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Taking Analytics Further

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Dilemmas in the Multiple Curve Approach

October 2013

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We discuss some complications arising from using a multiple curve approach to price financial derivatives. We consider using the multiple curve approach in single-currency swaps, cross-currency swaps and forward rate agreements. Some common methods in multiple curve construction are discussed, along with some concerns practitioners should keep in mind. Sample multiple curve constructions are compared to the corresponding single-curve constructions using Numerix software.

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1 Introduction

During and after the financial crisis in 2007-2008, the financial derivatives market became more volatile. As a result, Libor is no longer risk-free, causing the modeling to shift from a traditional Libor curve construction (the so-called "single-curve approach") into a so-called "dual-curve approach."[1] The latter approach refers to decoupling the discounting curve (typically an OIS curve) from the index forward curve (e.g., Libor 3m). Regulatory pushes on CSA standardization accelerated the transition to the dual-curve approach, which impacted even basic financial derivatives pricing and curve construction. Challenges in using the OIS curve post-crisis for discounting, collateralization and CVA are pointed out in [15]. In [2], the authors outlined the mathematical foundation regarding the impact of collateral agreements on derivatives pricing. Soon, the dual-curve approach was replaced by the multi-curve approach even for standard vanilla linear instruments like IRS, CCS, etc. Such application of multi-curve created confusion in pricing vanilla linear instruments and in corresponding curve construction. The goal of this article is to unveil the dilemmas in a multi-curve world and to show a few common practices as well as their corresponding pros and cons.

2 Single-Currency Swap Dilemmas

2.1 Background

The market practice to strip the forward curve is through bootstrapping from liquid instruments. For a single currency, the most common practice using the dual-curve approach would be the following steps:

1. Strip discounting curve (e.g., OIS curve).
2. Construct benchmark index forward curve (e.g., 3m Libor curve in USD, 6m Euribor curve in EUR, etc.) given previously boostraped discounting curve.
3. Contruct other single-currency forward curves or basis curves related to different indices (e.g., different Libor tenors curves like 1m, 12m, etc. or BMA curve for muni).

Therefore, the new forward curve (2curve) from step 2 would be slightly different from the old forward curve (1curve). From a modeling point of view, the new approach means a probability measure change (from one curve to another). This is not new for the modeling world and for most cross-currency derivatives trades; people already dealt with the headaches of the "arbitrage mess" from cross-currency basis 10+ years ago. We can use the cross-currency world as an analog to illustrate the dual-curve world, as tabulated below.

[1]The author would like to thank Meng Lu, Jie Zhu, Liang Wu, Alexander Antonov, Jim Crotinger, Serguei Issakov, Alan Brace, Anna Barbashova, Matthieu Arneguy, Igor Zaoutine, David Davis, Satyam Kancharla, Dennis Sadak, James Jockle, David Eliezer, Meng Zhao, Jon Zucker and other colleagues in various groups including FE, R&D, BA, FV and various seminars/conferences for insightful and useful discussion. Computed results shown in the paper were using Numerix Analytics. Any remaining errors and unintentional deviations from best practice are entirely the author's responsibility. The views expressed are entirely the author's personal own views.

	Cross-Currency World		Dual-Curve World	
	Domestic	Foreign	Discounting	Forward
	USD	JPY	OIS	XIBOR
2 Currency/Curve	USD Yield Curve	JPY Basis Curve *	OIS Curve	Xibor Forward Curve (2Curve) **
1 Currency/Curve		JPY Yield Curve		Xibor Forward Curve (1curve)

Note:

*Adjusted yield curve (typically from Domestic Yield curve and FX forward and/or Cross-Currency Basis swap.

**Adjusted forward curve (typically from OIS curve and swap).

2.2 Dilemmas in OIS Curve

In practice, one needs to determine the liquid instruments and then bootstrap the curve associated with the assumed interpolation and extrapolation. The dilemmas here are about how to construct the OIS curve, how to construct other floating index forward curves and what the dependencies for those curves are. For different (currency) markets, the trading conventions and instruments dependencies are slightly different. The table below shows a brief summary of curve construction dependencies for the four major currencies (EUR, GBP, JPY, USD).

Currency	**OIS Curve**	**Benchmark Curve**	**Forward Curve**	**Basis Curve (Single Currency)**
EUR	Eonia $\longrightarrow$	Euribor6m $\longrightarrow$	Euribor1m Euribor3m Euribor12m	Euribor3mvs6m Euribor12mvs6m
GBP	Sonia $\longrightarrow$	GBP Libor6m $\longrightarrow$	Libor1m Libor3m Euribor12m	 Libor12m
JPY	Tonar $\longrightarrow$	JPY Libor6m $\longrightarrow$	Libor1m Libor3m	Libor3mvs6m
USD	FedFund* $\longrightarrow$	USD Libor3m $\longrightarrow$	MuniSwap	Libor1mvs3m Libor6mvs3m Libor6mvs3m Munivs3m CPvsLibor3m FedFundvsLibor3m** PrimevsLibor3m T-BillvsLibor3m

Note: The list in the table is not a complete list, and new instruments are constantly evolving.

*The OIS curve in USD is a bit more complex; we will discuss more in the following section.

**Traditionally (under one curve), FedFundvsLibor3m basis swap is used to strip FedFund curve given Libor3m curve; now (under two curve), FedFund curve could be OIS curve itself.

From the table above, the most tricky OIS curve construction would be USD OIS curve. We observe a few typical ways (or combinations of ways) to construct the OIS curve below.

2.3 Bootstrap from OIS

This is the most common method for non-USD currencies. In the United States, the instruments would be Fed Funds Overnight Rate and Fed Funds Overnight Index Swap (from 1w up to 10y), with interest compounded daily. Detailed pricing formulae and conventions can be found in [11] and [10]). From a liquidity point of view, it is usually only liquid for a tenor of less than 2y.

2.4 Bootstrap from OIS Futures

In the United States, 30-Day Fed Funds Futures (pricing formulae and convention details can be found in [12]) and 3-month OIS Futures (www.cmegroup.com) are commonly used, though there are only 36 standard 30-day FF Futures contracts and eight standard 3M OIS Futures contracts, based on www.cmegroup.com. Therefore, one can only bootstrap the OIS curve up to 3y or 2y.

2.5 Bootstrap from OIS Basis Swap

In the United States, the instruments are typically Fed Funds Basis Swaps ranging from 1y to 30y, which are averaged index basis swaps. Pricing formula and convention details can be found in [10].

In order to capture longer term OIS discounting, one could then bootstrap the short end of OIS curve from Fed Funds OIS (and/or Fed Funds Futures) and then bootstrap the long end through Fed Funds Basis Swap (typically combined with IRS to get Libor information). This has become the common practice in the USD market; however, the actual implementation methodology could vary in several ways. We describe these in Method1, Method2 and Method3 below.

2.5.1 Method1: Approximation Approach

By ignoring discrepancy business day count adjustment and compounding crude adjustment (see [6]), we obtain

$$OIS(t)_{\text{adj}} \approx \left(\left(1 + \frac{OIS(t)_{\text{approximate}}}{360}\right)^{90} - 1\right) \times 4, \tag{2.1}$$

where

$$OIS(t)_{\text{approximate}} \approx \left(1 + \frac{r_Q - \text{FFBS}(t)}{4}\right)^4 - 1.$$

$$r_Q = \left(\left(1 + \frac{\text{IRS}(t) \times 360/365}{2}\right)^{\frac{2}{4}} - 1\right) \times 4.$$

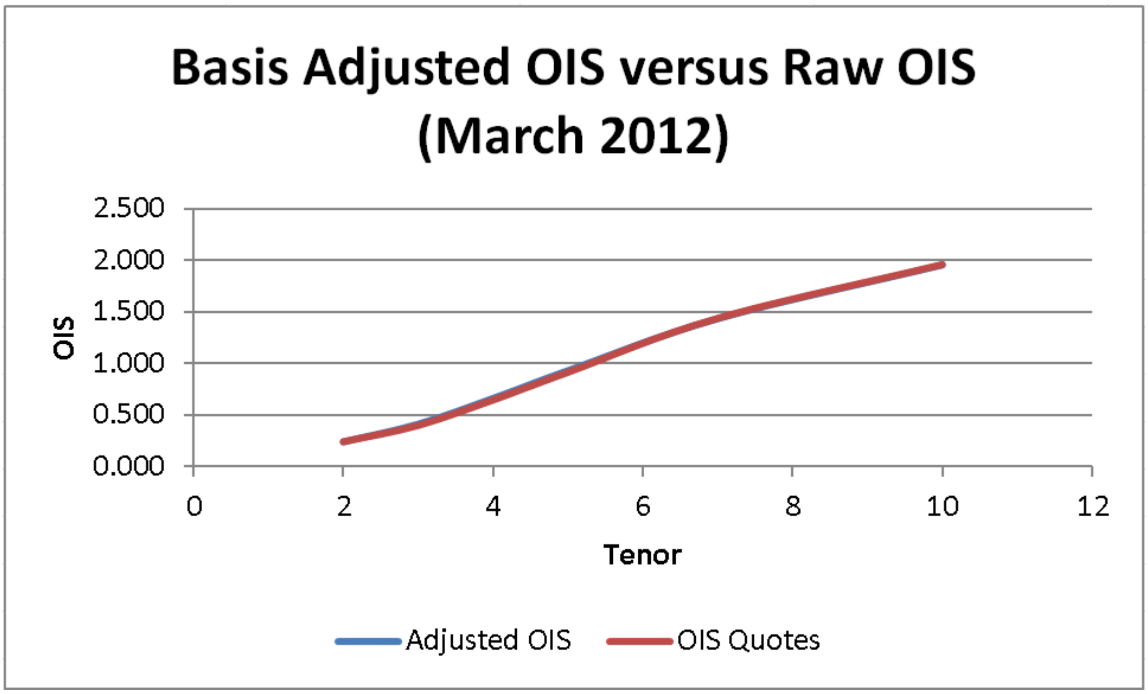
$\text{IRS}(t)$ is the market quote of swap rate.

$\text{FFBS}(t)$ is the market spread quote of Fed Funds Basis Swap.

Below we provide an example using real market quotes from March 2012 and implied adjusted spread and adjusted OIS from Method 1.

Tenor	2	3	4	5	7	10	12	15	20	25	30	40	50
IRS(t)	0.5760%	0.7510%	1.0030%	1.2730%	1.7685%	2.2655%	2.4890%	2.7065%	2.8770%	2.9590%	3.0060%	3.0140%	2.9780%
r_Q	0.5677%	0.7400%	0.9880%	1.2536%	1.7405%	2.2283%	2.4474%	2.6606%	2.8276%	2.9079%	2.9539%	2.9617%	2.9265%
FFBS(t)	0.3275%	0.3288%	0.3288%	0.3275%	0.3163%	0.2925%	0.2788%	0.2644%	0.2475%	0.2325%	0.2175%	0.2175%	0.2175%
OIS_approximate	0.2404%	0.4119%	0.6609%	0.9293%	1.4318%	1.9499%	2.1863%	2.4178%	2.6052%	2.7024%	2.7646%	2.7726%	2.7366%
OIS_adj	0.2405%	0.4121%	0.6614%	0.9304%	1.4343%	1.9546%	2.1922%	2.4250%	2.6136%	2.7114%	2.7741%	2.7821%	2.7461%
Adjusted Sprd	-33.55054	-33.89279	-34.15883	-34.26132	-33.41513	-31.09363	-29.67644	-28.14661	-26.34270	-24.75957	-23.19079	-23.19079	-23.19079
Adjusted OIS	0.24049	0.41207	0.66141	0.93039	1.43435	1.95456	2.19224	2.42503	2.61357	2.71140	2.77409	2.78209	2.74609

We can see from this table and from the graph below that the implied OIS rate has deterministic approximate spread to IRS and such an OIS has embedded market information from FFBS. Moreoever, the OIS is reasonably close to market OIS for long tenors, so using using this adjusted OIS for longer (say, 30y) tenors is justified and computationally efficient.



Note: The approximation method above makes the following assumptions that practitioners must recognize.

- Ignore conventions like business day count, calendar, roll convention, basis, spot lag, schedule mismatch, etc.
- Compounding approximation assumes “flat curve with rate equal to the difference between Libor and Fed Funds Basis and ignore weekends and holidays” ([6]).
- The error between adjusted OIS and OIS market quotes are combination of approximation errors and liquidity.
- Fastest method.

2.5.2 Method2: Brute Force Approach

The brute force approach jointly solves both Fed Fund Basis Swap and IRS to par. We know analytics for both Fed Fund Basis Swap and IRS, indicated below.

Fed Funds Basis Swap	
Pay LiborLeg -3mLibor	Receive FedFundLeg +WeightedAvgFedFund + BasisSprd
IRS	
Receive LiborLeg +3mLibor	Pay FixedLeg -FixedSwapRate

Note:

- This is the most accurate way because it solves discount factors for both OIS discounting curve and Libor3m forward curve jointly.
- Schedules among legs between Fed Funds Basis Swap and IRS may not align, which burdens the solver and is very sensitive to choice of interpolation method.
- Computationally expensive (especially the weighted average on daily forward Fed Funds effective rate feature).

2.5.3 Method3: Synthetic Approach

The synthetic approach is to bootstrap the OIS curve by repricing Fed Fund Basis Swap to par given IRS. The idea is represented in the flow chart below.

Fed Fund Basis Swap	
Pay LiborLeg -3mLibor	Receive FedFundLeg +WeightedAvgFedFund + BasisSprd
IRS	
Receive LiborLeg +3mLibor	Pay FixedLeg -FixedSwapRate

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Synthetic Fed Funds Swap	
Receive FedFundLeg +WeightedAvgFF + BasisSprd	Pay FixedLeg -FixedSwapRate

Through the synthetic construction in the graph above, we can have a single instrument (Fixed versus Floating (weighted average FF+BasisSprd)) per tenor, which would fit into most standard curve stripping frameworks as a regular bootstrapping procedure.

Note:

- This is faster than Method2.
- This is more accurate than Method1.

- Schedule alignment is still an issue.
- Tenors have to be matched so as to create a synthetic FF average swap.

2.6 Related Dilemmas

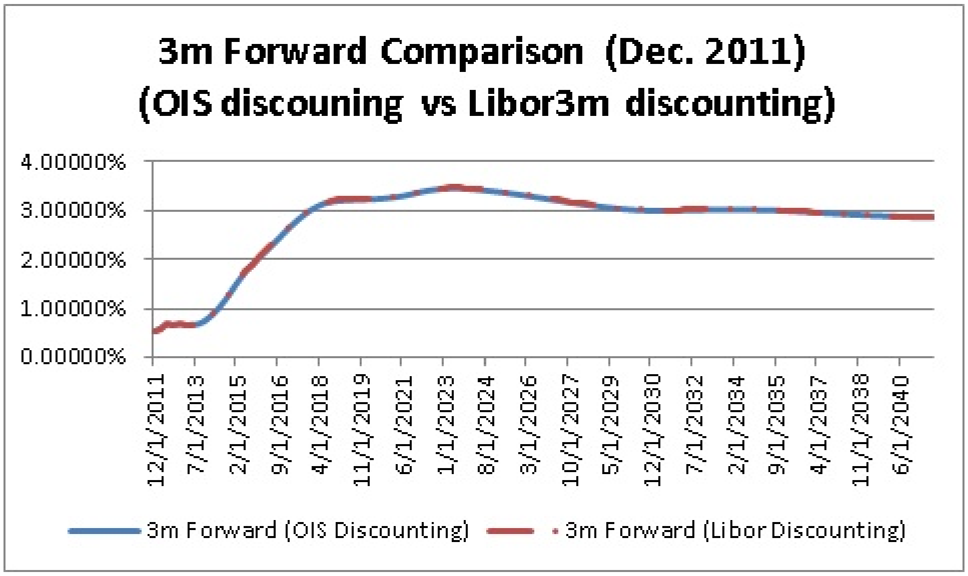
We consider now a few dilemmas that arise when one uses these methods.

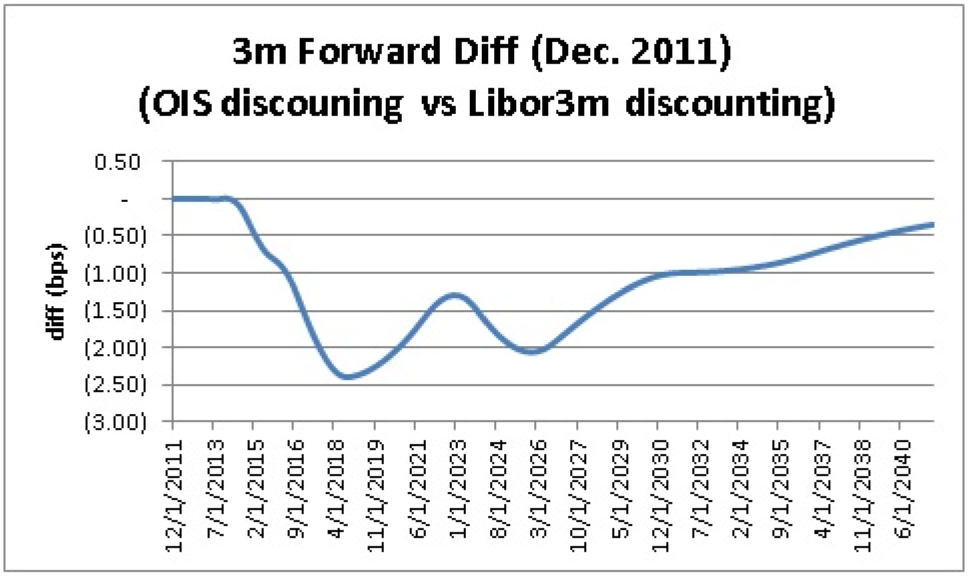
2.6.1 Wrong Instrument Choice?

This is related to the availability and choice of liquid instruments. Let us assume that we have an OIS curve (obtained from an above method) and we need to find our forward curve (e.g., USD Libor 3m curve). In the single-curve world, we typically bootstrap from combinations of cash (1w up to 1y), future or FRA (typically most liquid up to 3y or 5y) and swap (1y up to 50y). However, in the dual-curve world, each Libor has its own (different) risk, which means we cannot use, say, spot Libor cash rate like 1w, 2w, 3w, 1m, 2m, 4m, 5m, 6m, etc. to strip the 3m Libor curve. Therefore, one possible choice could be using FRA for short term and using swap for long term. Note that FRA itself also has a few dilemmas described in *Forward Rate Agreement (FRA) Dilemmas*, below. For futures, it is even trickier on the convexity part (detailed discussion can be found in [9]).

2.6.2 Libor Is Changed or Unchanged?

In the single-currency world, the standard curve construction logic would be to make discounting (e.g., OIS) agreed upon and fixed, then imply Libor forward. This implies that Libor forward depends on the discounting choice. However, intuitively Libor is a fixing floating index that should not depend on discounting curve choice. Different players may have different funding (cost) curves, and CSA may also require discounting (e.g., repo rate on the cheapest bond as collateral) other than OIS discounting which really refers to cash collateral within that single-currency market. Mathematically, it depends on discounting in our typical risk-neutral environment. This is because of the probability measure changes when discounting is changed. Prior to the crisis, the "standard" discounting curve was the "same" as (or almost equivalent to) the Libor (3m) forward curve in USD; post-crisis, the "standard" discounting has shifted into OIS discounting. OIS discounting and Libor discounting would produce nearly the same price for a 3m Libor forward, as the graphs below show.





Therefore, for the trading convention of a prevailing reference index like USD Libor 3m, termsheets usually define those in a concrete way by referring to something like:

> ***Reference rate****: for any reference date in the applicable interest period, the 3-month London Interbank Offered Rate (LIBOR) for deposits in U.S. dollars ("3-month USD LIBOR") as it appears on Reuters screen LIBOR01 page (or any successor or replacement service or page thereof) at 11:00 a.m., London time on such day, subject to adjustment as described on page….*

As such, the reference rate is fixed in the major market data vendors' weighted average of dealers'/brokers' quotes pool. Such market quotes would need canonical assumptions on discounting, which means that pre-crisis Libor fixings depend on Libor discounting itself (single-curve approach) since most dealers/brokers would have assumed the Libor curve was risk-free, even though it reflected only about an AA rating. Post-crisis Libor fixings shifted to OIS discounting (multi-curve approach), though dealers/brokers may not give the quotes in consistent ways. As CSA regulation tends to standardize collateral agreement ("ISDA Credit Support Annex (CSA) standardization in Q2 2012") as well as "London Clearing House uses OIS discounting for clearing swaps" (see [5]), the swap quotes and Libor fixings would be more reliable than during crisis period when Libor was criticized due to various reasons including collateral/funding discounting and liquidity.

2.6.3 Collateral in Different Currency

Another dilemma is about collateral currency. This is actually fairly common in the real world. Suppose I were a US-based hedge fund or bank that is trading (collateralized) vanilla EUR IRS with a EUR bank. Assume further that most of my business is in the US (also in USD) and that I own a lot of USD treasury bonds (long term) and cash in USD. Therefore, it would be very natural for me to put collateral in a USD cash or a USD cash equivalent treasury bond, rather than put collateral in EUR even if the trade is an EUR single-currency trade (IRS). In this situation, we are faced with the dilemma of how to discount the value of the trade price and the risks. In the next section, we will touch on it again and dive into more details, including "cheapest to deliver" (see [1]).

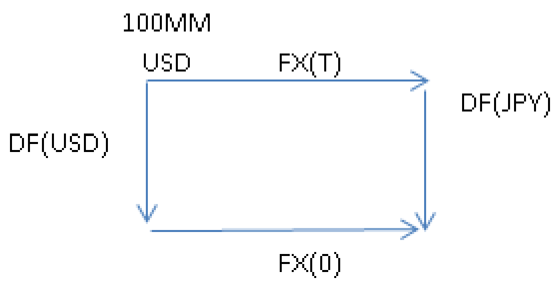
3 Cross-Currency Swap Dilemmas

3.1 Background

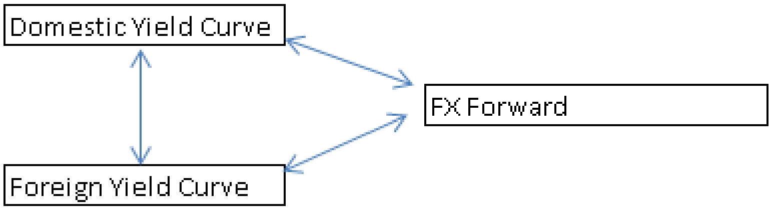
There are two popular types of cross-currency swaps.

- Floating versus Floating (commonly used for major currency pairs like USD/EUR, USD/JPY, EUR/JPY)
- Fixed versus Floating (commonly used for minor currency pairs like USD/TWD).

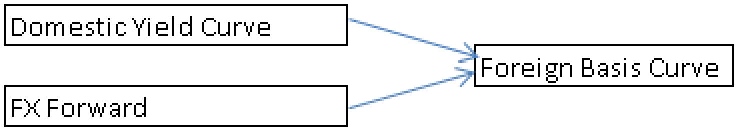
In this section, we focus mostly on Floating versus Floating cross-currency swaps (sometimes called Cross-Currency Basis Swaps, or CCBS) because we can see that the most liquid OIS markets and major multi-curve evolutions would start in those major currency markets (e.g., USD, EUR, GBP, JPY). Using USD versus JPY as a simple example to illustrate arbitrage-free relationships in interest rate and FX markets (sometimes called "interest rate parity" in [13]), let us suppose a JPY investor has a future cashflow of 100 million in USD at future time T. She could either discount it using USD discounting back to today T(0) and convert it into JPY using the spot FX rate (FX(0)), or she could convert the future cashflow at T through FX rate (FX(T)) and then discount back to today T(0) using JPY discounting. If there were no arbitrage, these two methods would yield the same amount. Such parity can be visualized in the following graph.



Therefore, in theory, we could see a perfect triangle relationship among domestic yield curve, foreign yield curve and FX forward. As the graph below illustrates, one can solve for one out of three given the other two.



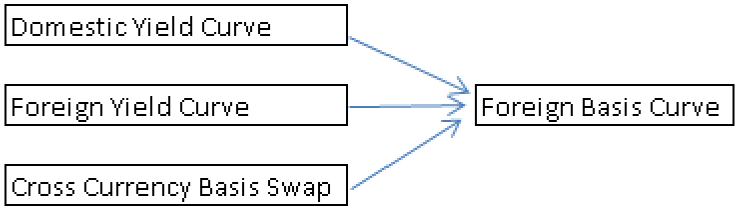
However, both interest rate (domestic and foreign) and FX Forward are independent actively traded markets. Therefore, the triangle won't be "perfect" even for a normal market due to different liquidity among those three markets. If one needs to value a cross-currency trade under domestic measure, then a typical way would be to bootstrap the "implied" Foreign Yield Curve (let's call it Foreign Basis Curve) from a set of FX forwards given a Domestic Yield Curve (see graph below).



Note that such a Foreign Basis Curve typically won't be same as the Foreign Local Yield Curve (e.g., bootstrapped from cash, FRA/future, swap), and it reflects that the foreign investor's (domestic currency's) funding interest rate (discounting) at local (foreign currency) market would be different from the local investor's. Such a basis curve would satisfy:

- Repricing each node of FxFwd back to par.

So far, everything is still consistent. Such a relationship implies that the Cross-Currency Basis is zero (which is what "perfect" means). However, if we look into Cross-Currency Basis Swap (CCBS) quotes, they are not zero! This implies that the CCBS actually breaks the above interest rate parity due to many reasons, including liquidity premium, currency strength, currency country's credit profile differentials, etc. Prior to the crisis, such bases were very small, and many practitioners simply ignored them . However, after the crisis such bases became much larger (50 bps-100 bps post-crisis, compared with to 1-5 bps pre-crisis). Therefore, such a basis can no longer be ignored in derivatives pricing and risk management. As with FX Forward, one can also bootstrap "implied" foreign basis curve from a set of cross-currency basis swaps given both domestic yield curve and foreign yield curve (which makes Libor forward consistent with local Libor forward). A clear formula and relationship for such parity is outlined in [13], wherein the authors also provide a detailed explanation of cross-currency basis swap (CCBS) mechanisms. Graphical illustration is shown below.

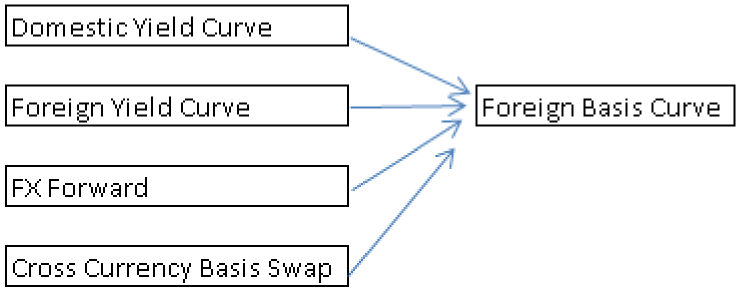


Such a foreign basis curve would satisfy:

- Repricing each node of CCBS back to par.
- Matching local Libor forward.

In short, the logic here is to solve for the discounting curve while keeping the prevailing index forward unchanged. Recall that in the previous section, we discussed how dilemmas for Libor forward projection would vary if different discounting (probability measures) were used. Therefore, the Libor definition is changed slightly due to different discounting. From a practical point of view, this dilemma can often be ignored due to the tiny effect on forward change which means one can still assume that basis swap's Libor forward in foreign currency (JPY) is the same as Libor forward in the JPY domestic market. As in the single-currency world, even if we change from Libor curve (single curve) to a very different discounting curve (e.g., OIS curve) when pricing a vanilla IRS, the Libor forwards in the IRS would be almost the same (max about 1-3 bps for 30y).

Furthermore, FX Forward may only last up to five years, depending on the market, while CCBS would last up to 30 years. Therefore, from a liquidity point of view, one may strip the foreign basis curve from a combination of FX Forward (for short term) and CCBS (for long term).



We now turn our attention from the traditional dilemma in the cross-currency world to the dillemas arising from a multi-curve environment.

3.2 OIS Curve Again

Recall that under single currency, the dual-curve logic is to bootstrap the OIS curve, then solve for the index forward curve while keeping OIS curve unchanged. In the cross-currency world, the logic is to solve for the implied foreign discounting curve while keeping the foreign Libor forward unchanged. Then in a Cross-Currency Basis Swap we involve four curves:

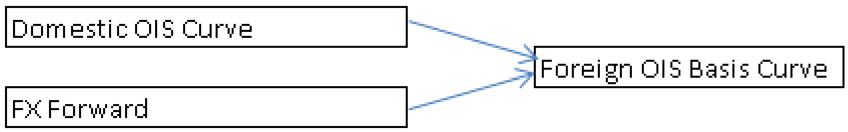
1. Domestic Discount Curve (e.g., FF OIS curve).

2. Domestic Forward Curve (e.g., USD 3M Libor curve).
3. Foreign Discount Curve.
4. Foreign Forward Curve (e.g., JPY 3m Libor).

The goal here is still to solve for a "cross-currency implied" foreign basis discounting curve (or say Foreign OIS Basis Curve) where discounting and forward projection are decoupled. A few common practices are described below

3.2.1 Bootstrap from Fx Fwd Given Domestic OIS Curve

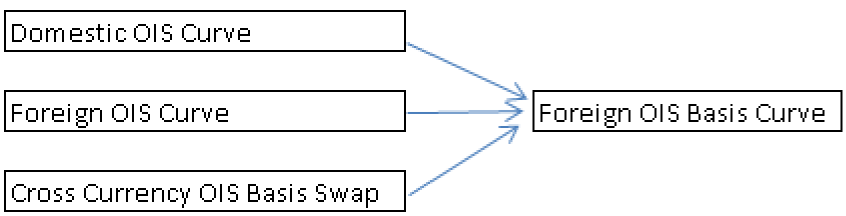
The process is same as the pre-crisis single-curve approach to solve the triangle relationship. The graphic flow is shown below.



One big assumption here is that the FX Forward market moves to OIS discounting for both domestic and foreign.

3.2.2 Bootstrap from Cross-Currency OIS Basis Swap

This practice reflects some new instrument innovation. One example could be USD Fed Funds versus JPY Tonar cross-currency swap (quarterly pay frequency with Act/360 day count). This would be very similar to single-curve CCBS. The graph below illustrates the dependency.



3.2.3 Bootstrap from Cross-Currency (Libor) Basis Swap

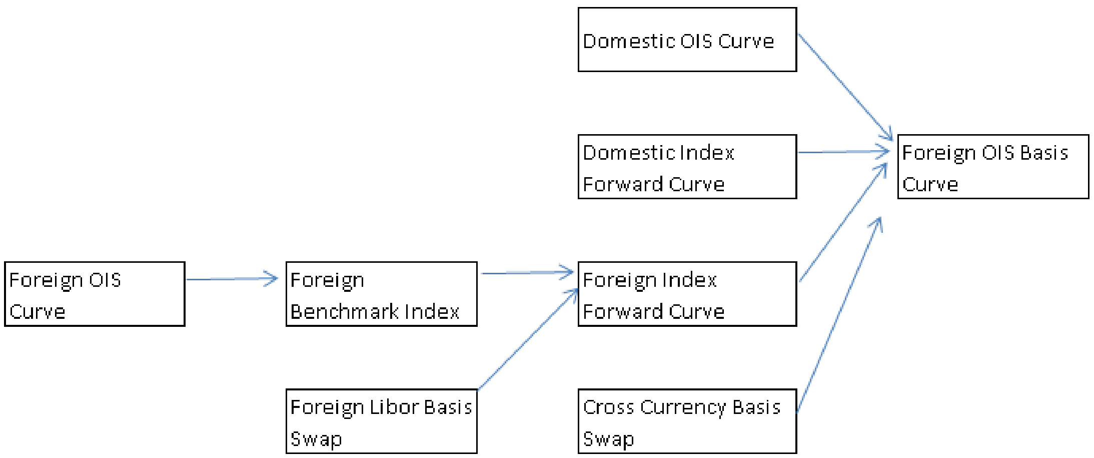
This practice is more complex depending on market conventions. Common practice would be the following steps (using USD 3m Libor versus JPY 3m Libor as an example):

1. Construct Domestic OIS Discounting Curve (e.g., FF OIS Curve from method mentioned in*Bootstrap from OIS Basis Swap*).
2. Construct Domestic Index Forward Curve (e.g., USD 3m Libor curve).
3. Construct Foreign OIS Discounting Curve (e.g., JPY OIS Curve from Tonar).
4. Construct Foreign Benchmark Index Forward Curve (e.g., JPY 6m Libor, which is the pre-vailing benchmark index curve).

5. Construct Foreign Index Forward Curve (e.g., JPY 3m Libor from Libor Basis Swap (3mvs6m) as shown in *Single-Currency Swap Dilemma*).

6. Solve for Implied Foreign Basis Curve (or, say, Foreign OIS Basis Curve) from CCBS given 1, 2 and 5 above.

Graphic illustration is shown below:



3.3 Collateral Currency

Revisiting the earlier example of EUR IRS with collateral in a different currency (USD) in *Collateral in Different Currency*, people would have two choices:

- Value IRS in EUR as usual, and convert PV (or P/L) into USD.
- Value IRS in USD (in which case, the trade becomes a cross-currency trade with all cash-flows in a foreign currency (EUR), which must be converted into USD at each payment dates).

This is very similar to interest rate parity. The first choice is more like real world trading activity: value the trade, convert values to collateral currency, then put the corresponding collaterals. The latter choice is more like calculating pricing and risks, and would therefore involve an “implied” OIS basis curve from different methods mentioned in *OIS Curve Again* above. The existence of such a basis curve may break the interest rate parity and may lead to discrepancies between actual collateral value and booked collateral value. This also implies that EUR IRS collateralized in USD would be different from EUR IRS collateralized in EUR or JPY. In real trades, there is a significant possibility of different collateralized currency, but also a possibility of a discrepancy in the funding or collateral curve (which may or may not be OIS curves, because collateral may not always be cash). I agree that this would be a big mess in derivatives valuations and risks. Risk management needs to have a consistent framework (e.g., under domestic risk-neutral measures). One way could be to force OIS as collateral discounting for each currency, and if the collaterals are bond or other cash equivalent one would need to revalue those theoretical values under the same consistent OIS discounting as well. Therefore, market quotes from market data vendors would need to be standardized with canonical discounting as well as currency. This would still be a mess; we would have EUR IRS collateralized in EUR, USD, JPY, GBP, etc. Moreover, different countries may argue for adding their country's currency to the standard market quotes.

3.4 Cheapest-to-Deliver (CTD)

This was driven by collateral agreement and CSA standardization. In the CSA, two counterparties may have rights to choose collateral on the fly among a few predefined currencies ([1]). This would create a huge headache in modeling, hedging and risk management., as explained in [14], shown below.

> *This becomes hugely complex when a trade is backed by a CSA that allows the counterparties to post collateral in a variety of currencies and assets. Most dealers agree the discount rate should in theory be based on the cheapest-to-deliver collateral. ...Market practice is altogether different, with even the major dealers taking a variety of approaches to pricing trades based on multi-currency CSAs.*
>
> *"Theoretically, it is not difficult to put together a model – it would be an extension of a stochastic basis model where you have more than one basis," says Vladimir Piterbarg, global head of quantitative research at Barclays Capital in London. "The huge question is whether you are able to execute the hedging strategy required."*

In my view, such disorder in CTD modelling and hedging would have much bigger model risks. CSA standardization and simplification become critical while the market is moving towards a consistently multi-curve approach.

4 Forward Rate Agreement (FRA) Dilemma

4.1 Background

This section is a complementary section on Forward Rate Agreements (FRAs) in a multi-curve world. An FRA is usually paid up front rather than in arrears (at the end of each Libor period). Therefore, the trading convention would assume that the Libor forward is calculated in a "fair" way by computing the discount factor from the fixing date to the payment date:

$$PV = P(t_f)\frac{(L(t_f,t_s,t_e)-K)\Delta(t_s,t_e)}{[1+L(t_f,t_s,t_e)\Delta(t_s,t_e)]}. \tag{4.1}$$

In the single-curve world, Libor is assumed to be risk-free (aka discounting curve = forward curve):

$$L(t_f,t_s,t_e)=\left(\frac{P(t_f,t_s)}{P(t_f,t_e)}-1\right)\frac{1}{\Delta(t_s,t_e)}. \tag{4.2}$$

Therefore, due to the martingale property (details can be found in [4] and [7]) of the traded asset $P(t_f,t)$, an FRA's pricing formula in the deterministic world could be simplified to

$$PV = P(t_e)(L(t_f,t_s,t_e)-K)\Delta(t_s,t_e). \tag{4.3}$$

The corresponding martingale property under the T-forward measure $\mathbb{Q}_T$ is

$$\mathbb{E}^{\mathbb{Q}_T}\left[F(t_f,t_s,t_e)\right] = L(0,t_s,t_e), \tag{4.4}$$

and the corresponding martingale property under risk-neutral measure $\mathbb{Q}$ is (see [7] for details)

$$\frac{\mathbb{E}^{\mathbb{Q}}\left[D(0,t_f)\,P(t_s,t_e)\,F(t_f,t_s,t_e)\right]}{\mathbb{E}^{\mathbb{Q}}\left[D(0,t_f)\,P(t_s,t_e)\right]} = L(0,t_s,t_e), \tag{4.5}$$

where

$\beta(t)$ is Money Market Account, $\beta(t) = e^{\int_0^t r(u)\,du}$, and $\beta(0) = 1$.

$D(t,T)$ is Stochastic Discount Factor, $D(t,T) = e^{-\int_t^T r(u)\,du} = \frac{\beta(t)}{\beta(T)}$, and $D(t,t) = 1$.

$P(t,T)$ is Zero Coupon Bond Price, $P(t,T) = \mathbb{E}^{\mathbb{Q}}\left[\beta(t)\,\frac{1}{\beta(T)}\right] = \mathbb{E}\left[D(t,T)\right]$.

In the dual-curve world, however, Libor becomes risky, which means that this approach would use the wrong discounting curve (Eq. (4.2) does not hold). As a result, the martingale property in Eq. (4.5) fails. The common practice following trading convention (ignoring convexity effect?) would

- Assume that $P(t_f)$ in Eq. (4.1) would be the collateral discounting (from OIS discounting curve),
- Assume that $L(t_f,t_s,t_e)$ in Eq. (4.1) would be risky. In this case, we would have

$$L(t_f,t_s,t_e) = \left(\frac{P_F(t_f,t_s)}{P_F(t_f,t_e)} - 1\right)\frac{1}{\Delta(t_s,t_e)},$$

  or, as written in [8],

$$P_F(t_s,t_e) = \frac{1}{1 + L(t_f,t_s,t_e)\Delta(t_s,t_e)}. \tag{4.6}$$

In other words, in normal practice the discounting part of Libor from t_e to t_s is also assumed to be as risky as Libor forward projection. The rationale here is that trading parties can only put daily collateral until t_s. Therefore, the period between t_s and t_e is only for theoretical Libor tenor, and there won't be any collateral transactions. Then the discounting determinant would leave to the market common trading convention like Eq. (4.6). This simple approach would assume that Libor forward is risky when it is settled, though such settlement amount would be still discounted at the risk-free rate (e.g., continuously full collateralization with zero minimum transfer amount).

4.2 Comparison

The figure below compares FRA under 2Curve, 1Curve(Libor) and 1Curve(OIS) as of December 2011. We can see that the 2Curve result is very close to the 1Curve(Libor) result, though they still differ on the same 6mx12 FRA instrument.

ID	FRA.P	FRA.P2	FRA.P3
OBJECT	ANALYTIC	ANALYTIC	ANALYTIC
TYPE	FRA	FRA	FRA
INDEX CURVE	SWAPCURVE.2CURVE		SWAPCURVE.1CURVE
NOWDATE	12/12/2011	12/12/2011	12/12/2011
FRA	FRA.6MX12M	FRA.6MX12M	FRA.6MX12M
DOMESTIC YIELD CURVE	EONIACURVE	EONIACURVE	SWAPCURVE.1CURVE
Updated	2334 @ 04:05:55 PM	2246 @ 04:05:55 PM	2339 @ 04:05:55 PM
Timer	0	0	0
ID	FRA.P	FRA.P2	FRA.P3
PRICE	(79,127.18)	(129,460.10)	(78,645.68)
RATE	1.428494%	0.441662%	1.428494%
SWAP BPV	0.506032047	0.506032047	0.500447935
FIX DATE	6/12/2012	6/12/2012	6/12/2012
STARTDATE	6/12/2012	6/12/2012	6/12/2012
END DATE	12/12/2012	12/12/2012	12/12/2012
NOTIONAL	10000000	10000000	10000000

The figure belows compares analytic pricer, script/kernel and manual calculation since we can verify that by defining payoff in script explicitly or by manual calculation.

ID	FRA.P		
OBJECT	ANALYTIC		
TYPE	FRA		
INDEX CURVE	SWAPCURVE		
NOWDATE	12/12/2011		
FRA	FRA.6MX12M		
DOMESTIC YIELD CURVE	EONIACURVE		
Updated	2241 @ 03:37:30 PM		
Timer	0		
ID	FRA.P	Manual Calc	Script
PRICE	(79,127.18)	(79,127.18)	(79,127.18)
RATE	1.428494%	1.428494%	1.428494%

A sample payoff script that reflect the above logic (following Eq. (4.3)) is shown below.


```
PRODUCTS
    DISCOUNTING Coupon, Coupon2, FRA, FRA2
    NONDISCOUNTING DCF, DF, L
    PAYMENTLOG CouponLog, ForwardLog
END PRODUCTS

PAYOFFSCRIPT
    IF ISACTIVE(FundDates) THEN


        //FRA Manual Calc
        DCF = FundDatesDCF
        DF = ZCB
        L =(1 / DF - 1) / DCF
        Coupon2 =(L - K) * FundDatesDCF * Notional
        FRA2 = 1 /(1 + L * FundDatesDCF) * Coupon2
    END IF
END PAYOFFSCRIPT
```

5 Conclusion

This article is obviously not a complete list of dilemmas only addresses only a few aspects of some simple vanilla linear products. Nevertheless, it suggests some best practices for OIS discounting curves from both single-currency world and cross-currency world. The problems presented by the multi-curve approach require more investigation, more proposals for best practices and perhaps regulatory standardization. The author would expect that more dilemmas and puzzles would arise as the market evolves. This article should help you with curve-stripping choices, decisions on best practices and rethinking the application of the multi-curve approach. Any more comments, feedbacks and discussions in any format are welcome.

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