Chapter 11

On financial decisions of the firm

11.1 Introduction

One may think of decisions of firms as divided into two categories: *real* decisions and *financial* decisions. The real decisions focus on which projects the firms should undertake, the financial decisions deal with how the firm should raise money to undertake the desired projects. The area of *corporate finance* tries to explain the financial decisions of firms.

This chapter gives a very short introduction to the most basic issues in this area. The goal is to understand a couple of famous *irrelevance propositions* set forth by Modigliani and Miller stating conditions under which the firms financing decisions are in fact of no consequence. The conditions are in fact very restrictive but very useful since any discussion on optimality and rationality of financing decisions must start by relaxing one or several of these conditions.

We will consider only two types of securities: bonds and stocks. In reality there are many other types of securities (convertible bonds, callable bonds, warrants,...) and an important area of research (*security design)* seeks to explain why the different types of financing even exist. But we will have enough to do just learning the basic terminology and the reader will certainly see how to include more types of securities into the analysis.

Finally, it should be noted that a completely rigorous way of analyzing the firm's financing decisions requires general equilibrium theory - especially a setup with incomplete markets - but such a rigorous analysis will take far more time than we have in this introductory course.

11.2 'Undoing' the firm's financial decisions

At the heart of the irrelevance propositions are the investors ability to 'undo' the firm's financial decision: If a firm changes the payoff profile of its debt and equity, the investor can under restrictive assumptions change his portfolio and have an unchanged payoff of his investments. We illustrate all this in a one-period, finite state space model.

Given two dates 0 and 1 and a finite state space with S states. Assume that markets are complete and arbitrage free. Let p_s denote the price of an Arrow-Debreu security for state s, i.e. a security which pays 1 if the state at date 1 is s and 0 in all other states. Assume that an investment policy has been chosen by the firm which costs I_0 to initiate at time 0 and which delivers a state contingent payoff at time 1 given by the vector (i.e. random variable) $x = (x_1, \dots, x_S)$.

The firm at date 0 may choose to finance its investment by issuing debt maturing at date 1 with face value D, and by issuing shares of stocks (equity). Assuming no bankruptcy costs, the payoff at the final date to equity and debt is given by the random variables

$$\begin{aligned} E_1 &= \max(x - D, 0) \\ B_1 &= \min(x, D) \end{aligned}$$

respectively. If we assume that there are N shares of stocks, the payoff to each stock is given by $S_1 = \frac{1}{N} E_1$. Note that the entire cash flow to the firm is distributed between debt and equity holders. If we define the value of the firm at time 0 as the value at time 0 of the cash flows generated at time 1 minus the investment I_0, it is clear that the value of the firm at time 0 is independent of the level of D. This statement is often presented as Modigliani-Miller theorem but as we have set it up here (and as it is often presented) it is not really a proposition but an assumption: The value of the firm is by assumption unaffected by D since by assumption the payoff on the investment is unaffected by the choice of D. As we shall see below, this changes for example when there are bankruptcy costs or taxes.

Consider two possible financing choices: One in which the firm chooses to be an all equity (*unlevered)* firm and have $D = 0$, and one in which the firm chooses a level of $D > 0$ (a *levered* firm). We let superscript U denote quantities related to the unlevered firm and let superscript L refer to the case of a levered firm. By assumption $V^L = V^U$ since the assumption of leverage only results in a different distribution of the 'pie' consisting of the firm's cash flows, not a change in the pie's size. Now consider an agent who in his optimal portfolio in equilibrium wants to hold a position of one stock in the unlevered firm. The payoff at date 1 of this security is given by

$$S_1^U = \frac{1}{N} E_1^U = \frac{1}{N} x.$$

If the firm decides to become a levered firm, the payoff of one stock in the firm becomes

$$S_1^L = \frac{1}{N} \max(x - D, 0)$$

which is clearly different from the unlevered case. However note the following: Holding $\frac{1}{N}$ shares in the levered firm *and* the fraction $\frac{1}{N}$ of the firms debt, produces a payoff equal to

$$\frac{1}{N} \max(x - D, 0) + \frac{1}{N} \min(x, D) = \frac{1}{N} x.$$

From this we see that even if the firm changes from an unlevered to a levered firm, the investor can adapt to his preferred payoff by changing his portfolio (something he can always do in a complete market). Similarly, we note from the algebra above that it is possible to create a position in the levered firm's stock by holding one share of unlevered stock and selling the fraction $\frac{1}{N}$ of the firm's debt. In other words, the investor is able to undo the firm's financial decision. In general equilibrium models this implies, that if there is an equilibrium in which the firm chooses no leverage, then there is also an equilibrium in which the firm chooses leverage and the investors choose portfolios to offset the change in the firm's financing decision. This means that the firm's capital structure remains unexplained in this case and more structure must be added to understand how a level of debt may be optimal in some sense.

It is important to note that something like complete markets is required and this is very restrictive. In real world terms, to imitate a levered stock in the firm, the investor must be able to borrow at the same conditions as the firm (highly unrealistic) and furthermore have the debt contract structured in such a way, that it imitates the payoffs of the firm when it is in bankruptcy.

We now consider another financing decision at time 0, namely the dividend decision. We consider for simplicity a firm which is all equity financed. To give this a somewhat more realistic setup, imagine that we are in fact considering the last period of a firm's life and that it carries with it an 'endowment' of cash W_0 from previous periods, which you can also think of as 'earnings' from previous activity. Also, imagine that the firm has N shares of stock outstanding initially. The value of the firm at time 0 is given by the value of the cash flows that the firm delivers to shareholders:

$$V_0 = Div_0 - \Delta E_0 + \sum_s p_s x_s$$

where Div_0 is the amount of dividends paid at time 0 to the shareholders and ΔE_0 is the amount of new shares issued (*repurchased* if negative) at time 0. It must be the case that

$$W_0 + \Delta E_0 = I_0 + Div_0$$

i.e. the initial wealth plus money raised by issuing new equity is used either for investment or dividend payout. If the firm's investment decision has been fixed at I_0 and W_0 is given, then $Div_0 - \Delta E_0 = W_0 - I_0$ is fixed, and substituting this into the equation for firm value tells us that firm value is independent of dividends when the investment decision is given. The dividend payment can be financed with issuing stocks. This result is also sometimes referred to as the Modigliani-Miller theorem.

But you might think that if the firm issues new stock to pay for a dividend payout, it dilutes the value of the old stocks and possibly causes a loss to the old shareholders. In the world with Arrow-Debreu prices this will not happen:

Consider a decision to issue new stocks to finance a dividend payment of Div_0. Assume for simplicity that $I_0 = W_0$. The number of stocks issued to raise Div_0 amount is given by M where

$$Div_0 = \frac{M}{N+M}\sum_s p_s x_s.$$

The total number of stocks outstanding after this operation is $M + N$ and the value that the old stockholders are left with is the sum of the dividend and the diluted value of the stocks, i.e.

$$\begin{aligned} & Div_0 + \frac{N}{N+M}\sum_s p_s x_s \\ = \; & \frac{M}{N+M}\sum_s p_s x_s + \frac{N}{N+M}\sum_s p_s x_s \\ = \; & \sum_s p_s x_s \end{aligned}$$

which is precisely the value before the equity financed dividend payout. This means that the agent who depends in his optimal portfolio choice on no dividends can undo the firm's decision to pay a dividend by taking the dividend and investing it in the firms equity. Similarly, if a dividend is desired at time 0 but the firm does not provide one, the investor can achieve it by selling the appropriate fraction of his stock position. The key observation is that as long as the value of the shareholders position is unchanged by the dividend policy, the investor can use complete financial markets to design the desired cash flow.

A critical assumption for dividends to have no effect on the value of the firm and on the shareholder's wealth is that income from dividend payouts and income from share repurchases are taxed equally - something which is not true in many countries. If there is a lower tax on share repurchases it would be optimal for investors to receive no dividends and have any difference between W_0 and I_0 paid out by a share repurchase by the firm. Historically, one has observed dividend payouts even when there is lower taxes on share repurchases. Modigliani-Miller then tells us that something must be going on in the real world which is not captured by our model. The most important real world feature which is not captured by our model is asymmetric information. Our model assumes that everybody agrees on what the cash flows of the firm will be in each state in the future. In reality, there will almost always be insiders (managers and perhaps shareholders) who know more about the firm's prospects than outsiders (potential buyers of stocks, debtholders) and both the dividend policy and the leverage may then be used to signal to the outside world what the prospects of the firm really are. Changing the outsider's perception of the firm may then change the value of the firm.

11.3 Tax shield

If we change the model a little bit and assume that there are corporate taxes but that equity and debt financing are treated differently in the tax code then the capital structure becomes important. Change the model by assuming that the cash flows at time 1 are taxed at a rate of τ_c but that interest payments on debt can be deducted from taxable income. Let rD denote the part of the debt repayment which is regarded as 'interest'. The after tax cash flow of an unlevered firm at time 1 is given by

$$V_1^U = (1 - \tau_c)x$$

whereas the after tax cash flow of the levered firm (assuming full deduction of interest in all states) is given by

$$V_1^L = rD + (1 - \tau_c)(x - rD).$$

The difference in the cash flows is therefore

$$V_1^L - V_1^U = \tau_c rD$$

which means that the levered firm gets an increased value of

$$V_0^L - V_0^U = \tau_c r D \sum_s p_s.$$

As D increases, so does the value of this tax shield. Hence, in this setup financing the firm's operations with debt only would be optimal. But of course, it would be hard to convince tax authorities that a 100% debt financing was not actually a 100% equity financing! On the other hand, it should at least be the case that a significant fraction of debt was used for financing when there is a tax shield.

11.4 Bankruptcy costs

However, using a very high level of debt also increases the probability of bankruptcy. And it would add realism to our model if we assumed that when bankruptcy occurs, lawyers and accountants receive a significant fraction of the value left in the firm. This means that the total remaining value of the firm is no longer distributed to debtholders, and the debtholders will therefore have an interest in making sure that the level of debt issued by a firm is kept low enough to reduce the risk of bankruptcy to an acceptable level.

The trade-off between gains from leverage resulting from a tax shield and losses due to the increased likelihood of bankruptcy gives a first shot at defining an optimal capital structure. This is done in one of the exercises.

11.5 Financing positive NPV projects

We have seen earlier that in a world of certainty, one should only start a project if it has positive NPV. When uncertainty enters into our models the NPV criterion is still interesting but we need of course to define an appropriate concept of NPV. Both the arbitrage-free pricing models and the CAPM models gave us ways of defining present values of uncertain income streams.

We consider a one-period, finite state space model in which there is a complete, arbitrage-free market. Denote state prices (Arrow-Debreu prices) by $p = (p_1, \ldots, p_S)$. Assume that a firm initially (because of previous activity, say) has a net cash flow at time 1 given by the vector $x = (x_1, \ldots, x_S)$. The firm is financed partly by equity and partly by debt, and firm value, equity value and debt value at time 0 are therefore given as

$$\begin{aligned} V_0 &= \sum_s p_s x_s \\ E_0 &= \sum_s p_s (x_s - D)^+ \\ B_0 &= \sum_s p_s \min(x_s, D). \end{aligned}$$

We want to consider some issues of financing new investment projects in this very simple setup. We first note that projects with identical net present values may have very different effects on debt and equity. Indeed, a positive NPV project may have a negative effect on one of the two. This means that the ability to renegotiate the debt contract may be critical for the possibility of carrying out a positive NPV project.

Consider the following setup in which three projects a, b, c are given, all assumed to cost 1$ at time 0 to initiate, and with no possibility of scaling. Also shown is the cash flow x which requires no initial investment and the corresponding values of debt and equity when the debt has a face value of 30 :

	p	a	b	c	x	E_1	B_1
state 1	0.4	2	17	2	20	0	20
state 2	0.3	-20	0	4	40	10	30
state 3	0.3	30	-10	6	60	30	30
PV	-	3.8	3.8	3.8	38	12	26

Hence there are three projects all of which have an NPV of 2.8. Therefore, the increase in overall firm value will be 2.8. But how should the projects be financed? Throughout this chapter, we assume that all agents involved know and agree on all payouts and state prices. This is an important situation to analyze to develop a terminology and to get the 'competitive' situation straight first.

One possibility is to let the existing shareholders finance it out of their own pockets, i.e. pay the one dollar to initiate a project and do nothing about the terms of the debt: Here is what the value of equity looks like in the three cases at time 0:

	E_0^{new}	$E_0^{new} - E_0^{old}$	B_0^{new}	$B_0^{new} - B_0^{old}$
$x + a$	18	6	23.8	-2.2
$x + b$	11.8	-0.2	30	4
$x + c$	15	3	26.8	0.8

Clearly, only projects a and c will be attractive to the existing shareholders. Project a is however not something the bondholders would want carried through since it actually redistributes wealth over to the shareholders. What if a project is instead financed by issuing new stocks to other buyers? Let us check when this will be attractive to the old shareholders: This we can handle theoretically without actually working out the numbers:

To raise one dollar by issuing new equity, the new shareholders must acquire m shares, where m satisfies

$$\frac{m}{n+m}E_0^{new} = 1$$

and where n is the existing number of outstanding shares and E_0^{new} is the value of the equity after a new project has been carried out. Note that in this way new shareholders are by definition given a return on their investment consistent with the state prices. The old shareholders will be happy about the project as long as

$$\frac{n}{n+m}E_0^{new} - E_0^{old} > 0$$

i.e. as long as they have a capital gain on their shares. But this is equivalent to requiring that

$$\begin{aligned}\left(1 - \frac{m}{n+m}\right) E_0^{new} - E_0^{old} &> 0 \text{ i.e.} \\ E_0^{new} - E_0^{old} &> 1\end{aligned}$$

where we have used the definition of m to get to the last inequality. Note that this requirement is precisely the same as the one stating that in the case of financing by the old shareholders, the project should cost less to initiate than the capital gain. Note the similarity with the dividend irrelevance argument. In the argument we have just given, the shareholders decide whether to get a capital gain of $E_0^{new} - E_0^{old}$ and have a negative dividend of 1$, whereas in the other case, the capital gain is $\frac{n}{n+m}E_0^{new} - E_0^{old}$ but there is no dividend.

Now consider debt financing. There are many ways one could imagine this happening: One way is to let the debtholders finance the projects by having so much added to face value D that the present value of debt increases by 1. This requires three very different face values:

	new face value	E_0^{new}	B_0^{new}
a	40.67	14.8	27
b	27	14.8	27
c	30,33	14.8	27

An interesting special case is the following: Assume that existing bondholders are not willing to do any renegotiation of the debt terms. One could imagine for example, that the bondholders consisted of a large group of individuals who cannot easily be assembled to negotiate a new deal. Now, if project b were the only available project, then the shareholders would not enter into this project since the benefits of the project would go to the bondholders exclusively. This is the so-called *debt overhang* problem where it is impossible to finance a positive NPV project by issuing debt which is junior to the existing debt. To be able to carry through with project b, the shareholders would have to talk the debtholders into reducing the face value of the debt.

In general, it is easy to see that if a project has positive NPV there exists a way of financing the project which will benefit both debt holders and equity holders (can you show this?).

A special case which one often sees mentioned in textbooks is the case where the new project is of the same 'risk class' as the firm before entering into the project. This is true of project c. Such a project can always be financed by keeping the same debt-equity ratio after the financing as before. This is also left to the reader to show.