CHAPTER 15

Option Valuation

Just what is an option worth? Actually, this is one of the more difficult questions in finance. Option valuation is an esoteric area of finance since it often involves complex mathematics. Fortunately, just like most options professionals, you can learn quite a bit about option valuation with only modest mathematical tools. But no matter how far you might wish to delve into this topic, you must begin with the Black-Scholes-Merton option pricing model. This model is the core from which all other option pricing models trace their ancestry.

The previous chapter introduced to the basics of stock options. From an economic standpoint, perhaps the most important subject was the expiration date payoffs of stock options. Bear in mind that when investors buy options today, they are buying risky future payoffs. Likewise, when investors write options today, they become obligated to make risky future payments. In a competitive financial marketplace, option prices observed each day are collectively agreed on by buyers and writers assessing the likelihood of all possible future payoffs and payments and setting option prices accordingly.

In this chapter, we discuss stock option prices. This discussion begins with a statement of the fundamental relationship between call and put option prices and stock prices known as put-call parity. We then turn to a discussion of the Black-Scholes-Merton option pricing model. The Black-Scholes-Merton option pricing model is widely regarded by finance professionals as the premiere model of stock option valuation.

(*margin def.* **put-call parity** Thereom asserting a certain parity relationship between call and put prices for European style options with the same strike price and expiration date.

15.1 Put-Call Parity

Put-call parity is perhaps the most fundamental parity relationship among option prices. **Put-call parity** states that the difference between a call option price and a put option price for European-style options with the same strike price and expiration date is equal to the difference between the underlying stock price and the discounted strike price. The put-call parity relationship is algebraically represented as

$$C - P = S - Ke^{-rT}$$

where the variables are defined as follows:

C = call option price,	P = put option price,
S = current stock price,	K = option strike price,
r = risk-free interest rate,	T = time remaining until option expiration.

The logic behind put-call parity is based on the fundamental principle of finance stating that two securities with the same riskless payoff on the same future date must have the same price. To illustrate how this principle is applied to demonstrate put-call parity, suppose we form a portfolio of risky securities by following these three steps:

1. buy 100 stock shares of Microsoft stock (MSFT),
2. write one Microsoft call option contract,
3. buy one Microsoft put option contract.

Both Microsoft options have the same strike price and expiration date. We assume that these options are European style, and therefore cannot be exercised before the last day prior to their expiration date.

Table 15.1 Put-Call Parity

	Expiration Date Payoffs	
Expiration Date Stock Price	$S_T > K$	$S_T < K$
Buy stock	S_T	S_T
Write one call option	$-(S_T - K)$	0
Buy one put option	0	$(K - S_T)$
Total portfolio expiration date payoff	K	K

Table 15.1 states the payoffs to each of these three securities based on the expiration date stock price, denoted by S_T. For example, if the expiration date stock price is greater than the strike price, that is, $S_T > K$, then the put option expires worthless and the call option requires a payment from writer to buyer of $(S_T - K)$. Alternatively, if the stock price is less than the strike price, that is, $S_T < K$, the call option expires worthless and the put option yields a payment from writer to buyer of $(K - S_T)$.

In Table 15.1, notice that no matter whether the expiration date stock price is greater or less than the strike price, the payoff to the portfolio is always equal to the strike price. This means that the portfolio has a risk-free payoff at option expiration equal to the strike price. Since the portfolio is risk-free, the cost of acquiring this portfolio today should be no different than the cost of acquiring any other risk-free investment with the same payoff on the same date. One such riskless investment is a U.S. Treasury bill.

The cost of a U.S. Treasury bill paying *K* dollars at option expiration is the discounted strike price Ke^{-rT}, where *r* is the risk-free interest rate, and *T* is the time remaining until option expiration, which together form the discount factor e^{-rT}. By the fundamental principle of finance stating that two riskless investments with the same payoff on the same date must have the same price, it follows that this cost is also equal to the cost of acquiring the stock and options portfolio. Since this portfolio is formed by (1) buying the stock, (2) writing a call option, and (3) buying a put option, its cost is the sum of the stock price, plus the put price, less the call price. Setting this portfolio cost equal to the discounted strike price yields this equation.

$$S + P - C = Ke^{-rT}$$

By a simple rearrangement of terms we obtain the originally stated put-call parity equation, thereby validating our put-call parity argument.

$$C - P = S - Ke^{-rT}$$

The put-call parity argument stated above assumes that the underlying stock paid no dividends before option expiration. If the stock does pay a dividend before option expiration, then the put-call parity equation is adjusted as follows, where the variable *D* represents the present value of the dividend payment.

$$C - P = S - D - Ke^{-rT}$$

The logic behind this adjustment is the fact that a dividend payment reduces the value of the stock, since company assets are reduced by the amount of the dividend payment. When the dividend payment occurs before option expiration, investors adjust the effective stock price determining option payoffs to be made after the dividend payment. This adjustment reduces the value of the call option and increases the value of the put option.

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15.1a The argument supporting put-call parity is based on the fundamental principle of finance that two securities with the same riskless payoff on the same future date must have the same price. Restate the demonstration of put-call parity based on this fundamental principle. (Hint: Start by recalling and explaining the contents of Table 15.1.)

15.1b Exchange-traded options on individual stock issues are American style, and therefore put-call parity does not hold exactly for these options. In the "LISTED OPTIONS QUOTATIONS" page of the *Wall Street Journal*, compare the differences between selected call and put option prices with the differences between stock prices and discounted strike prices. How closely does put-call parity appear to hold for these American-style options?

15.2 The Black-Scholes-Merton Option Pricing Model

Option pricing theory made a great leap forward in the early 1970s with the development of the Black-Scholes option pricing model by Fischer Black and Myron Scholes. Recognizing the important theoretical contributions by Robert Merton, many finance professionals knowledgeable in the history of option pricing theory refer to an extended version of the model as the Black-Scholes-Merton option pricing model. In 1997, Myron Scholes and Robert Merton were awarded the Nobel prize in Economics for their pioneering work in option pricing theory. Unfortunately, Fischer Black had died two years earlier and so did not share the Nobel Prize, which cannot be awarded posthumously. The nearby Investment Updates box presents the *Wall Street Journal* story of the Nobel Prize award.

Investment Updates: Nobel prize

The Black-Scholes-Merton option pricing model states the value of a stock option as a function of these six input factors:

1. the current price of the underlying stock,
2. the dividend yield of the underlying stock,
3. the strike price specified in the option contract,
4. the risk-free interest rate over the life of the option contract,
5. the time remaining until the option contract expires,
6. the price volatility of the underlying stock.

The six inputs are algebraically defined as follows:

S = current stock price, y = stock dividend yield,

K = option strike price, r = risk-free interest rate,

T = time remaining until option expiration, and

σ = sigma, representing stock price volatility.

In terms of these six inputs, the Black-Scholes-Merton formula for the price of a call option on a single share of common stock is

$$C = Se^{-yT}N(d_1) - Ke^{-rT}N(d_2)$$

The Black-Scholes-Merton formula for the price of a put option on a share of common stock is

$$P = Ke^{-rT}N(-d_2) - Se^{-yT}N(-d_1)$$

In these call and put option formulas, the numbers d_1 and d_2 are calculated as

$$d_1 = \frac{\ln(S/K) + (r - y + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

In the formulas above, call and put option prices are algebraically represented by C and P, respectively. In addition to the six input factors S, K, r, y, T, and σ, the following three mathematical functions are used in the call and put option pricing formulas:

1) e^x, or $exp(x)$, denoting the natural exponent of the value of x,

2) $ln(x)$, denoting the natural logarithm of the value of x,

3) $N(x)$, denoting the standard normal probability of the value of x.

Clearly, the Black-Scholes-Merton call and put option pricing formulas are based on relatively sophisticated mathematics. While we recommend that the serious student of finance make an effort to understand these formulas, we realize that this is not an easy task. The goal, however, is to understand the economic principles determining option prices. Mathematics is simply a tool for strengthening this understanding. In writing this chapter, we have tried to keep this goal in mind.

Many finance textbooks state that calculating option prices using the formulas given here is easily accomplished with a hand calculator and a table of normal probability values. We emphatically disagree. While hand calculation is possible, the procedure is tedious and subject to error. Instead, we suggest that you use the Black-Scholes-Merton Options Calculator computer program included with this textbook (or a similar program obtained elsewhere). Using this program, you can easily and conveniently calculate option prices and other option-related values for the Black-Scholes-Merton option pricing model. We encourage you to use this options calculator and to freely share it with your friends.

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15.2a Consider the following inputs to the Black-Scholes-Merton option pricing model.

$S = \$50$	$y = 0\%$
$K = \$50$	$r = 5\%$
$T = 60$ days	$\sigma = 25\%$

These input values yield a call option price of \$2.22 and a put option price of \$1.82. Verify the above option prices using the options calculator. (Note: The options calculator computes numerical values with a precision of about three decimal points, but in this textbook prices are normally rounded to the nearest penny.)

Table 15.2 Six Inputs Affecting Option Prices

Input	Sign of input effect: Call	Put	Common Name
Underlying stock price (S)	+	–	Delta
Strike price of the option contract (K)	–	+	
Time remaining until option expiration (T)	+	+	Theta
Volatility of the underlying stock price (σ)	+	+	Vega
Risk-free interest rate (r)	+	–	Rho
Dividend yield of the underlying stock (y)	–	+	

15.3 Varying the Option Price Input Values

An important goal of this chapter is to provide an understanding of how option prices change as a result of varying each of the six input values. Table 15.2 summarizes the sign effects of the six inputs on call and put option prices. The plus sign indicates a positive effect and the minus sign indicates a negative effect. Where the magnitude of the input impact has a commonly used name, this is stated in the rightmost column.

The two most important inputs determining stock option prices are the stock price and the strike price. However, the other input factors are also important determinants of option value. We next discuss each input factor separately.

Figure 15.1 about here

Varying the Underlying Stock Price

Certainly, the price of the underlying stock is one of the most important determinants of the price of a stock option. As the stock price increases, the call option price increases and the put option price decreases. This is not surprising, since a call option grants the right to buy stock shares and a put option grants the right to sell stock shares at a fixed strike price. Consequently, a higher stock price at option expiration increases the payoff of a call option. Likewise, a lower stock price at option expiration increases the payoff of a put option.

For a given set of input values, the relationship between call and put option prices and an underlying stock price is illustrated in Figure 15.1. In Figure 15.1, stock prices are measured on the horizontal axis and option prices are measured on the vertical axis. Notice that the graph lines describing relationships between call and put option prices and the underlying stock price have a convex (bowed) shape. Convexity is a fundamental characteristic of the relationship between option prices and stock prices.

Varying the Option's Strike Price

As the strike price increases, the call price decreases and the put price increases. This is reasonable, since a higher strike price means that we must pay a higher price when we exercise a call option to buy the underlying stock, thereby reducing the call option's value. Similarly, a higher strike price means that we will receive a higher price when we exercise a put option to sell the underlying stock, thereby increasing the put option's value. Of course this logic works in reverse also; as the strike price decreases, the call price increases and the put price decreases.

Figure 15.2 about here

Varying the Time Remaining until Option Expiration

Time remaining until option expiration is an important determinant of option value. As time remaining until option expiration lengthens, both call and put option prices normally increase. This is expected, since a longer time remaining until option expiration allows more time for the stock price to move away from a strike price and increase the option's payoff, thereby making the option more valuable. The relationship between call and put option prices and time remaining until option expiration is illustrated in Figure 15.2, where time remaining until option expiration is measured on the horizontal axis and option prices are measured on the vertical axis.

Figure 15.3 about here

Varying the Volatility of the Stock Price

Stock price volatility (sigma, σ) plays an important role in determining option value. As stock price volatility increases, both call and put option prices increase. This is as expected, since the more volatile the stock price, the greater is the likelihood that the stock price will move farther away from a strike price and increase the option's payoff, thereby making the option more valuable. The relationship between call and put option prices and stock price volatility is graphed in Figure 15.3, where volatility is measured on the horizontal axis and option prices are measured on the vertical axis.

Figure 15.4 about here

Varying the Interest Rate

Although seemingly not as important as the other inputs, the interest rate still noticeably affects option values. As the interest rate increases, the call price increases and the put price decreases. This is explained by the time value of money. A higher interest rate implies a greater discount, which lowers the present value of the strike price that we pay when we exercise a call option or receive when we exercise a put option. Figure 15.4 graphs the relationship between call and put option prices and interest rates, where the interest rate is measured on the horizontal axis and option prices are measured on the vertical axis.

Varying the Dividend Yield

A stock's dividend yield has an important effect on option values. As the dividend yield increases, the call price decreases and the put price increases. This follows from the fact that when a company pays a dividend, its assets are reduced by the amount of the dividend, causing a like decrease in the price of the stock. Then, as the stock price decreases, the call price decreases and the put price increases.

(*margin def.* **delta** Measure of the dollar impact of a change in the underlying stock price on the value of a stock option. Delta is positive for a call option and negative for a put option.)

(*margin def.* **eta** Measures of the percentage impact of a change in the underlying stock price on the value of a stock option. Eta is positive for a call option and negative for a put option.)

(*margin def.* **vega** Measures of the impact of a change in stock price volatility on the value of a stock option. Vega is positive for both a call option and a put option.)

15.4 Measuring the Impact of Input Changes on Option Prices

Investment professionals using options in their investment strategies have standard methods to state the impact of changes in input values on option prices. The two inputs that most affect stock option prices over a short period, say, a few days, are the stock price and the stock price volatility. The approximate impact of a stock price change on an option price is stated by the option's **delta**. In the Black-Scholes-Merton option pricing model, expressions for call and put option deltas are stated as follows, where the mathematical functions e^x and $N(x)$ were previously defined.

$$Call\ option\ Delta = e^{-yT}N(d_1) > 0$$

$$Put\ option\ Delta = -e^{-yT}N(-d_1) < 0$$

As shown above, a call option delta is always positive and a put option delta is always negative. This corresponds to Table 15.2, where a + indicates a positive effect for a call option and a – indicates a negative effect for a put option resulting from an increase in the underlying stock price.

The approximate percentage impact of a stock price change on an option price is stated by the option's **eta**. In the Black-Scholes-Merton option pricing model, expressions for call and put option etas are stated as follows, where the mathematical functions e^x and $N(x)$ were previously defined.

$$Call\ option\ Eta = e^{-yT}N(d_1)S/C > 1$$

$$Put\ option\ Eta = -e^{-yT}N(-d_1)S/P < -1$$

In the Black-Scholes-Merton option pricing model, a call option eta is greater than +1 and a put option eta is less than -1.

The approximate impact of a volatility change on an option's price is measured by the option's **vega**.[1] In the Black-Scholes-Merton option pricing model, vega is the same for call and put options and is stated as follows, where the mathematical function $n(x)$ represents a standard normal density.

$$Vega \quad = \quad Se^{-yT}n(d_1)\sqrt{T} > 0$$

As shown above, vega is always positive. Again this corresponds with Table 15.2, where a + indicates a positive effect for both a call option and a put option from a volatility increase.

As with the Black-Scholes-Merton option pricing formula, these so-called "greeks" are tedious to calculate manually; fortunately they are easily calculated using an options calculator.

Interpreting Option Deltas

Interpreting the meaning of an option delta is relatively straightforward. Delta measures the impact of a change in the stock price on an option price, where a one dollar change in the stock price causes an option price to change by approximately delta dollars. For example, using the input values stated immediately below, we obtain a call option price of $2.22 and a put option price of $1.81. We also get a call option delta of +.55 and a put option delta of -.45.

S = $50	y = 0%
K = 50	r = 5%
T = 60 days	σ = 25%

[1] Those of you who are scholars of the Greek language recognize that "vega" is not a Greek letter like the other option sensitivity measures. (It is a star in the constellation Lyra.) Alas, the term vega has still entered the options professionals vocabulary and is in widespread use.

Now if we change the stock price from $50 to $51, we get a call option price of $2.81 and a put option price of $1.41. Thus a +$1 stock price change increased the call option price by $.59 and decreased the put option price by $.40. These price changes are close to, but not exactly equal to the original call option delta value of +.55 and put option delta value of -.45.

Interpreting Option Etas

Eta measures the percentage impact of a change in the stock price on an option price, where a 1 percent change in the stock price causes an option price to change by approximately eta percent. For example, the input values stated above yield a call option price of $2.22, and a put option price of $1.81, a call option eta of 12.42, and a put option eta of -12.33. If the stock price changes by 1 percent from $50 to $50.50, we get a call option price of $2.51 and a put option price of $1.60. Thus a 1 percent stock price change increased the call option price by 11.31 percent and decreased the put option price by 11.60 percent. These percentage price changes are close to the original call option eta value of +12.42 and put option eta value of -12.33.

Interpreting Option Vegas

Interpreting the meaning of an option vega is also straightforward. Vega measures the impact of a change in stock price volatility on an option price, where a 1 percent change in sigma changes an option price by approximately the amount vega. For example, using the same input values stated earlier we obtain call and put option prices of $2.22 and $1.82, respectively. We also get an option vega of +.08. If we change the stock price volatility to $\sigma = 26\%$, we then get call and put option prices of $2.30 and $1.90. Thus a +1 percent stock price volatility change increased both call and put option prices by $.08, exactly as predicted by the original option vega value.

(*margin def.* **gamma** Measure of delta sensitivity to a stock price change.)

(*margin def.* **theta** Measure of the impact on an option price of time remaining until option expiration lengthening by one day.)

(*margin def.* **rho** Measure of option price sensitivity to a change in the interest rate.)

Interpreting an Option's Gamma, Theta, and Rho

In addition to delta, eta, and vega, options professionals commonly use three other measures of option price sensitivity to input changes: gamma, theta, and rho.

Gamma measures delta sensitivity to a stock price change, where a one dollar stock price change causes delta to change by approximately the amount gamma. In the Black-Scholes-Merton option pricing model, gammas are the same for call and put options.

Theta measures option price sensitivity to a change in time remaining until option expiration, where a one-day change causes the option price to change by approximately the amount theta. Since a longer time until option expiration normally implies a higher option price, thetas are usually positive.

Rho measures option price sensitivity to a change in the interest rate, where a 1 percent interest rate change causes the option price to change by approximately the amount rho. Rho is positive for a call option and negative for a put option.

(*margin def.* **implied standard deviation (ISD)** An estimate of stock price volatility obtained from an option price. **implied volatility (IVOL)** Another term for implied standard deviation.)

15.5 Implied Standard Deviations

The Black-Scholes-Merton stock option pricing model is based on six inputs: a stock price, a strike price, an interest rate, a dividend yield, the time remaining until option expiration, and the stock price volatility. Of these six factors, only the stock price volatility is not directly observable and must be estimated somehow. A popular method to estimate stock price volatility is to use an implied value from an option price. A stock price volatility estimated from an option price is called an **implied standard deviation** or **implied volatility**, often abbreviated as *ISD* or *IVOL*, respectively. Implied volatility and implied standard deviation are two terms for the same thing.

Calculating an implied volatility requires that all input factors have known values, except sigma, and that a call or put option value be known. For example, consider the following option price input values, absent a value for sigma.

S = \$50	y = 0%
K = 50	r = 5%
T = 60 days	

Suppose we also have a call price of C = \$2.22. Based on this call price, what is the implied volatility? In other words, in combination with the input values stated above, what sigma value yields a call price of C = \$2.22? The answer is a sigma value of .25, or 25 percent.

Now suppose we wish to know what volatility value is implied by a call price of C = \$3. To obtain this implied volatility value, we must find the value for sigma that yields this call price. If you use the options calculator program, you can find this value by varying sigma values until a call option price of $3 is obtained. This should occur with a sigma value of 34.68 percent. This is the implied standard deviation (ISD) corresponding to a call option price of $3.

You can easily obtain an estimate of stock price volatility for almost any stock with option prices reported in the *Wall Street Journal*. For example, suppose you wish to obtain an estimate of stock price volatility for Microsoft common stock. Since Microsoft stock trades on Nasdaq under the ticker MSFT, stock price and dividend yield information are obtained from the "Nasdaq National Market Issues" pages. Microsoft options information is obtained from the "Listed Options Quotations" page. Interest rate information is obtained from the "Treasury Bonds, Notes and Bills" column.

The following information was obtained for Microsoft common stock and Microsoft options from the *Wall Street Journal*.

Stock price = $89 Dividend yield = 0%

Strike price = $90 Interest rate = 5.54%

Time until contract expiration = 73 days

Call price = $8.25

To obtain an implied standard deviation from these values using the options calculator program, first set the stock price, dividend yield, strike, interest rate, and time values as specified above. Then vary sigma values until a call option price of $8.25 is obtained. This should occur with a sigma value of 52.1 percent. This implied standard deviation represents an estimate of stock price volatility for Microsoft stock obtained from a call option price.

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15.5a In a recent issue of the *Wall Street Journal*, look up the input values for the stock price, dividend yield, strike price, interest rate, and time to expiration for an option on Microsoft common stock. Note the call price corresponding to the selected strike and time values. From these values, use the options calculator to obtain an implied standard deviation estimate for Microsoft stock price volatility. (Hint: When determining time until option expiration, remember that options expire on the Saturday following the third Friday of their expiration month.)

15.6 Hedging A Stock Portfolio With Stock Index Options

Hedging is a common use of stock options among portfolio managers. In particular, many institutional money managers make some use of stock index options to hedge the equity portfolios they manage. In this section, we examine how an equity portfolio manager might hedge a diversified stock portfolio using stock index options.

To begin, suppose that you manage a $10 million diversified portfolio of large-company stocks and that you maintain a portfolio beta of 1 for this portfolio. With a beta of 1, changes in the value of your portfolio closely follow changes in the Standard and Poor's 500 index. Therefore, you decide to use options on the S&P 500 index as a hedging vehicle. S&P 500 index options trade on the Chicago Board Options Exchange (CBOE) under the ticker symbol SPX. SPX option prices are reported daily in the “Index Options Trading” column of the *Wall Street Journal*. Each SPX option has a contract value of 100 times the current level of the S&P 500 index.

SPX options are a convenient hedging vehicle for an equity portfolio manager because they are European style and because they settle in cash at expiration. For example, suppose you hold one SPX call option with a strike price of 910 and at option expiration, the S&P 500 index stands at 917. In this case, your cash payoff is 100 times the difference between the index level and the strike price, or 100 × (917 - 910) = $700. Of course, if the expiration date index level falls below the strike price, your SPX call option expires worthless.

Hedging a stock portfolio with index options requires first calculating the number of option contracts needed to form an effective hedge. While you can use either put options or call options to construct an effective hedge, we here assume that you decide to use call options to hedge your $10 million equity portfolio. Using stock index call options to hedge an equity portfolio involves writing a certain number of option contracts. In general, the number of stock index option contracts needed to hedge an equity portfolio is stated by the equation

$$\text{Number of option contracts} = \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{Option contract value}}$$

In your particular case, you have a portfolio beta of 1 and a portfolio value of $10 million. You now need to calculate an option delta and option contract value.

The option contract value for an SPX option is simply 100 times the current level of the S&P 500 index. Checking the "Index Options Trading" column in the *Wall Street Journal* you see that the S&P 500 index has a value of 928.80, which means that each SPX option has a current contract value of $92,880.

To calculate an option delta, you must decide which particular contract to use. You decide to use options with an October expiration and a strike price of 920, that is, the October 920 SPX contract. From the "Index Options Trading" column, you find the price for these options is 35-3/8, or 35.375. Options expire on the Saturday following the third Friday of their expiration month. Counting days on your calendar yields a time remaining until option expiration of 70 days. The interest rate on Treasury bills maturing closest to option expiration is 5 percent. The dividend yield on the S&P 500 index is not normally reported in the *Wall Street Journal*. Fortunately, the S&P 500 trades in the form of depository shares on the American Stock Exchange (AMEX) under the ticker SPY. SPY shares represent a claim on a portfolio designed to match as closely as possible the S&P 500. By looking up information on SPY shares on the Internet, you find that the dividend yield is 1.5 percent.

With the information now collected, you enter the following values into an options calculator: S = 928.80, K = 920, T = 70, r = 5%, and y = 1.5%. You then adjust the sigma value until you get the call price of C = 35.375. This yields an implied standard deviation of 17 percent, which represents a current estimate of S&P 500 index volatility. Using this sigma value 17 percent then yields a call option delta of .599. You now have sufficient information to calculate the number of option contracts needed to effectively hedge your equity portfolio. By using the equation above, we can calculate the number of October 920 SPX options that you should write to form an effective hedge.

$$\frac{1.0 \times \$10{,}000{,}000}{.599 \times \$92{,}880} \approx 180 \text{ contracts}$$

Furthermore, by writing 180 October 920 call options, you receive 180 × 100 × 35.375 = $636,750.

To assess the effectiveness of this hedge, suppose the S&P 500 index and your stock portfolio both immediately fall in value by 1 percent. This is a loss of $100,000 on your stock portfolio. After the S&P 500 index falls by 1 percent its level is 919.51, which then yields a call option price of

C = 30.06. Now, if you were to buy back the 180 contracts, you would pay $180 \times 100 \times 30.06 = \$541{,}080$. Since you originally received \$636,750 for the options, this represents a gain of \$636,750 - \$541,080 = \$95,670, which cancels most of the \$100,000 loss on your equity portfolio. In fact, your final net loss is only \$4,330, which is a small fraction of the loss that would have been realized on an unhedged portfolio.

To maintain an effective hedge over time, you will need to rebalance your options hedge on, say, a weekly basis. Rebalancing simply requires calculating anew the number of option contracts needed to hedge your equity portfolio, and then buying or selling options in the amount necessary to maintain an effective hedge. The nearby Investment Update box contains a brief *Wall Street Journal* report on hedging strategies using stock index options.

Investment Updates: Hedging

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15.6a In the hedging example above, suppose instead that your equity portfolio had a beta of 1.5. What number of SPX call options would be required to form an effective hedge?

15.6b Alternatively, suppose that your equity portfolio had a beta of .5. What number of SPX call options would then be required to form an effective hedge?

Table 15.3. Volatility Skews for IBM Options

Strikes	Calls	Call ISD (%)	Puts	Put ISD (%)
115	17-1/4	58.14	4-5/8	58.62
120	13-1/8	51.77	5-3/4	53.92
125	9-3/4	48.28	7-3/8	50.41
130	6-7/8	45.27	9-3/4	48.90
135	4-5/8	43.35	11-3/4	42.48
140	2-7/8	40.80	15-3/8	42.83

Other information: S = 127.3125, y = 0.07%,

T = 43 days, r = 3.6%

(*margin def.* **volatility skew** Description of the relationship between implied volatilities and strike prices for options. Volatility skews are also called *volatility smiles*.)

15.7 Implied Volatility Skews

We earlier defined implied volatility (IVOL) and implied standard deviation (ISD) as the volatility value implied by an option price and stated that implied volatility represents an estimate of the price volatility (sigma, σ) of the underlying stock. We further noted that implied volatility is often used to estimate a stock's price volatility over the period remaining until option expiration. In this section, we examine the phenomenon of **implied volatility skews** - the relationship between implied volatilities and strike prices for options.

To illustrate the phenomenon of implied volatility skews, Table 15.3 presents option information for IBM stock options observed in October 1998 for options expiring 43 days later in

November 1998. This information includes strike prices, call option prices, put option prices, and call and put implied volatilities calculated separately for each option. Notice how the individual implied volatilities differ across different strike prices. Figure 15.5 provides a visual display of the relationship between implied volatilities and strike prices for these IBM options. The steep negative slopes for call and put implied volatilities might be called volatility skews.

Figure 15.5 about here

(*margin def.* **stochastic volatility** The phenomenon of stock price volatility changing randomly over time.)

Logically, there can be only one stock price volatility since price volatility is a property of the underlying stock, and each option's implied volatility should be an estimate of a single underlying stock price volatility. That this is not the case is well known to options professionals, who commonly use the terms volatility smile and volatility skew to describe the anomaly. Why do volatility skews exist? Many suggestions have been proposed regarding possible causes. However, there is widespread agreement that the major factor causing volatility skews is **stochastic volatility**. Stochastic volatility is the phenomenon of stock price volatility changing over time, where the price volatility changes are largely random.

The Black-Scholes-Merton option pricing model assumes that stock price volatility is constant over the life of the option. Therefore, when stock price volatility is stochastic the Black-Scholes-Merton option pricing model yields option prices that may differ from observed market prices. Nevertheless, the simplicity of the Black-Scholes-Merton model makes it an excellent working model of option prices and many options professionals consider it an invaluable tool for analysis and decision making. Its simplicity is an advantage because option pricing models that account for stochastic volatility can be quite complex, and therefore difficult to work with. Furthermore, even when volatility is stochastic, the Black-Scholes-Merton option pricing model yields accurate option prices for options with strike prices close to a current stock price. For this reason, when using implied volatility to estimate an underlying stock price volatility it is best to use at-the-money options - that is, options with a strike price close to the current stock price.

CHECK THIS

15.7a Using information from a recent *Wall Street Journal*, calculate IBM implied volatilities for options with at least one month until expiration.

15.8 Summary and Conclusions

In this chapter, we examined stock option prices. Many important aspects of option pricing were covered, including the following:

1. Put-call parity states that the difference between a call price and a put price for European style options with the same strike price and expiration date is equal to the difference between the stock price less a dividend adjustment and the discounted strike price. Put-call parity is based on the fundamental principle that two securities with the same riskless payoff on the same future date must have the same price today.

2. The Black-Scholes-Merton option pricing formula states that the value of a stock option is a function of the current stock price, stock dividend yield, the option strike price, the risk-free interest rate, the time remaining until option expiration, and the stock price volatility.

3. The two most important determinants of the price of a stock option are the price of the underlying stock and the strike price of the option. As the stock price increases, call prices increase and put prices decrease. Conversely, as the strike price increases, call prices decrease and put prices increase.

4. Time remaining until option expiration is an important determinant of option value. As time remaining until option expiration lengthens, both call and put option prices normally increase. Stock price volatility also plays an important role in determining option value. As stock price volatility increases, both call and put option prices increase.

5. Although less important, the interest rate can noticeably affect option values. As the interest rate increases, call prices increase and put prices decrease. A stock's dividend yield also affects option values. As the dividend yield increases, call prices decrease and put prices increase.

6. The two input factors that most affect stock option prices over a short period, say, a few days, are the stock price and the stock price volatility. The impact of a stock price change on an option price is measured by the option's delta. The impact of a volatility change on an option's price is measured by the option's vega.

7. A call option delta is always positive and a put option delta is always negative. Delta measures the impact of a stock price change on an option price, where a one dollar change in the stock price causes an option price to change by approximately delta dollars.

8. Vega measures the impact of a change in stock price volatility (sigma σ) on an option price, where a one percent change in volatility changes an option price by approximately the amount vega.

9. Of the six input factors to the Black-Scholes-Merton option pricing model, only the stock price volatility is not directly observable and must be estimated somehow. A stock price volatility estimated from an option price is called an implied volatility or an implied standard deviation, which are two terms for the same thing.

10. Options on the S&P 500 index are a convenient hedging vehicle for an equity portfolio because they are European style and because they settle for cash at option expiration. Hedging a stock portfolio with index options requires calculating the number of option contracts needed to form an effective hedge.

11. To maintain an effective hedge over time, you should rebalance the options hedge on a regular basis. Rebalancing requires recalculating the number of option contracts needed to hedge an equity portfolio, and then buying or selling options in the amount necessary to maintain an effective hedge.

12. Volatility skews, or volatility smiles occur when individual implied volatilities differ across call and put options with different strike prices. Volatility skews commonly appear in implied volatilities for stock index options and also appear in implied volatilities for options on individual stocks. The most important factor causing volatility skews is stochastic volatility, the phenomenon of stock price volatility changing over time in a largely random fashion.

13. The Black-Scholes-Merton option pricing model assumes a constant stock price volatility, and yields option prices that may differ from stochastic volatility option prices. Nevertheless, even when volatility is stochastic the Black-Scholes-Merton option pricing model yields accurate option prices for options with strike prices close to a current stock price. Therefore, when using implied volatility to estimate an underlying stock price volatility, it is best to use at-the-money options.

Key Terms

put-call parity	**delta**
vega	**gamma**
theta	**eta**
rho	**implied volatility (IVOL)**
volatility smile	**implied standard deviation (ISD)**
volatility skew	**stochastic volatility**

Get Real!

This chapter began by introducing you to the put-call parity condition, one of the most famous pricing relationships in finance. Using it, we can establish the relative prices of puts, calls, the underlying stock, and a T-bill. In practice, if you were to use closing prices and rates published in, say, *The Wall Street Journal*, you would find numerous apparent violations. However, if you tried to execute the trades needed to profit from these violations, you would essentially always find that you can't get the printed prices. One reason for this is that the prices you see are not even contemporaneous because the markets close at different times. Thus, trying to make money pursuing put-call parity violations is probably not a good idea.

We next introduced the Nobel-Prize-winning Black-Scholes-Merton option pricing formula. We saw that the formula and its associated concepts are fairly complex, but, despite that complexity, the formula is very widely used by traders and money managers.

To learn more about the real-world use of the concepts we discussed, you should purchase a variety of stock and index options and then compare the prices you pay to the theoretical prices from the option pricing formula. You will need to come up with a volatility estimate. A good way to do this is to calculate some implied standard deviations from a few days earlier. You should also compute the various "greeks" and observe how well they describe what actually happens to some of your option prices. How well do they work?

Another important use for option pricing theory is to gain some insight into stock market volatility, both on an overall level and for individual stocks. Remember that in Chapter 1 that we discussed the probabilities associated with returns equal to the average plus or minus a particular number of standard deviations for a small number of indexes. Implied standard deviations (ISDs) provide a means of broadening this analysis to anything with traded options. Try calculating a few ISDs for both stock index options and some high-flying, technology-related stocks. You might be surprised how volatile the market thinks individual stocks can be. The ISDs on high-tech stocks serve as a warning to investors about the risks of loading up on such investments compared to investing in a broadly diversified market index.

Chapter 15
Option Valuation

Questions and problems

Review Problems and Self-Test

1. **Put-Call Parity** A call option sells for $4. It has a strike price of $40 and six months to maturity. If the underlying stock currently sells for $30 per share, what is the price of a put option with a $40 strike and six months to maturity? The risk-free interest rate is 5 percent.

2. **Black-Scholes** What is the value of a call option if the underlying stock price is $200, the strike price is $180, the underlying stock volatility is 40 percent, and the risk-free rate is 4 percent? Assume the option has 60 days to expiration.

Answers to Self-Test Problems

1. Using the put-call parity formula, we have

$$C - P = S - Ke^{-rT}$$

Rearranging to solve for *P*, the put price, and plugging in the other numbers get us:

$$\begin{aligned} P &= C - S + Ke^{-rT} \\ &= \$4 - \$30 + \$40e^{-.05(.5)} \\ &= \$13.01 \end{aligned}$$

2. We will simply use the options calculator supplied with this book to calculate the answer to this question. The inputs are

S	= current stock price	= \$200
K	= option strike price	= \$180
r	= risk free interest rate	=.04
σ	= stock volatility	= .40
T	= time to expiration	= 60 days.

Notice that, absent other information, the dividend yield is zero. Plugging these values into the options calculator produces a value of $25.63.

Test Your IQ (Investment Quotient)

1. **Put-Call Parity** According to put-call parity, a risk-free portfolio is formed by buying 100 stock shares and

a. writing one call contract and buying one put contract
b. buying one call contract and writing one put contract
c. buying one call contract and buying one put contract
d. writing one call contract and writing one put contract

2. **Black-Scholes-Merton Model** In the Black-Scholes-Merton option pricing model, the value of an option contract is a function of six input factors. Which of the following is not one of these factors?

a. the price of the underlying stock
b. the strike price of the option contract
c. the expected return on the underlying stock
d. the time remaining until option expiration

3. **Black-Scholes Formula** In the Black-Scholes option valuation formula, an increase in a stock's volatility: (*1992 CFA exam*)

a. increases the associated call option value
b. decreases the associated put option value
c. increases or decreases the option value, depending on the level of interest rates
d. does not change either the put or call option value because put-call parity holds

4. **Option Prices** Which of the following variables influence the value of options? (*1990 CFA exam*)

I. level of interest rates
II. time to expiration of the option
III. dividend yield of underlying stock
IV. stock price volatility

a. I and IV only
b. II and III only
c. I, III and IV only
d. I, II, III and IV

5. **Option Prices** Which of the following factors does not influence the market price of options on a common stock? (*1989 CFA exam*)

a. expected return on the underlying stock
b. volatility of the underlying stock
c. relationship between the strike price of the options and the market price of the underlying stock
d. option's expiration date

6. **Option Prices** Which one of the following will increase the value of a call option? (*1993 CFA exam*)

a. an increase in interest rates
b. a decrease in time to expiration of the call
c. a decrease in the volatility of the underlying stock
d. an increase in the dividend rate of the underlying stock

7. **Option Prices** Which one of the following would tend to result in a high value of a call option? (*1988 CFA exam*)

a. interest rates are low
b. the variability of the underlying stock is high
c. there is little time remaining until the option expires
d. the exercise price is high relative to the stock price

8. **Option Price Factors** Which of the following incorrectly states the signs of the impact of an increase in the indicated input factor on call and put option prices?

		Call	Put
a.	risk-free interest rate	+	-
b.	underlying stock price	+	-
c.	dividend yield of the underlying stock	-	+
d.	volatility of the underlying stock price	+	-

9. **Option Price Factors** Which of the following incorrectly states the signs of the impact of an increase in the indicated input factor on call and put option prices?

		Call	Put
a.	strike price of the option contract	+	-
b.	time remaining until option expiration	+	+
c.	underlying stock price	+	-
d.	volatility of the underlying stock price	+	+

10. **Option Price Sensitivities** Which of the following measures the impact of a change in the stock price on an option price?

a. vega
b. rho
c. delta
d. theta

11. **Option Price Sensitivities** Which of the following measures the impact of a change in time remaining until option expiration on an option price?

a. vega
b. rho
c. delta
d. theta

12. **Option Price Sensitivities** Which of the following measures the impact of a change in the underlying stock price volatility on an option price?

a. vega
b. rho
c. delta
d. theta

13. **Option Price Sensitivities** Which of the following measures the impact of a change in the interest rate on an option price?

a. vega
b. rho
c. delta
d. theta

14. **Hedging with Options** You wish to hedge a stock portfolio, where the portfolio beta is 1, the portfolio value is $10 million, the hedging index call option delta is .5, and the hedging index call option contract value is $100,000. Which of the following hedging transactions is required to hedge the portfolio?

a. write 200 call option contracts
b. write 100 call option contracts
c. buy 200 call option contracts
d. buy 100 call option contracts

15. **Hedging with Options** You wish to hedge a stock portfolio, where the portfolio beta is .5, the portfolio value is $10 million, the hedging index call option delta is .5, and the hedging index call option contract value is $100,000. Which of the following hedging transactions is required to hedge the portfolio?

a. write 200 call option contracts
b. write 100 call option contracts
c. buy 200 call option contracts
d. buy 100 call option contracts

Questions and Problems

Core Questions

1. **Option Prices** What are the six factors that determine an option's price?

2. **Options and Expiration Dates** What is the impact of lengthening the time to expiration on an option's value? Explain.

3. **Options and Stock Price Volatility** What is the impact of an increase in the volatility of the underlying stock on an option's value? Explain.

4. **Options and Dividend Yields** How do dividend yields affect option prices? Explain.

5. **Options and Interest Rates** How do interest rates affect option prices? Explain.

6. **Put-Call Parity** A call option is currently selling for $10. It has a strike price of $80 and three months to maturity. What is the price of a put option with a $80 strike and three months to maturity? The current stock price is $85, and the risk-free interest rate is 6 percent.

7. **Put-Call Parity** A call option currently sells for $10. It has a strike price of $80 and three months to maturity. A put with the same strike and expiration date sells for $8. If the risk-free interest rate is 4 percent, what is the current stock price?

8. **Call Option Prices** What is the value of a call option if the underlying stock price is $100, the strike price is $70, the underlying stock volatility is 30 percent, and the risk-free rate is 5 percent? Assume the option has 30 days to expiration.

9. **Call Option Prices** What is the value of a call option if the underlying stock price is $20, the strike price is $22, the underlying stock volatility is 50 percent, and the risk-free rate is 4 percent? Assume the option has 60 days to expiration and the underlying stock has a dividend yield of 2 percent.

10. **Call Option Prices** What is the value of a put option if the underlying stock price is $60, the strike price is $65, the underlying stock volatility is 25 percent, and the risk-free rate is 5 percent? Assume the option has 180 days to expiration.

<u>Intermediate Questions</u>

11. **Put-Call Parity** A put and a call option have the same maturity and strike price. If both are at the money, which is worth more? Prove your answer and then provide an intuitive explanation.

12. **Put-Call Parity** A put and a call option have the same maturity and strike price. If they also have the same price, which one is in the money?

13. **Put-Call Parity** One thing the put-call parity equation tells us that given any three of a stock, a call, a put, and a T-bill, the fourth can be synthesized or replicated using the other three. For example, how can we replicate a share of stock using a put, a call, and a T-bill?

14. **Delta** What does an option's delta tell us? Suppose a call option with a delta of .60 sells for $2.00. If the stock price rises by $1, what will happen to the call's value?

15. **Eta** What is the difference between an option's delta and its eta? Suppose a call option has an eta of 12. If the underlying stock rises from $100 to $102, what will be impact on the option's price?

16. **Vega** What does an option's vega tell us? Suppose a put option with a vega of .60 sells for $12.00. If the underlying volatility rises from 50 to 51 percent, what will happen to the put's value?

17. **American Options** A well-known result in option pricing theory is that it will never pay to exercise a call option on a non-dividend-paying stock before expiration. Why do you suppose this is so? Would it ever pay to exercise a put option before maturity?

18. **ISDs** A call option has a price of $2.57. The underlying stock price, strike price, and dividend yield are $100, $120, and 3 percent, respectively. The option has 100 days to expiration, and the risk-free interest rate is 6 percent. What is the implied volatility?

19. **Hedging with Options** Suppose you have a stock market portfolio with a beta of 1.4 that is currently worth $150 million. You wish to hedge against a decline using index options. Describe how you might do so with puts and calls. Suppose you decide to use SPX calls. Calculate the number of contracts needed if the contract you pick has a delta of .50, and the S&P 500 index is at 1200.

20. **Calculating the Greeks** Using an options calculator, calculate the price and the following "greeks" for a call and a put option with one year to expiration: delta, gamma, rho, eta, vega, and theta. The stock price is $80, the strike price is $75, the volatility is 40 percent, the dividend yield is 3 percent, and the risk-free interest rate is 5 percent.

Chapter 15
Option Valuation
Answers and solutions

Answers to Multiple Choice Questions

1. **A**
2. **C**
3. **A**
4. **D**
5. **A**
6. **A**
7. **B**
8. **D**
9. **A**
10. **C**
11. **D**
12. **A**
13. **B**
14. **A**
15. **B**

Answers to Questions and Problems

Core Questions

1. The six factors are the stock price, the strike price, the time to expiration, the risk-free interest rate, the stock price volatility, and the dividend yield.

2. Increasing the time to expiration increases the value of an option. The reason is that the option gives the holder the right to buy or sell. The longer the holder has that right, the more time there is for the option to increase in value. For example, imagine an out-of-the-money option that is about to expire. Because the option is essentially worthless, increasing the time to expiration obviously would increase its value.

3. An increase in volatility acts to increase both put and call values because greater volatility increase the possibility of favorable in-the-money payoffs.

4. An increase in dividend yields reduces call values and increases put values. The reason is that, all else the same, dividend payments decrease stock prices. To give an extreme example, consider a company that sells all its assets, pays off its debts, and then pays out the remaining cash in a final, liquidating dividend. The stock price would fall to zero, which is great for put holders, but not so great for call holders.

5. Interest rate increases are good for calls and bad for puts. The reason is that if a call is exercised in the future, we have to pay a fixed amount at that time. The higher is the interest rate, the lower is the present value of that fixed amount. The reverse is true for puts in that we receive a fixed amount.

6. Rearranging the put-call parity condition to solve for *P*, the put price, and plugging in the other numbers get us:

$$\begin{aligned} P &= C - S + Ke^{-rT} \\ &= \$10 - \$85 + \$80e^{-.06(.25)} \\ &= \$3.81 \end{aligned}$$

7. Rearranging the put-call parity condition to solve for *S*, the stock price, and plugging in the other numbers get us:

$$\begin{aligned} S &= C - P + Ke^{-rT} \\ &= \$10 - \$8 + \$80e^{-.04(.25)} \\ &= \$81.20 \end{aligned}$$

8. Using the option calculator with the following inputs:

S	= current stock price	= $100,
K	= option strike price	= $70,
r	= risk-free rate	= .05,
σ	= stock volatility	= .30, and
T	= time to expiration	= 30 days

results in a call option price of $30.29.

9. Using the option calculator with the following inputs:

S	= current stock price	= \$20,
K	= option strike price	= \$22,
r	= risk-free rate	=.04,
σ	= stock volatility	= .50,
T	= time to expiration	= 60 days, and
y	= dividend yield	= 2 percent

results in a call option price of \$.90.

10. Using the option calculator with the following inputs:

S	= current stock price	= \$60,
K	= option strike price	= \$65,
r	= risk-free rate	=.05,
σ	= stock volatility	= .25, and
T	= time to expiration	= 180 days

results in a put option price of \$6.24.

11. The call is worth more. To see this, we can rearrange the put-call parity condition as follows:

$$C - P = S - Ke^{-rT}$$

If the options are at the money, $S = K$, so the right-hand side of this expression is equal to the strike minus the present value of the strike price. This is necessary positive. Intuitively, if both options are at the money, the call option offers a much bigger potential payoff, so it's worth more.

12. Looking at the previous answer, if the call and put have the same price (i.e., $C - P = 0$), it must be that the stock price is equal to the present value of the strike price, so the put is in the money.

13. Looking at Question 7 above, a stock can be replicated by a long call (to capture the upside gains), a short put (to reflect the downside losses), and a T-bill (to capture the time-value component–the "wait" factor).

14. An option's delta tells us the (approximate) dollar change in the option's value that will result from a change in the stock price. If a call sells for \$2.00 with a delta of .60, a \$1 stock price increase will add \$.60 to option price, increasing it to \$2.60.

15. The delta relates dollar changes in the stock to dollar changes in the option. The eta relates percentage changes. So, the stock price rises by 2 percent (\$100 to \$102), an eta of 12 implies that the option price will rise by 24 percent.

16. Vega relates the change in volatility in percentage points to the dollar change in the option's price. If volatility rises from 50 to 51 percent, a 1 point rise, and vega is .60, then the option's price will rise by 60 cents.

17. The reason is that a call option on a non-dividend-paying stock is always worth more alive than dead, meaning that you will always get more from selling it than exercising it. If you exercise it, you only get the intrinsic value. If you sell it, you get instrinsic value at a minimum plus any remaining time value. For a put, however, early exercise can be optimal. Suppose, for example, the stock price drops to zero. That's as good as it gets, so we would like to go ahead and exercise. It will actually pay to exercise early for some stock price greater than zero, but no general formula is known for the critical stock price.

18. We have to use trial and error to find the answer (there is no other way). Using the option calculator with the following inputs:

S	= current stock price	= \$100,
K	= option strike price	= \$120,
r	= risk-free rate	=.06,
σ	= stock volatility	= ??,
T	= time to expiration	= 100 days,
y	= dividend yield	= 3 percent,

we try different values for the volatility until a call price of \$2.57 results. Verify that σ = 40% does the trick.

19. You can either buy put options or sell call options. In either case, gains or losses on your stock portfolio will be offset by gains or losses on your option contracts. To calculate the number of contracts needed to hedge a \$150 million portfolio with a beta of 1.4 using an option contract value of \$120,000 (100 times the index) and a delta of .50, we use the formula from the chapter:

$$\text{Number of option contracts} = \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{Option contract value}}$$

Filling in the numbers, we need to sell 1.4 × \$150M/(.5 × \$120,000) = 3,500 contracts.

20. Using the option calculator with the following inputs:

S	= current stock price	= \$80,
K	= option strike price	= \$75,
r	= risk-free rate	=.05,
σ	= stock volatility	= .40,
T	= time to expiration	= 365 days, and
y	= dividend yield	= 3 percent,

results in the following prices and "greeks:"

	Call	Put
Value	$15.21	$8.92
Delta	.640	-.330
Gamma	.011	.011
Rho	.360	-.353
Eta	3.366	-2.963
Vega	.285	.285
Theta	.016	.015

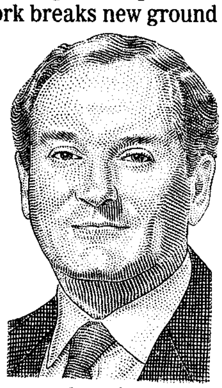
Two U.S. Economists Win Nobel Prize

Merton and Scholes Share Award for Breakthrough In Pricing Stock Options

By MICHAEL M. PHILLIPS
Staff Reporter of THE WALL STREET JOURNAL

Two economists with close ties to Wall Street, Robert C. Merton and Myron S. Scholes, won the Nobel Memorial Prize in Economic Science for pathbreaking work that helped spawn the $148 billion stock-options industry.

The Nobel economics prize is given to innovators whose work breaks new ground and sires whole bodies of economic research. But this year, the prize committee chose laureates not only with distinguished academic records, but also with especially pragmatic bents, to split the $1 million award. Prof. Merton, 53 years old, teaches at Harvard Business School, while Prof. Scholes, 56, has emeritus status from the Stanford Graduate School of Business.



Robert C. Merton

Both are turning theory into action as founding principals in **Long-Term Capital Management** LP of Greenwich, Conn., a highly successful three-year-old hedge fund headed by former traders at Salomon Brothers Inc. The success of Long-Term Capital, where the partners have shared more than $1 billion in profits, makes the two men among the richest Nobel laureates ever.

The Black-Scholes Formula

In their paper, Black and Scholes obtained exact formulas for pricing options.

$$C = SN(d) - Le^{-rt}N(d - \sigma\sqrt{t})$$

According to the formula, the value of the call option C is given by the difference between the expected share value (the first term on the right-hand side of the equation) and the expected cost (the second term) if the option is exercised at maturity.

In the early 1970s, Prof. Scholes, with the late mathematician Fischer Black, invented an insightful method of pricing options and warrants at a time when most investors and traders still relied on educated guesses to determine the value of various stock-market products. Prof. Merton later demonstrated the broad applicability of the Black-Scholes options-pricing formula, paving the way for the incredible growth of markets in options and other derivatives.

"Thousands of traders and investors now use this formula every day to value stock options in markets throughout the world," the Royal Swedish Academy of Sciences said yesterday.

The Black-Scholes option-pricing model "is really the classic example of an academic innovation that has been adopted widely in practice," said Gregg Jarrell, professor of economics at the University of Rochester's William E. Simon Business School and former chief economist at the Securities and Exchange Commission. "It is one of the most elegant and precise models that any of us has ever seen."

Options allow investors to trade the future rights to buy or sell assets – such as stocks – at a set price. An investor who holds 100 shares of International Business Machines Corp. stock today, for example, might buy an option giving them the right to sell 100 IBM shares at a fixed price in three months' time. The investor is therefore partially protected against a fall in the stock price during the life of the option.

Until the Black-Scholes model gained acceptance, the great minds of economics and finance were unable to develop a method of putting an accurate price on those options. The problem was how to evaluate the risk associated with options, when the underlying stock price changes from moment to moment. The risk of an option depends on the price of the stock underlying the option.

Myron S. Scholes

Prof. Scholes and Dr. Black, while working together in an office near the Massachusetts Institute of Technology, suddenly realized that the risk of the option was reflected in the stock price itself. The stock price already includes market participants' expectations about the future of the company that issued the stock. At the time, Prof. Scholes was a junior faculty member at MIT and Dr. Black worked for the Arthur D. Little consulting firm. "That was a great insight for us when we realized we had figured out a way to value these types of instruments by getting rid of the uncertainty," recalled Prof. Scholes, a Canadian-born U.S. citizen.

That breakthrough allowed the economists to create a pricing formula that included the stock price, the agreed sale or "strike" price of the option, the stock's volatility, the risk-free interest rate offered with a secure bond, and the time until the option's expiration. They published their work in 1973, the same year the

Please Turn to Page A6, Column 4

Investment Update (10/15/97)

Two U.S. Economists Win the Nobel Prize For Work on Options

Continued From Page A2

Chicago Board Options Exchange turned the scattered world of options trading into a more formal market.

Prof. Merton once wrote that the Black-Scholes formula, "virtually on the day it was published, brought the field to closure on the subjects of option and corporate-liability pricing."

Prof. Merton himself forged a formal theoretical framework for the Black-Scholes formula, and extended the analysis to other derivative products – financial instruments in which the value of the security depends on the value of another indicator, such as mortgage, interest or exchange rates. More broadly, his work allowed economists and financial professionals to view a wide variety of commonly traded financial instruments – such as corporate bonds – as derivatives and to price them using the ideas first expounded by Dr. Black and Prof. Scholes. "For the most part, the thing was conceived entirely in theory," said Prof. Merton.

The practical implications soon became apparent, however, as market participants flocked to the Black-Scholes-Merton approach to determine how much options are worth. "It's just a terrific yardstick for investors to help make that judgment," said Bill Kehoe, vice president and manager of the options marketing group at Merrill Lynch & Co., and an options trader since 1961.

Options markets have grown astronomically in the quarter century since the formula reached trading floors around the country. The value of U.S. exchange-traded options in 1995 was $118 billion. Last year, it surged to $148 billion, and in the first nine months of 1997, the figure hit $155 billion. More than 100,000 options series are now available. "Even now, we calculate the value of options world-wide using the Black-Scholes formula," said Yair Orgler, chairman of the Tel Aviv Stock Exchange.

"There's this enormous industry that's sprung up from the formula," said Prof. Jarrell, who studied under Prof. Scholes at the University of Chicago. "It has been an enormous generator of wealth, not just for Myron, but for society."

Prof. Merton said that one of his reasons for participating in Long-Term Capital Management was his eagerness to apply his research into both securities pricing and business-management issues. "What better way to do it than to be involved in creating an institution from scratch?" he said. Recently, however, faced with falling returns, the firm decided to return nearly half its $6 billion in capital to investors.

The Nobel announcement was greeted with ebullience by financial economists, who had long argued among themselves that the options-pricing formula was worthy of the top academic honor. "That your work is good enough to warrant considering is the most you can hope for," Prof. Merton said in an interview last year. "Beyond that, it's luck."

The day of celebration, however, did carry a touch of melancholy. Dr. Black died in 1995 after a long battle with cancer. At the time, he was a partner at Goldman, Sachs & Co., where he had worked since 1984. While Nobel rules prohibit posthumous awards, the prize committee mentioned his name frequently enough to make clear that he would have been among the winners had he survived.

"The first thing that came to mind when I heard I had won was I thought of Fischer Black and wished he were sharing the award with us," said Prof. Scholes, who plans to contribute some of his prize money to help fund the MIT chair that bears Dr. Black's name.

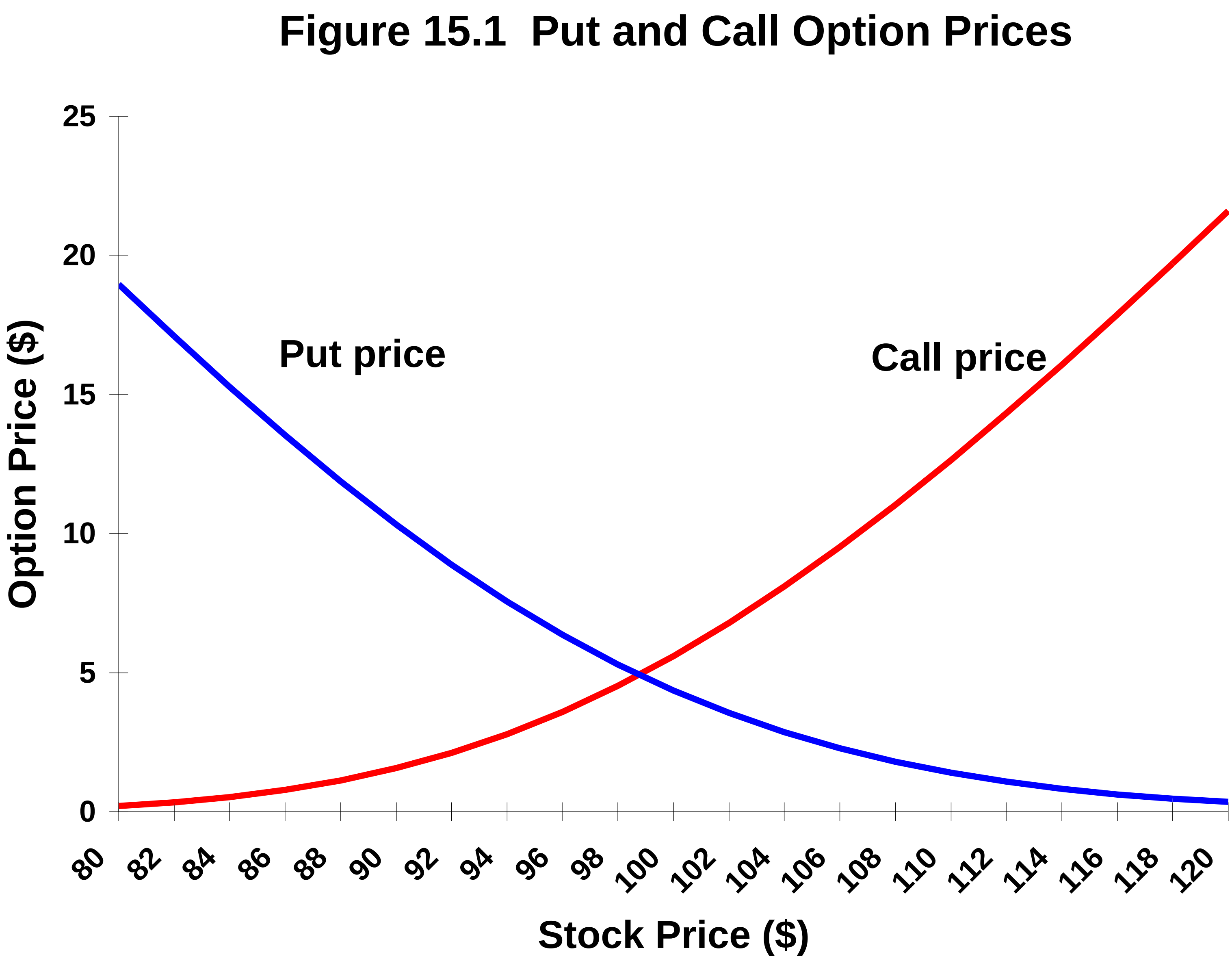


Figure 15.1 Put and Call Option Prices
Option Price ($)
25
20
15
10
5
0
Put price
Call price
80
82
84
86
88
90
92
94
96
98
100
102
104
106
108
110
112
114
116
118
120
Stock Price ($)

Figure 15.2 Option Prices and Time to Expiration

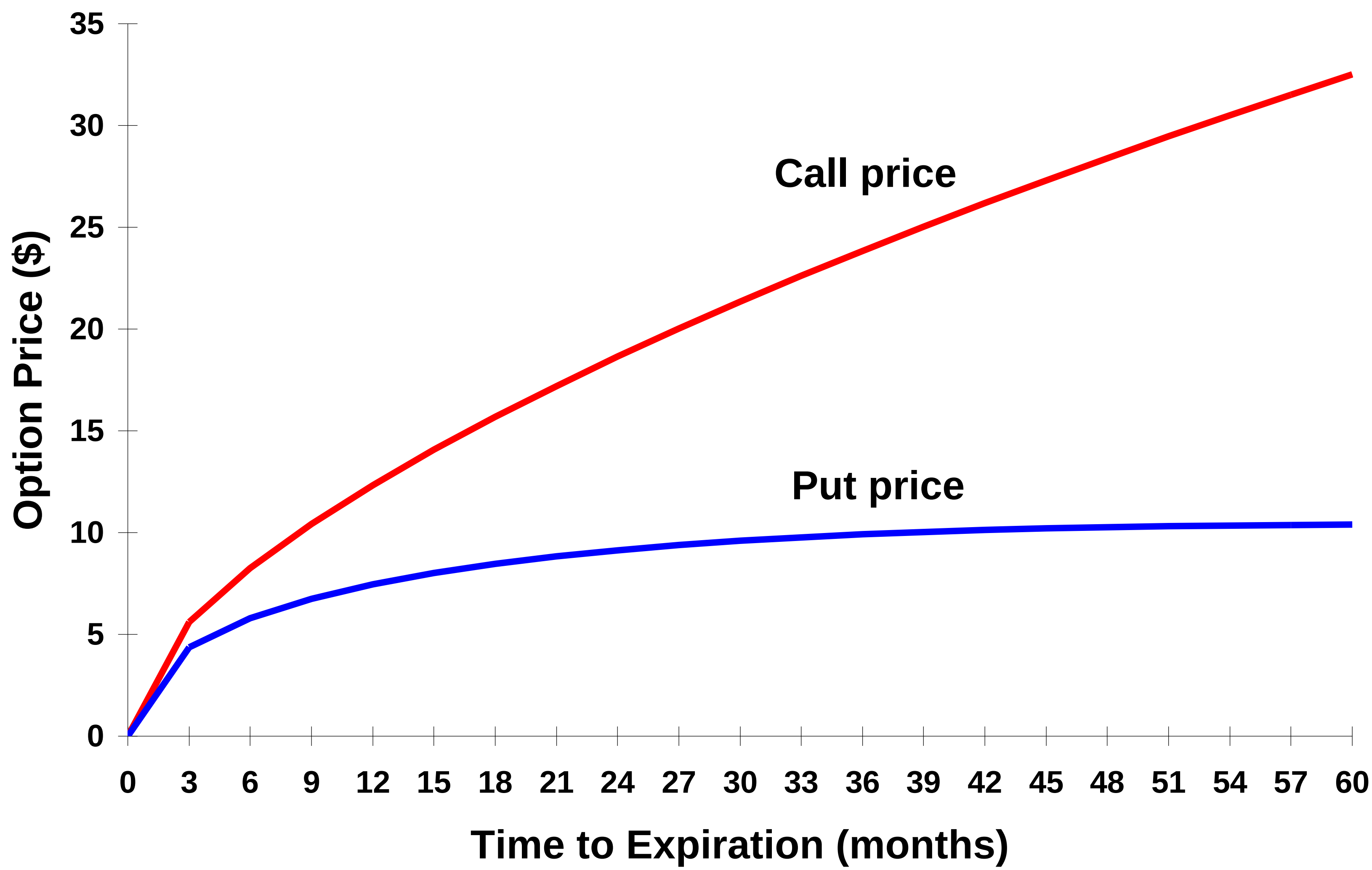


Figure 15.3 Option Prices and Sigma

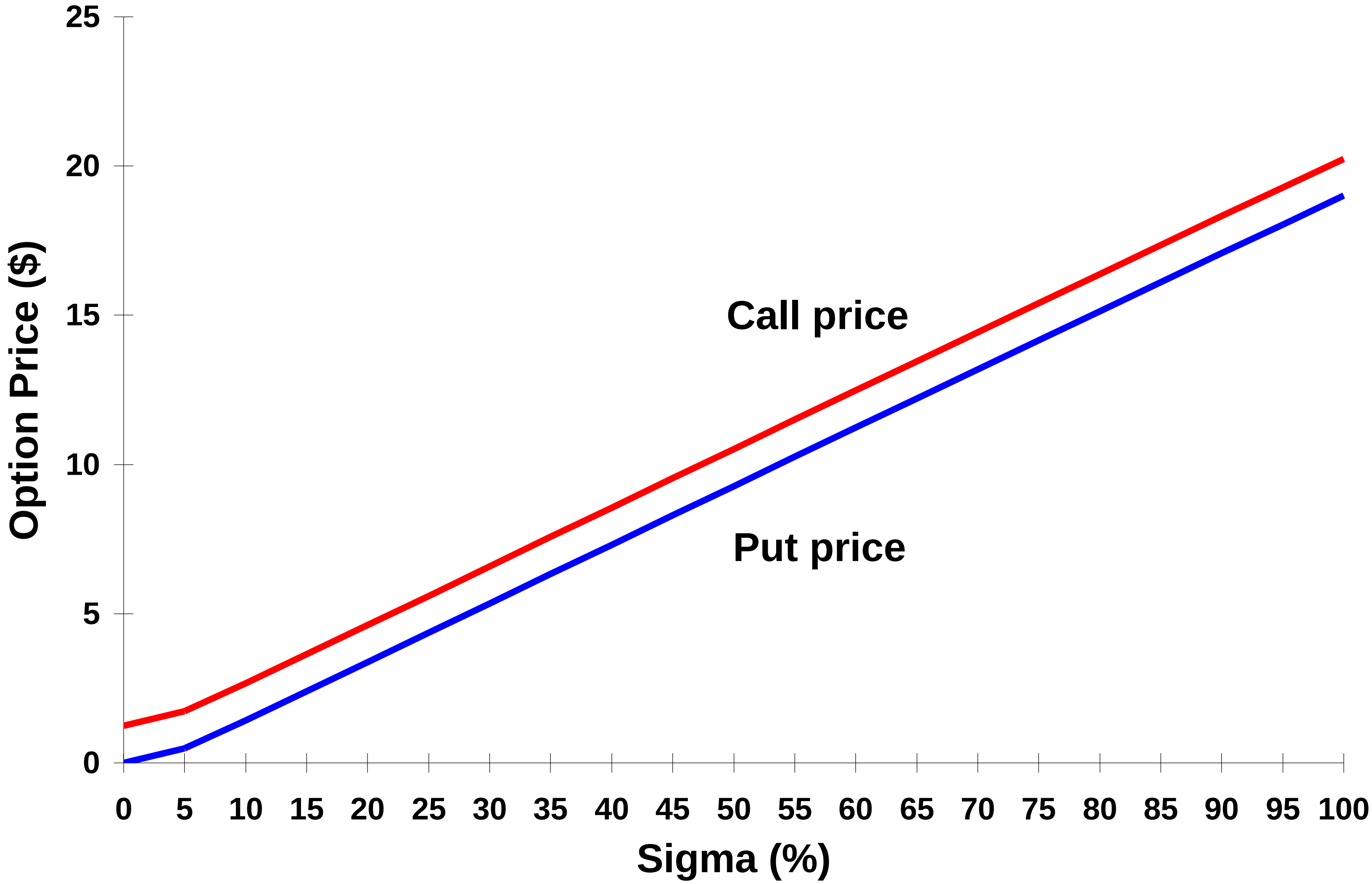
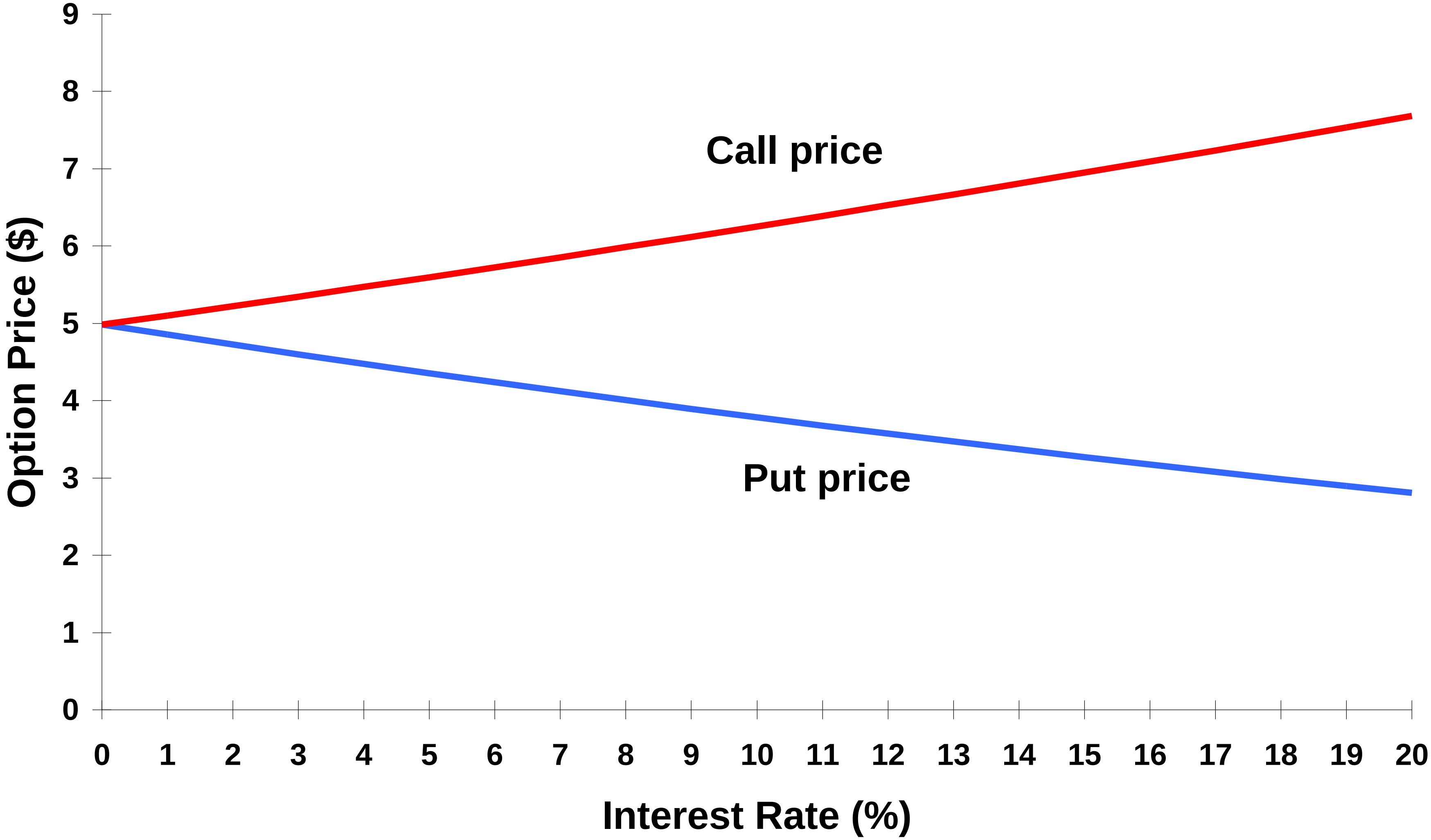


Figure 15.4 Options Prices and Interest Rates



Investment Update (7/29/98)

Money Managers Finally Use Options to Hedge Portfolios as the Stock Market Takes Tumble

By STEVEN M. SEARS
Dow Jones Newswires

NEW YORK—Traders and money managers began using options to hedge their portfolios yesterday after spending the past week ignoring defensive strategies to speculate on earnings and stock-price movements.

The turning point came late in the morning when the Standard & Poor's 500 index slid below 1140. This wiped out many S&P 500 index futures positions and market professionals responded by buying S&P 500 index options to protect their portfolios from the market's volatility.

This hedging activity marked a change in the approach they have taken to the market. Many professionals recently stopped hedging their portfolios because the stock market has quickly corrected in the past. They spent money for hedges they ultimately didn't need.

"A lot of people were completely unhedged when the decline began," said Leon Gross, Salomon Smith Barney's options strategist. He noted that the S&P 500 index's rise to 1186 from 1086 took six weeks, while it dropped 50 points in only four days.

The fear in the options market spiked higher as the S&P index fell along with the Dow Jones Industrial Average.

The option market's fear gauge, the Chicago Board Options Exchange Volatility Index, rose 1.72, or 7.5%, to 24.66. "This is an indication that people are getting nervous and paying for puts," Mr. Gross said.

Options prices reflected this discomfort, which made hedging portfolios even more expensive than normal. For more aggressive traders, such as hedge funds, high options prices created opportunities to short sell puts and sectors.

The Nasdaq index of the 100 largest nonfinancial stocks was a popular way to short the technology sector. Other traders sold put options because they think the fear is overdone and they'll be able to buy the contracts back for less money.

Figure 15.5 Volatility Skews for IBM Options