be the settlement day of the option (option expiry plus settlement days).

The strike of swaptions should always be given as the corresponding fixed coupon in the swap. The value given in the option definition will override the value defined in the swap.

The volatility defined will always be interpreted as the volatility of the forward par rate.

Swaptions: example

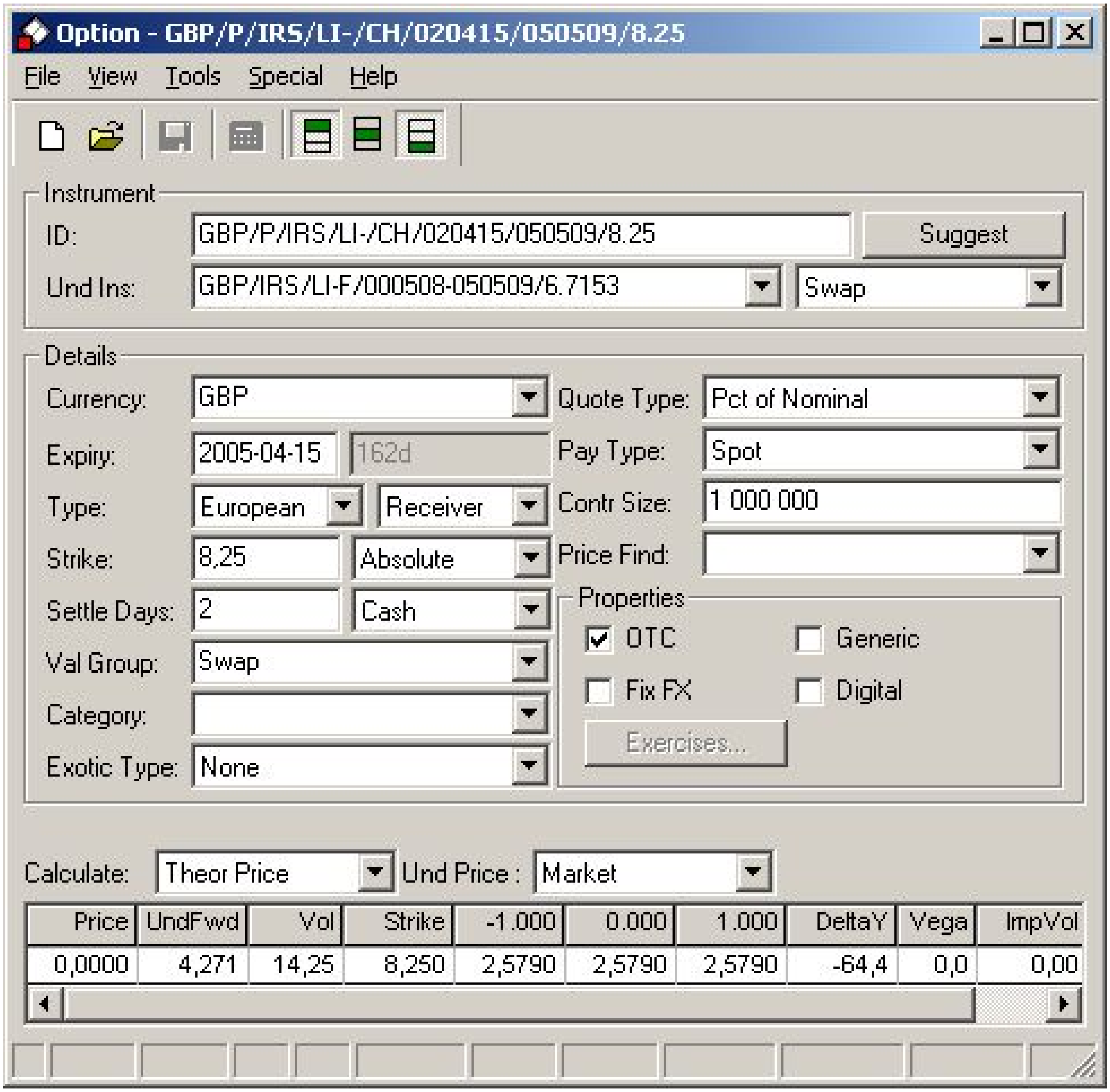
The underlying swap has the following details:



We are going to calculate the theoretical price of a swaption expiring 2005-04-15 with the following instrument data:

Quote type:	*Pct of Nominal*
Generic:	*No*
OTC:	*Yes*
Option Type:	*Receiver*
Contract Size:	*1 000 000*
Strike:	*6.25*
Settlement:	*Cash*
Exercise Type:	*European*
Pay Type:	*Spot*
Settlement days:	*2*

The theoretical price is GBP 2.5790.



Credit default swaptions: theory

For credit default Swaptions, the forward rate used is the forward par rate of the underlying credit default swap (CDS).

If a generic CDS is selected as the underlying, the start date of that CDS will be the settlement day of the option (option expiry plus settlement days).

The strike of a credit default swaption should always be given as the corresponding fixed coupon in the CDS. The value given in the option definition will override the value defined in the underlying CDS. The underlying CDS will be considered to be a contract where you pay a fixed coupon for credit protection (a payer CDS), regardless of the chosen underlying.

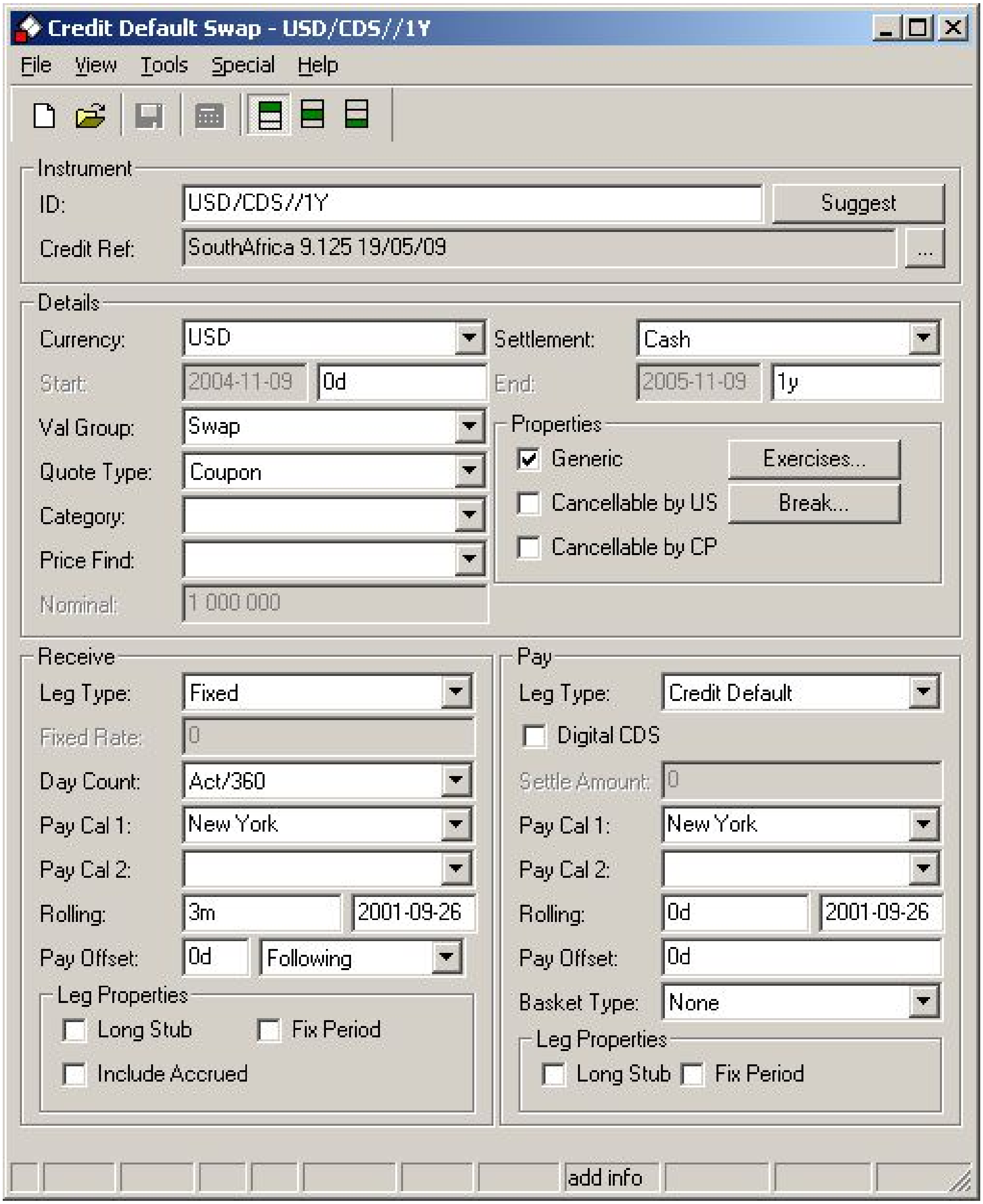
This means that a call option on a CDS gives you the right to enter into a contract where you pay a fixed coupon (the strike) for credit protection. A put option will accordingly give you the right to receive a fixed coupon for providing credit protection.

The volatility defined will always be interpreted as the volatility of the forward par rate.

Since the forward and the strike are given in rate terms, the value from the Black-Scholes formula must be converted to price terms using a scale factor. The scale factor will be the sensitivity of the forward CDS's present value to a unit change in the fixed coupon.

These options are valued as credit contingent options. This means that if the issuer defaults before the expiry of the option, the option is void. This contingent feature is captured by using the credit curve from the credit reference instead of the risk free curve, for the discounting in the Black–Scholes formula.

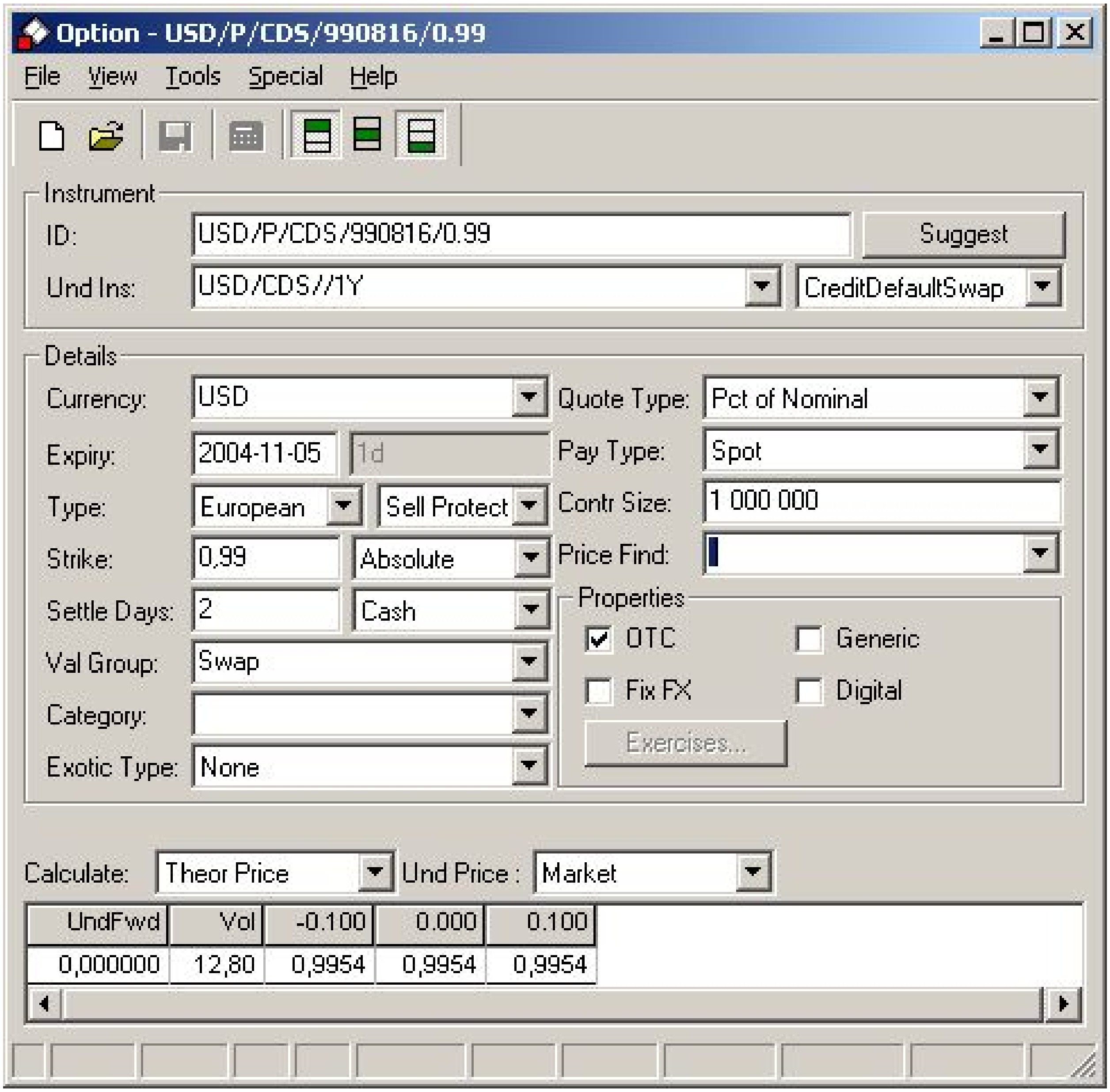
Credit default swaptions: example


Credit Default Swap - USD/CDS//1Y
File View Tools Special Help
Instrument
ID: USD/CDS//1Y
Suggest
Credit Ref: SouthAfrica 9.125 19/05/09
...
Details
Currency: USD
Settlement: Cash
Start: 2004-11-09 0d
End: 2005-11-09 1y
Val Group: Swap
Quote Type: Coupon
Category:
Price Find:
Nominal: 1 000 000
Properties
Generic
Cancellable by US
Cancellable by CP
Exercises...
Break...
Receive
Leg Type: Fixed
Fixed Rate: 0
Day Count: Act/360
Pay Cal 1: New York
Pay Cal 2:
Rolling: 3m 2001-09-26
Pay Offset: 0d Following
Leg Properties
Long Stub
Fix Period
Include Accrued
Pay
Leg Type: Credit Default
Digital CDS
Settle Amount: 0
Pay Cal 1: New York
Pay Cal 2:
Rolling: 0d 2001-09-26
Pay Offset: 0d
Basket Type: None
Leg Properties
Long Stub
Fix Period
add info

The underlying CDS is shown in figure. We calculate the theoretical price of a credit default swaption expiring 2004-11-05 with the following instrument data:

Quote type:	*Pct of Nominal*
Generic:	*No*
OTC:	*Yes*
Option Type:	*Call*
Contract Size:	*1 000 000*
Strike:	*0.99*
Settlement:	*Cash*
Exercise Type:	*European*
Pay Type:	*Spot*
Settlement days:	*2*

The theoretical price is \$0.9954.



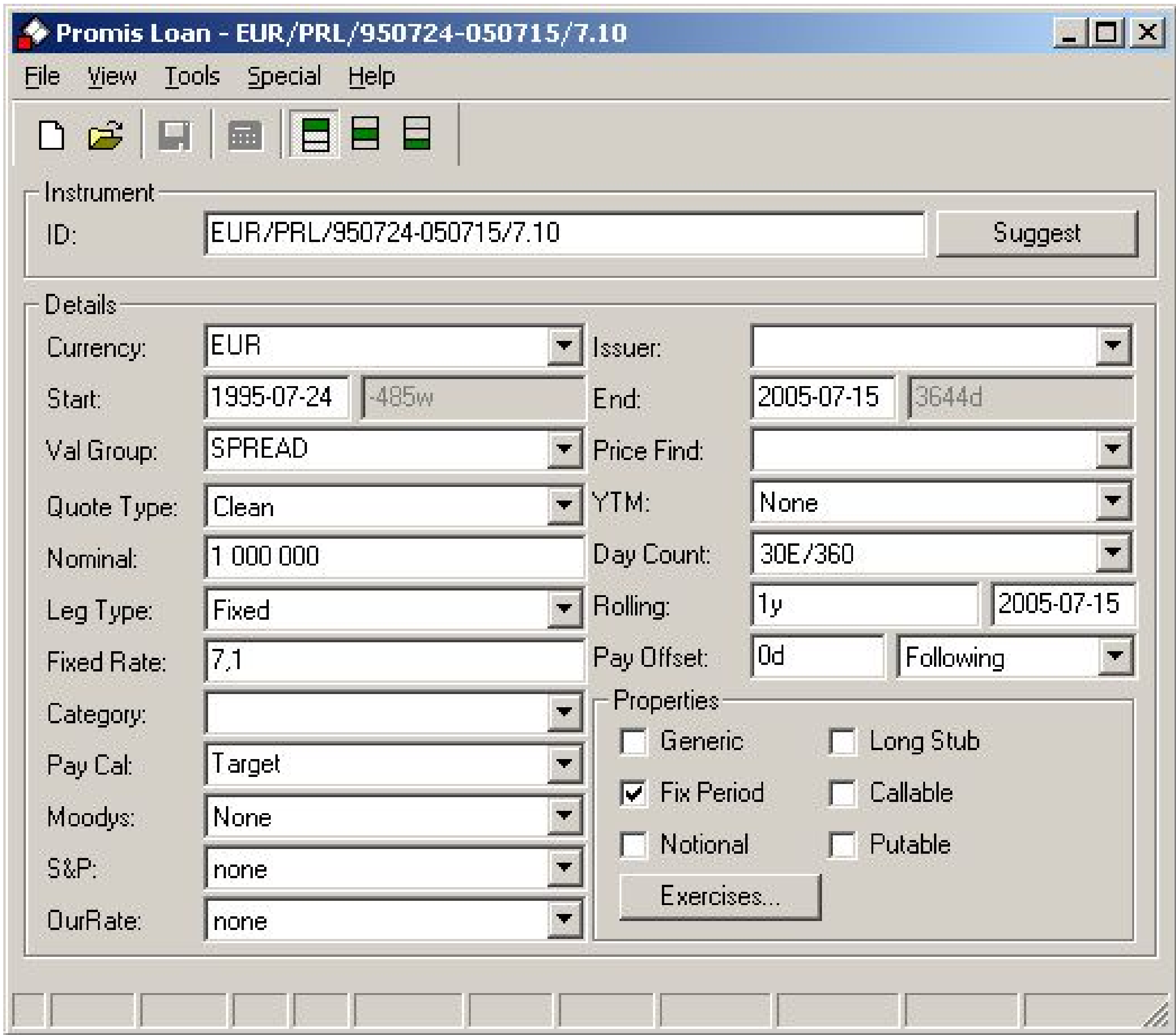
Options on promissory loans: theory

Options on Promissory Loans are treated separately. They have some characteristics similar to bonds and others like swaptions. The forward price is calculated in the same way as for bonds, based on the current price of the promissory loan and the Repo rate. This forward price is then converted into a forward yield. The promissory loan option is then priced as the similar swaption, with a forward par rate equal to the forward yield. Thus an option on a Promissory Loan is valued with a Black & Scholes yield formula similar to the formula used for pricing of Swaptions. However, using the Black & Scholes yield formula approach, for a deep in-the-money option (strike below par and option close to maturity) the value will be zero, which is incorrect. The problem lies in the conversion from forward price to forward yield. Sometimes the forward yield will be negative and this produces zero figures.

Promissory Loans are very similar, in terms of structure, to an accrued-in-arrears Bond.

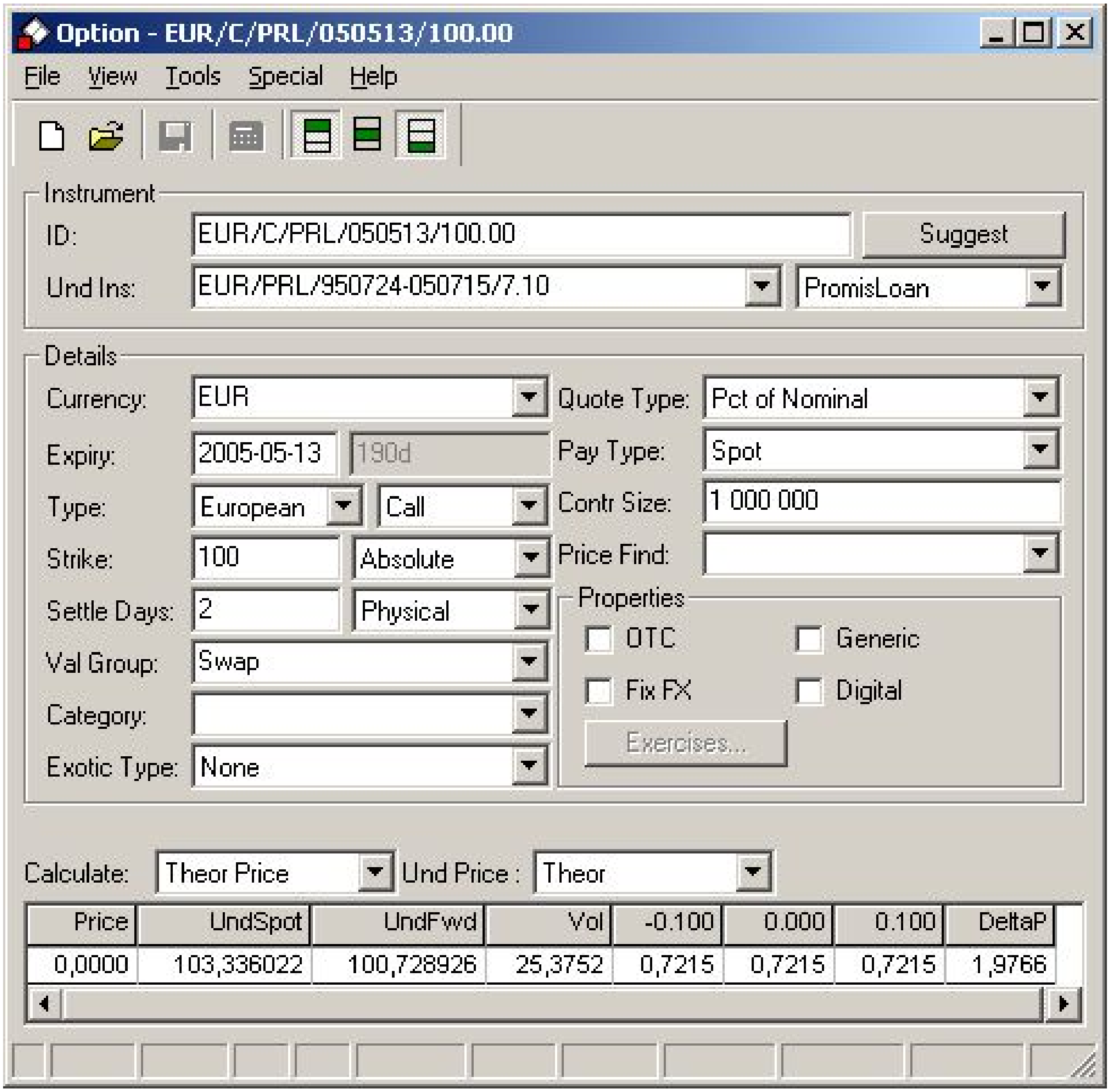
Options on promissory loans: example

The underlying promissory loan has the following data:



Consider the option expiring 2005-05-13 with the following instrument data:

Quote type:	*Pct of Nominal*
Option Type:	*Call*
Contract Size:	*1 000 000*
Strike:	*100*
Settlement:	*Physical Delivery*
Exercise Type:	*European*
Pay Type:	*Spot*
Settlement days:	*2*



The theoretical price is €0.7215.

Interest rate options: theory

For options on rate indices, the implied forward rate is used as forward price. The scale factor is then

$$S = \frac{\tau}{1 + F\tau},$$

where time period τ is the length of the rate index in years, calculated with the correct day count convention. The denominator comes from the fact that the payday of the option is at the beginning of the period (like FRAs), and the numerator comes from the fact that the option price expressed in annual yield terms is only applicable for the rate index period.

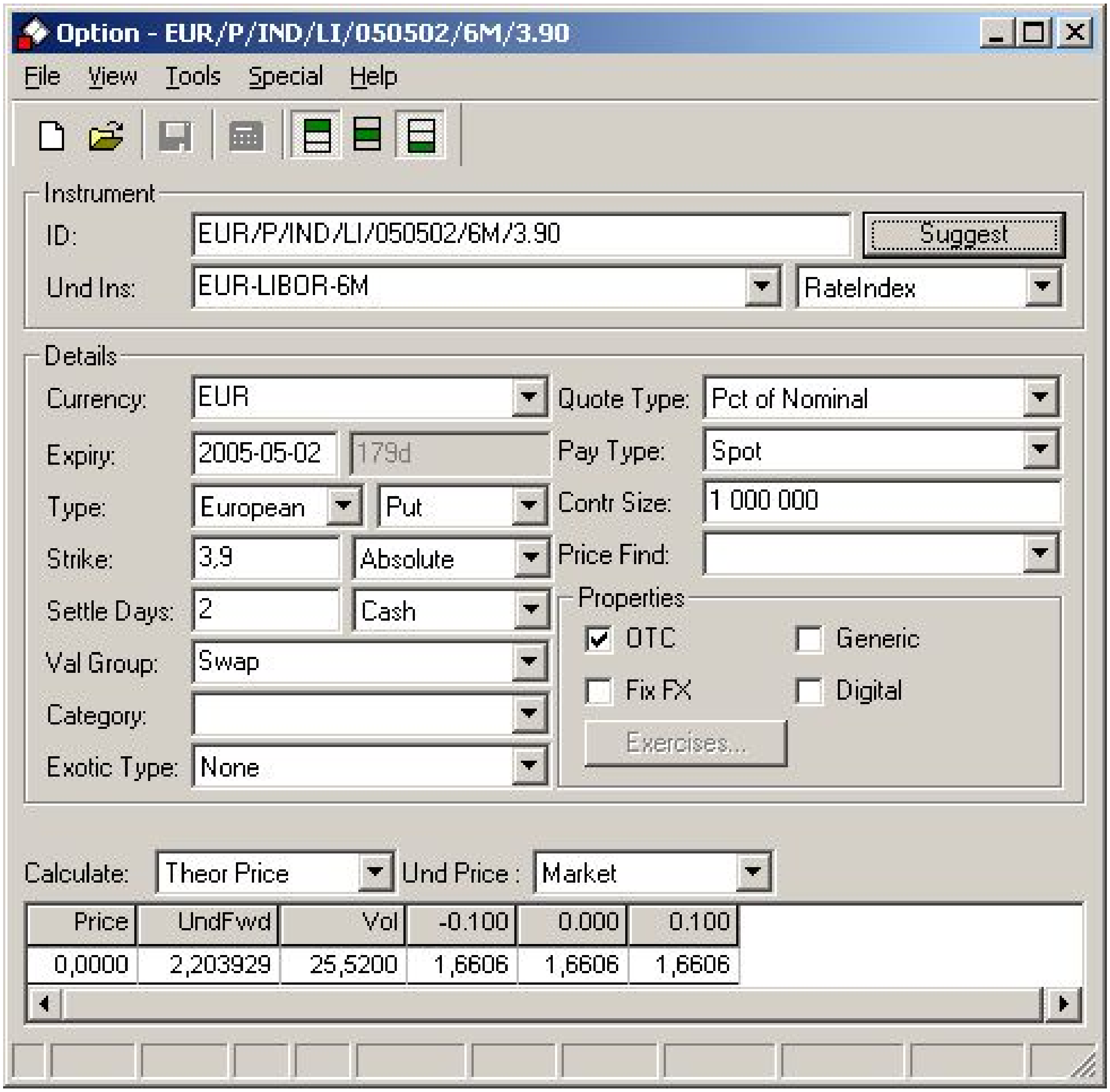
Volatility is always interpreted as the volatility of the implied forward rate.

Interest rate options: example

Consider the interest rate option with the following instrument data:

Quote type:	*Pct of Nominal*
Expiry:	*2005-05-02*
Option Type:	*Put*
Contract Size:	*1 000 000*
Underlying Instrument:	*EUR-LIBOR-6M*
Strike:	*3.9*
Settlement:	*Cash*
Exercise Type:	*European*
Pay Type:	*Spot*
Settlement days:	*2*

The theoretical price is €1.6606.



Currency options: theory

Currency options are valued in ATLAS using the Garman–Kolhagen formula. The value of a call option is

$$c = \exp(-r_d\tau)[\exp(r_d r_f)S_1 N(d) - XN(d - \sigma\sqrt{\tau})],$$

where

$$d = \frac{\ln(S_1/S_2) + (r_d r_f + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}$$

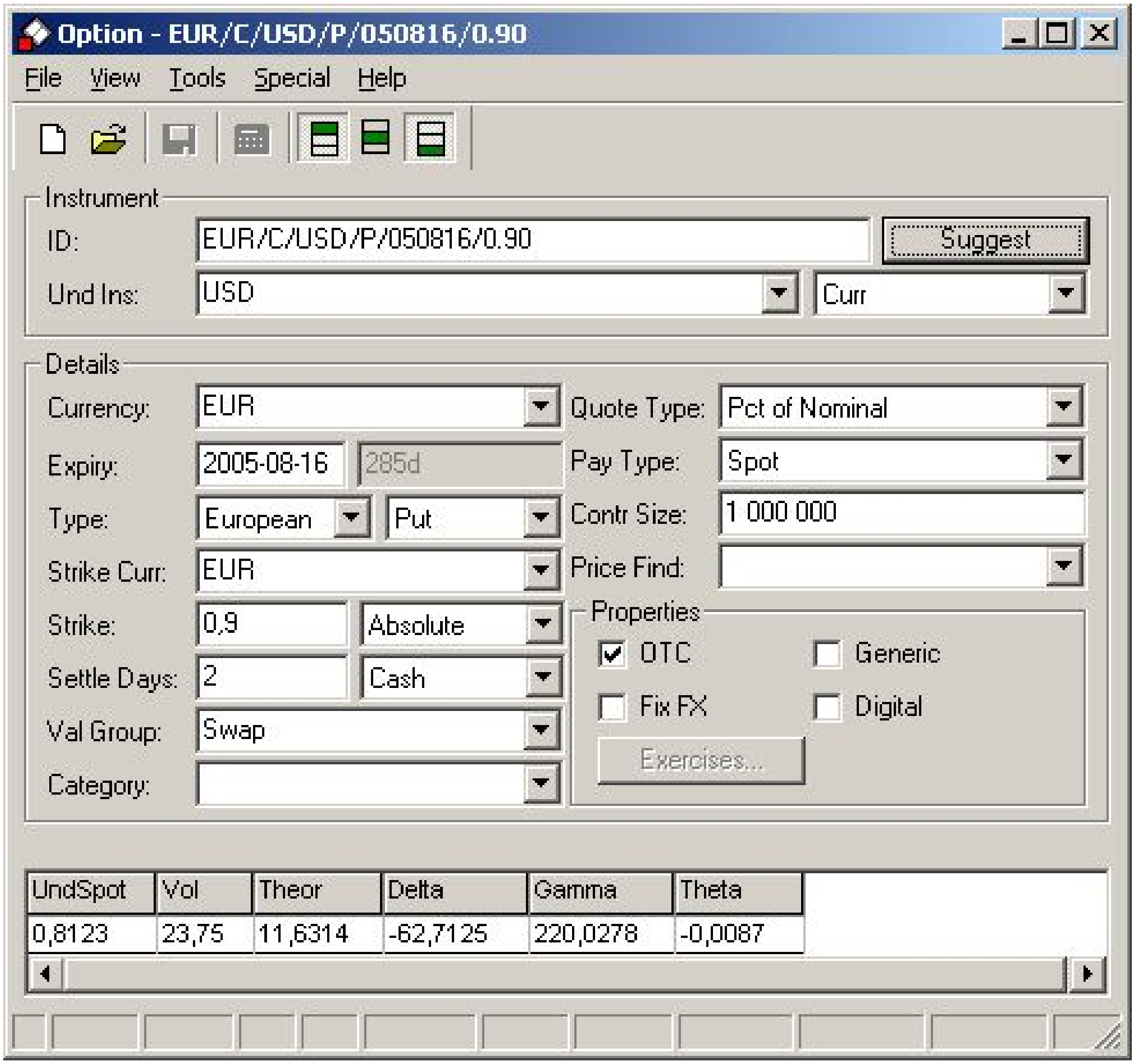
r_d	=	domestic rate
r_f	=	foreign rate
S_1	=	spot FX rate
S_2	=	strike FX rate

Currency options: example

We are going to calculate the theoretical price of a EUR/USD currency option with the following data:

Quote type:	*Pct of Nominal*
Currency:	*EUR*
Expiry:	*2005-08-16*
Option Type:	*Put*
Contract Size:	*1 000 000*
Underlying Instrument:	*Currency*
Strike:	*0.9*
Settlement:	*Cash*
Exercise Type:	*European*
Pay Type:	*Spot*
Settlement days:	*2*

The theoretical price is €11.6314.



Caps and floors: theory

Valuation of caps and floors is made by adding the values of the individual caplets or floorlets. Valuation of caplets and floorlets using Black–Scholes model is made using the following formulas: the price of a caplet is

$$c = P(t)[FN(d) - XN(d - \sigma\sqrt{\tau})]N_m T,$$

and the price of a floorlet is

$$p = P(t)[XN(\sigma\sqrt{\tau} - d) - FN(-d)]N_m T,$$

where

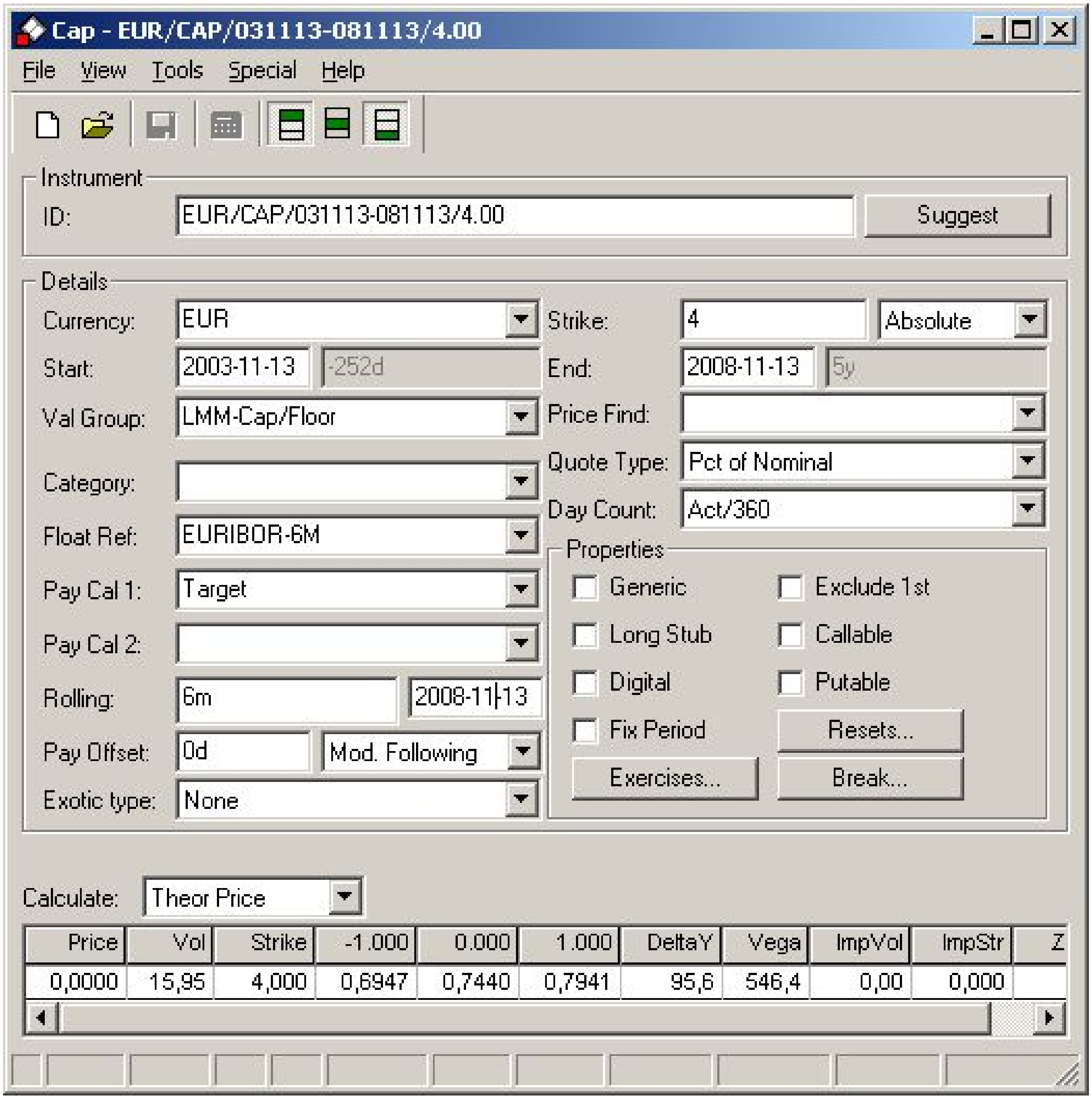
F	=	forward rate
X	=	strike rate
τ	=	time to maturity of option
σ	=	volatility
$P(t)$	=	discount factor until pay day
$N(d)$	=	normal distribution function
d	=	$\dfrac{\ln(F/X)}{\sigma\sqrt{\tau}} + \dfrac{\sigma}{2}\sqrt{\tau}$
N_m	=	nominal amount of caplet
T	=	time period of cashflow

Caps and floors: example

Consider the cap with the following instrument data:

Start day:	*2003-11-13*
End day:	*2008-11-13*
Float Ref:	*EURIBOR-6M*
Strike:	*4*
Day Count:	*Act/360*
Pay Method:	*Mod. following*
Rolling Period:	*6m*
Rolling Base Day:	*2008-11-13*

The theoretical price is €0.7440.



Exercises

① An underlying bond has the following instrument data:

Fixed Rate:	*6*
Start:	*2004-12-10*
End:	*2014-12-10*
Quote Type:	*Clean*
Day Count:	*30E/360*
YTM:	*None*
Rolling:	*1y*
Rolling Base Day:	*2014-12-10*
Pay Offset:	*0d*
Pay Method:	*Following*
Fix Period:	*Yes*
Notional:	*Yes*

Calculate the theoretical price of the futures with the following instrument data:

Expiry:	*2004-12-17*
Quote Type:	*Pct of Nominal*
Val Group:	*Swap*
Contract size:	*250 000*
Settle days:	*2*
Settlement:	*Physical delivery*

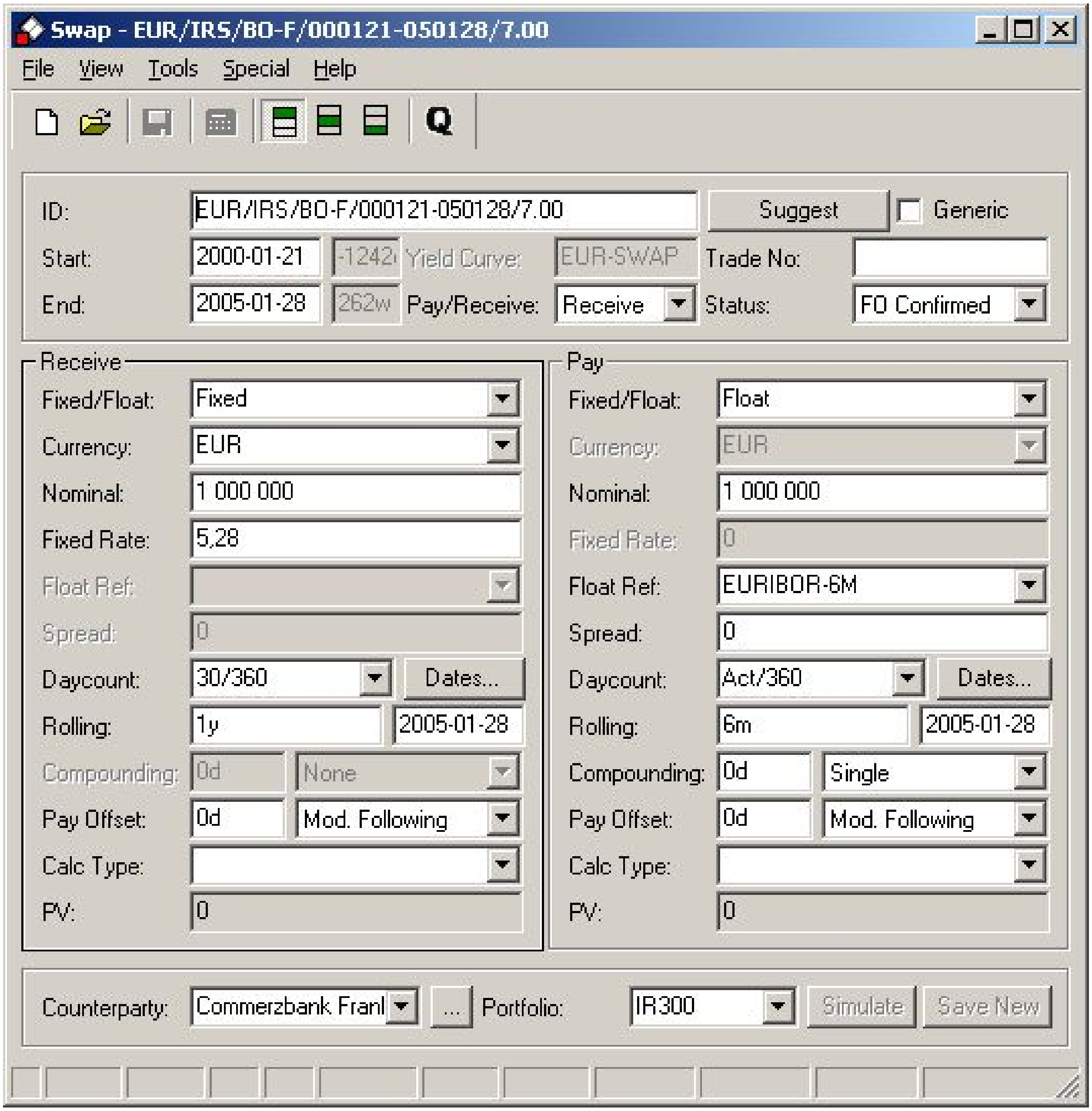
② An underlying futures on the rate index has the following instrument data:

Underlying Index:	*EURIBOR-3M*
Currency:	*EUR*
Expiry:	*2008-12-18*
Quote Type:	*100-rate*
Val Group:	*Swap*
Contract Size:	*1 000 000*
Settle Days:	*2*
Settlement:	*Cash*

Calculate the theoretical price of the option with the following instrument data:

Expiry:	*2005-08-16*
Quote Type:	*Pct of Nominal*
Pay Type:	*Spot*
Type:	*European Put*
Contract Size:	*1 000 000*
Strike	*100*
Strike Type	*Absolute*
Settle Days:	*2*
Settlement:	*Cash*
Val Group:	*Swap*
OTC	*Yes*

③ An underlying swap has the following instrument data:



Calculate the theoretical price of an option with the following instrument data:

Expiry:	*2005-12-16*
Quote Type:	*Pct of Nominal*
Pay Type:	*Spot*
Type:	*European Payer*
Contract Size:	*1 000 000*
Strike	*0*
Strike Type	*Absolute*
Settle Days:	*2*
Settlement:	*Cash*
Val Group:	*Swap*
OTC	*Yes*

④ Calculate the theoretical price of the floor with the following instrument data:

Currency:	*EUR*
Strike:	*4*
Strike type of the leg:	*Absolute*
Start:	*2003-12-05*
End:	*2006-12-05*
Val Group	*LMM-Cap/Floor*
Quote Type	*Pct of Nominal*
Float Ref:	*EURIBOR-6M*
Day Count:	*Act/360*
Rolling:	*6m*
Rolling Start Date	*2006-12-05*
Pay Offset:	*0d*
Pay Method:	*Mod. Following*
Exotic Type: *Ratchet*	
Spread:	*0.5*
Limit:	*3*
Exclude 1st	*Yes*