Stochastic Calculus for Finance
Michaelmas Term 1998

Outline

The aim of these lectures is to introduce some of the techniques from stochastic analysis that are employed in mathematical finance. This is a huge area, so we can certainly do no more than scratch the surface, but we will see that mathematics has been of fundamental importance in the revolution that has taken place in the financial markets over the last twenty-five years.

Although we use financial examples for motivation, Brownian motion and stochastic calculus play an important rôle in almost every area of modern mathematics.

As time permits we will cover some or all of the following topics:

1. **Basic examples of financial derivatives:** Examples of financial instruments, a first example of 'arbitrage pricing'.
2. **Discrete time models I:** Single period models, pricing a European option, characterising no arbitrage, risk neutral probabilities.
3. **Discrete time models II:** Multiperiod binary models, discrete parameter martingales, risk-neutral pricing, Cox-Ross-Rubinstein.
4. **Brownian motion:** Definition of Brownian motion (motivated via a rescaling of simple random walk), Lévy's construction.
5. **The reflection principle and hitting times:** reflection principle, hitting times, scaling properties.
6. **Martingales in continuous time:** filtrations, adapted processes, Optional Sampling Theorem.
7. **Stochastic integration and Itô's formula:** variation and quadratic variation, quadratic variation of Brownian motion, outline of construction of the Itô stochastic integral and the Itô isometry, the chain rule and integration by parts for stochastic calculus. The Martingale Representation Theorem, Lévy's characterisation of Brownian motion, Girsanov's Theorem.
8. **The Black-Scholes model:** self-financing strategies, equivalent martingale measures, the risk-neutral pricing formula.
9. **Pricing and hedging European options:** Examples of European options. Explicit pricing formula. Evaluation of price and hedging strategies for European calls and puts.
10. **Valuation of some exotic options.** Digital options, barrier options etc.

There are no formal prerequisites for the course, but a knowledge of (or willingness to learn) some basic probability would be a distinct advantage. (It will be assumed that the reader is familiar with the notions of probability distribution, mean and variance and conditional expectation. The central limit theorem will be quoted without proof and we will talk about stochastic processes without dwelling on what they are.)

References

We shall not be following any particular book. The following are useful references for different aspects of the course. If you need to revise some basic probability theory then try Grimmett & Welsh or for stochastic processes Grimmett & Stirzaker. Of the books below, those by Björk and Lamberton & Lapeyre are probably closest to the course. *There will be a complete set of typed notes.*

M Baxter & A Rennie. *Financial Calculus.* CUP 1997.

T Björk. *Arbitrage theory in continuous time.* OUP 1998.

K L Chung & R J Williams. *Introduction to Stochastic Integration.* Second edition. Birkhäuser 1990.

D Duffie. *Dynamic Asset Pricing Theory.* Princeton 1996.

G Grimmett & D Welsh. *Probability, an introduction.* OUP 1985.

G Grimmett & D Stirzaker. *Probability and Random processes.* Oxford 1982.

John Hull. *Options, Futures and Other Derivative Securities.* Second edition, Prentice Hall 1993.

I Karatzas & S E Shreve. *Brownian motion and stochastic calculus.* Springer 1991.

F B Knight. *Essentials of Brownian motion and diffusion.* American Mathematical Society 1981.

L Lamberton & B Lapeyre. *Stochastic calculus applied to finance.* Chapman & Hall 1996.

M Musiela & M Rutkowski. *Martingale methods in financial modelling.* Springer 1997.

P Wilmott, S Howison & J Dewynne. *The Mathematics of Financial Derivatives.* CUP 1995.