Do Swedes Smile?

On Implied Volatility Functions

Malin Engström*

Department of Corporate Finance

School of Business, Stockholm University

S-106 91 Stockholm, Sweden

Phone: + 46 8 16 19 75

E-mail: mem@fek.su.se

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Abstract

An examination of implied volatilities for Swedish equity options shows a rather U-shaped smile pattern when the volatilities are averaged within groups according to their moneyness. The detected volatility smile makes the use of at-the-money implied volatilities for valuation of in- or out-of-the-money options questionable. The at-the-money implied volatilities work well for valuing at-the-money options, but the smile pattern might make them inappropriate for other options. This paper investigates whether some kind of deterministic volatility function could lead to more accurate model values than the at-the-money implied volatilities, which are used as a benchmark. Different specifications of volatility functions are fitted over a six months estimation period. These functions are then used during a one-month evaluation period to value options as a test of the out-of-sample fit. Although the benchmark model performs best for the at-the-money options, other models are much better for deep in- and to some extent out-of-the money options. However, no single model works very well for options of all moneyness levels.

1. Introduction

When valuing an option, the volatility of the underlying asset is an important factor. Unfortunately, the volatility is not readily observable on the market and therefore some kind of estimate is needed. One possible estimate is the implied volatility according to some pricing model, i.e. the volatility that makes the model value equal to the market price. The volatility implied by the model can be seen as the market participants' assessment of the volatility and it is dependent on the underlying price process and other assumptions of the model in question. In e.g. the Black-Scholes (1973) model it is assumed that the asset returns are log-normally distributed with constant volatility, and that there are no transaction costs. If these assumptions are correct, all options written on an asset should yield the same implied volatility. However, several studies have found that implied volatilities tend to differ across moneyness and time to expiration. The pattern of implied volatilities across time to expiration is usually referred to as the term structure of implied volatilities, whereas the "skew" or the "smile" refers to the pattern across moneyness levels. However, in the following the expression "smile", used interchangeably with "skew", will include the pattern across both time and moneyness, the so called volatility surface.

The reason for the expression "smile" is the empirical findings for S&P 500 options before the 1987 crash. The implied volatilities for deep in- and out-of-the-money options were found to be higher than the at-the-money options, thus creating a smile-shaped pattern. After the crash, the smile in S&P 500 options has changed to look more like a "sneer" with monotonically decreasing implied volatilities for increasing exercise prices, but the implied volatility pattern is often referred to as the smile regardless of its actual shape. The actual shape of the smile differs between different markets, between different underlying assets and between time periods. One example is that the smile of the implied volatilities for currency options tends to be much more U-shaped than for other underlying assets. This could perhaps be explained by differences in market microstructure.

When computing implied volatilities, there may be problems with differences in liquidity for options of varying moneyness. Options close to being at-the-money are usually traded much more frequently than deep in- or out-of-the-money options. If the prices are stale, they may lead to biases in the volatility estimates for the latter options. In addition, the prices given the Black-Scholes model have shown to be more accurate for some options than for others, and some options are also more sensitive to volatility than others. Therefore, some kind of weighting scheme might be called for. In a review article, Mayhew (1995) discusses different weighting schemes for implied volatilities from options of different moneyness. The simplest way is to estimate the implied volatility for the option that is closest to being at-the-money, and then use this estimate to price all contracts with the same time to expiration regardless of moneyness. Moreover, according to e.g. Beckers (1981) the implied volatility for the nearest-the-money option appears to do as well as a weighted average. However, when there is a pronounced smile pattern in implied volatilities this may not be the case.

One of the assumptions underlying the Black-Scholes model is that the distribution of the price of the underlying asset is lognormal. Hull (1999) discusses two of the conditions for an asset price to have a lognormal distribution; the volatility of the asset should be constant and the price of the asset should change smoothly with no jumps. Usually this is not the case, and both factors tend to make extreme outcomes more likely. However, the impact depends on the time left to expiration. The percentage impact of stochastic volatility on option prices increases as the maturity increases, whereas the volatility smile created by stochastic volatility usually becomes less pronounced as the time to expiration increases. The effect of jumps however, is less pronounced for longer options because the jumps over a longer time period tend to get “averaged out”. Excess kurtosis in the underlying return distribution may be one explanation to the existence of smile patterns. There will then be a higher probability of extreme observations than in the Black-Scholes case, which in turn will increase the value of out-of and in-the-money options relative to at-the-money options. The result would be a U-shaped smile in implied volatilities. However, the underlying distribution may also be skewed, and this skewness will make one side of the smile more pronounced. The result would then be an asymmetric smile, which has also been found in several studies.

When pricing options, the volatility smile has to be incorporated in some way. This is done by incorporating for example stochastic volatility, see e.g. Scott (1987), Ball and Roma (1994), and Hull and White (1999), stochastic interest rates, see e.g. Amin and Jarrow (1992) or jumps in the price of the underlying asset, see e.g. Merton (1976). As discussed in Mayhew (1995), if the price of the asset underlying the option is assumed to follow a sufficiently complicated process, nearly any type of volatility smile can be generated. However, these multifactor models include several parameters that have to be estimated. In addition, there may be problems since there are no securities with which to directly hedge the volatility or jump risk.

The purpose of this paper is to examine the volatility smile implied for Swedish equity options. Differences in e.g. market microstructure may cause the smile pattern for Swedish options to be quite different from those detected in previous studies on other markets. A pronounced smile makes the use of at-the-money implied volatilities for valuation of in- or out-of-the-money options questionable. These volatilities work well when valuing at-the-money options, but the smile pattern might make them inappropriate for other options. It is therefore analysed whether implied volatility functions could lead to more accurate option valuation than the implied volatilities from at-the-money options.[1] Like many other options, the Swedish equity options are American. Many studies analysing American options choose to exclude options were early exercise might be optimal, and use European pricing models to compute the implied volatility. However, in this study, an American pricing model is used and options of all moneyness levels are included. For deep in-the-money options, where the early exercise premium is high, it can be hard to find an implied volatility. For these options, the use of an implied volatility function could be especially fruitful, and in the paper, deep in-the-money options with high early exercise premiums are therefore examined especially. The call options should not be exercise early during the analysed period since the underlying stocks do not pay any dividends. The paper will therefore focus on put options.

The American version of the Cox et al. (1979) binomial model is used in the iterations for the implied volatility. A U-shaped pattern in implied volatilities is indicated when these are averaged within groups according to the moneyness of the options. In accordance with this pattern, different specifications of deterministic volatility functions are fitted over a six months estimation period. These functions are then used to value options during a one-month evaluation period as a test of the out-of-sample fit. To compare the different methods, the at-the-money implied volatility is used as a benchmark. Unfortunately, the results of for example Rubinstein (1985) and Dumas et al. (1998) indicate that volatility functions implied

[1] Engström and Nordén (2000) find that some American option pricing models work rather poorly for deep in-the-money options. However, at-the-money implied volatilities are used in the valuation, and with a pronounced volatility smile, this could perhaps explain the poor fit for these options.

by option prices are non-stationary. The functions fitted to the estimation period data might therefore not be very good in describing the volatility for the evaluation period. In addition, therefore, it is tested how well implied volatility functions estimated over the last month of the estimation period value options in the evaluation period.

The results of using volatility functions rather than at-the-money implied volatilities are mixed, and as expected they depend on the moneyness of the options that are valued. For rather deep in-the-money options, some of the volatility functions outperform the benchmark model. Overall, the benchmark is outperformed for up to 50% of the observations depending on model. However, no single model works very well for options of all moneyness levels. Since the volatility functions that perform well for deep in-the-money options perform very badly for at-the-money options, it is hard to say that the use of a function rather than the at-the-money volatility leads to an improvement.

The paper is divided into eight sections. In the following section, some previous studies of implied volatility functions are discussed. In the third section, the institutional setting and the data is presented, whereas the methodology is discussed in section 4. The fifth section discusses the shape of the smile and in section 6 possible specifications of the volatility functions are presented. The results of the out-of-sample tests are in section 7, where also some discussion about robustness can be found. Finally, in section 8, the study is ended with some concluding remarks.

2. Previous studies

Black (1975) discusses the possible need for different volatilities for options with different remaining time to expiration. If e.g. the volatility has been unusually high lately, a gradual decline back to more normal levels may be expected. The reverse may be expected in periods of unusually low volatility. This mean reversion in volatility levels might therefore explain some of the term structure patterns in implied volatility detected in other studies.

MacBeth and Merville (1979) analyse market prices of equity call options written on six different stocks, and compare these to the prices predicted by the Black-Scholes model. The implied volatility is found to be different across moneyness and time to expiration. For all options, there is a tendency for the implied volatility to decrease as the option becomes less in-the-money. The time to expiration relationship however, depends on the moneyness of the calls. For example in-the-money options with a short remaining time to expiration tend to have higher implied volatilities than corresponding options with a longer time to expiration. For out-of-the-money options, the relationship is reversed.

Rubinstein (1985) examines implied volatilities for equity call options traded on the CBOE using a very rich data set. Six different model specifications besides the Black-Scholes model are used, but none of them seem to be able to capture all of the observed biases from the Black-Scholes model. Although the biases were found to be statistically significant, there was no evidence of economic significance. The biases across exercise prices from the Black-Scholes values are also found to be non-stationary. The direction of the bias is usually the same for most of the underlying stocks at a certain time, but the direction is different for different periods in time. For out-of-the-money calls, the conclusion is that shorter time to expiration corresponds to a higher implied volatility for the entire analysed period. For at-the-money calls, however, this is true only for the second half of the analysed period, whereas the opposite is true for the first half. In the first half, it is usually the case that a lower exercise price leads to a higher implied volatility if the options have the same time to expiration, whereas the opposite is true for the second period.

Geske and Roll (1984) demonstrate that an improper treatment of the early exercise possibility of American call options could explain some of the previously detected biases. If the “American version” of the Black-Scholes model (the so-called Black’s approximation) rather than e.g. the corrected version of the Roll (1977) American call option model is used, the probability of early exercise may influence the relation between implied volatility and for example exercise price. In time periods when the levels of interest rates and dividend payments make early exercise more likely, the implied volatility is expected to be directly related to the exercise price. During periods when the probability of early exercise is less likely, implied volatility is inversely related to the exercise price. This finding could perhaps explain the results of for example MacBeth and Merville (1979) and Rubinstein (1985).

Rubinstein (1994) analyses S&P500 index options for the latter part of the eighties, and finds that the crash of 1987 appears to have increased the market participants’ view of the possibilities of another market crash. This “crash-o-phobia” causes a slightly bimodal implied distribution to be quite common after the crash. The precrash “smile” pattern in the implied volatilities also appears to have changed into a “sneer” in the postcrash period.

Duque and Paxson (1994) examine the smile using the most actively traded equity call options on LIFFE in London. A European model is used to find the implied volatility after excluding options with possible early exercise. The shape of the smile is usually a relatively high implied volatility for in-the-money options, but there is also a “wry grin” (the implied volatilities for in-the-money options are smaller than for out-of-the-money options) or “reverse grin” (in-the-money volatilities are higher than for out-of-the-money options). Different trading strategies based on the relative implied volatilities are then tested as pseudo arbitrage of the smile. However, after transaction costs, most abnormal returns are eliminated.

Bates (1996) surveys different schemes to aggregate implied volatilities for option with different exercise prices and maturities into one assessment of volatility proposed in the literature. Most involve some kind of weighting scheme, where options of different moneyness and/or maturity get different weights. Also the effect on the implied volatility of the choice of short term interest rate in the pricing formula is discussed, as well as the choice of stochastic interest rate models. The findings concerning the forecasting abilities of implied volatilities are mixed. However, almost all studies find that the implied volatility contains information about future volatility. Evidence of both more and less positively skewed distributions than the lognormal has also been found in several studies. The results suggest that no single alternative hypothesis about the underlying distribution can eliminate the found biases across exercise prices. Bates (1996) suggests that time-varying skewness might be used to complement models of time-varying volatility.

Corrado and Su (1996) derive and empirically test a model that extends the Black-Scholes model in that non-normal skewness and kurtosis is taken into account. The adjustments are found to remove most of the biases across exercise prices for S&P 500 index options. In addition to implied standard deviations, implied coefficients for skewness and kurtosis are computed simultaneously.

Bakshi et al. (1997) examine the pricing and hedging performance of different option pricing models compared to the Black-Scholes model for S&P 500 call options. These other models allow for stochastic volatility, stochastic interest rates and jumps in different combinations. Because a common motivation for newer models is the smile pattern found using the Black-Scholes model, the smile pattern in the implied volatility found given the different models is analysed especially. The Black-Scholes implied volatility exhibits a clear U-shaped pattern across moneyness, with the most distinguished smile evident for options near expiration.

Dumas et al. (1998) develop a deterministic volatility function (DVF) model for option pricing. This model is fitted to different specifications of the volatility, and evaluated using S&P500 index options. The simplest model uses the implied volatility from the Black-Scholes model, whereas the others also try to capture the effect of moneyness and time to expiration. A relatively parsimonious volatility function works well for describing the observed volatility structure, whereas the Black-Scholes constant volatility model is better for determining hedge ratios. However, the results indicate that the volatility functions implied by the options prices are not stable over time. An “ad hoc” model, of a kind often used in practice, that simply smoothes the implied volatility across moneyness and maturity gives rise to pricing errors that are smaller than those of the DVF approach. Overall, the authors conclude, “simpler is better”.

Peña et al. (1999) analyse the determinants of the smile pattern in implied volatility using data on the Spanish IBEX-35 stock exchange index. The volatility function is estimated by fitting the implied volatility to six alternative models. Regardless of specification, a model including moneyness by both a linear and a quadratic term is preferred to one with just a linear term, which implies that the Spanish smile is characterised by curvature. The minimum implied volatility is found to be very close to the at-the-money implied volatility. A day-of-the-week pattern in the volatility function is also found. The curvature of the smile on Mondays is significantly different from the smiles in the end of the week. However, this seasonality appears to disappear when some economic variables are included in the analysis. A bi-directional Granger causality between transaction costs (proxied by the bid-ask spread) and the curvature of the volatility smile is also found. However, the results suggest that market conditions and transaction costs are relatively more important for options with a short remaining time to expiration. In a related study, Peña et al. (2001) test the influence of the relative bid-ask spread (as a measure of liquidity costs) on the volatility smile by fitting different models for the implied volatility incorporating also the bid-ask spread. However, these models are not found to be superior to the basic Black-Scholes implied volatilities.

Derman (1999) discusses how the future skew, given the current one, will vary with the underlying index level. It is argued that it often is easier to model change by describing what does not change - in options trading referred to as “sticky”. Three suggestions about what in the skew that is considered sticky when the underlying changes are presented; sticky-strike, sticky-delta, and sticky implied tree. Implied volatilities of S&P 500 index options over the period September 1997 to October 1998 are examined, as well as the appropriate rule followed by the market. In the sticky-strike approach, it is assumed that the volatility of an option with a particular exercise price stays the same regardless of changes in the underlying. The at-the-money volatility will therefore change as the underlying changes. This not very realistic model could at most be used for approximating the smile for exercise prices that are close to each other. The sticky-delta rule implies that the current level of at-the-money implied volatility should remain the same regardless of moves in the underlying. An option that is 10% out-of-the-money when the underlying has moved should have the same implied volatility as the option that was 10% out-of-the-money before the change. In the sticky-implied-tree approach, the at-the-money implied volatility decreases twice as rapidly as the level of the underlying increases. The first two approaches are more of heuristic rules than actual theories, whereas the implied-tree model provides a consistent, although not necessarily accurate, view.

Analysing equity call options traded on LIFFE, Duque and Lopes (1999) use the Hull and White (1998) stochastic volatility option pricing model to find the theoretical relations that should exist between shape of the smile, the level of volatility and the time to expiration. If early exercise could be optimal, the option is excluded. Cubic B-splines are used to model the implied volatilities for specific moneyness degrees using observed implied volatilities for other moneyness degrees. As the time to expiration decreases, the magnitude of the smile is found to increase. This gives empirical support for the notion that options tend to die smiling. However, the changes in the smile pattern are asymmetric. The “wry grin” found for longer-term options is converted into a “reverse grin” for options near expiration. For medium term options, the smile is more symmetric. Furthermore, a statistically significant positive relation between the smile and the volatility of the underlying stock is also found.

Hafner and Wallmeier (2000) analyse the pattern of DAX implied volatilities across exercise prices, and its determinants. They find that, on average, 95% of the cross-sectional variation in implied volatilities could be explained by variation in moneyness. The great impact of the 1987 crash on the implied volatility pattern indicates that market participants' assessment of crash risk is affecting the smile. However, because of lack of good proxies, this assessment is difficult to incorporate into a model.

Dennis and Mayhew (2001) investigate systematic and firm specific factors that could explain the volatility skew observed for equity options traded on the CBOE. The implied volatilities are computed using a 100-time steps binomial model, which account for early exercise. They find that options written on high beta stocks have more negative volatility skews than options written on stocks with lower betas, indicating that market risk is an important factor in explaining the skew. However, beta is only a significant factor in the post-crash of 1987 period, which indicates that option holders became more concerned with market risk after the crash. They also find that skews on individual equity options tend to be more negative when the skew on index options is more negative. However, the variation in firm specific factors appears to be more important than variations in systematic factors in explaining the variation in the skew.

3. Institutional setting and data

During the analysed period, options on 27 individual stocks were listed on OM. The underlying stocks were in turn listed on the Stockholm Stock Exchange (StSE). The average daily trading volume for these options was around 51,000 contracts, and approximately 70% of these were calls.[2] Comparisons with CBOE shows that the average daily trading volume

[2] During the period in question, the average daily trading volume on the StSE for the underlying stocks was SEK 2.6 billion for the on average 10,470 transactions per day. The market capitalisation for the 228 listed Swedish stocks at the end of year 1995 was SEK 1,179 billion (which at that time was equal to $177 billion).

was around 305,000 equity option contracts at that time, of which around 90,000 were put options.[3]

The trading at the StSE, as well as the option trading at OM, took place between 10.00 a.m. and 4.00 p.m. CET during the period in question. Series of call and put option contracts with an initial time to expiration of six months were listed every three months. The option expiration day is the third Friday of the expiration month. After receipt of exercise notices from the option holders, OM assigns the exercises randomly among the options writers. OM thus functions both as the exchange and the clearinghouse.

The data set used in the study consists of prices of put and call options during the sample period July 1, 1995, to February 1, 1996. Only the 10 underlying stocks with the most frequently traded put options on the market at that time are used.[4] Daily closing bid/ask quotes of the options are obtained from OM, whereas daily closing bid/ask quotes and transaction prices of the underlying stocks are obtained from the StSE. Dividends are paid only once a year and most dividend payouts occur around May. Hence, no dividend payments are made during the sample period. Finally, daily rates of the Swedish one-month Treasury bills are used as a proxy for the riskless interest rate.

The sample period is divided into an estimation period: July 1 to December 31, 1995, and an evaluation period: January 1 to February 1, 1996. Since the relation between implied volatilities for puts and corresponding calls is analysed in the estimation period, only days for which non-zero quotes for both puts and calls exist are included in the sample. To avoid possibly disturbing effects of the trading immediately before expiration, options with less than a week left to expiration are omitted. After a screening procedure, observations that do not satisfy the American put-call parity boundary condition are excluded. In addition, observations are omitted if the option value according to the binomial model is less than SEK 0.01, the lowest tick size allowed at OM. Moreover, for some contracts (when the exercise value of the option exceeds both the model value without exercise and the market value) it is not possible to obtain an implied volatility. These contracts are removed from the database. Only put options are included in the evaluation sample, and the only restrictions for this

[3] Source: OM Factbook and Futures and Options World, February 1996 respectively.
[4] The stocks are Astra A, Electrolux B, Ericsson B, Investor B, MoDo B, SCA B, Skandia, SEB A, Trelleborg B, and Volvo B.

sample is that the options should have at least a week left to expiration and the binomial model prices should not be less than SEK 0.01. In total, this leaves us with an estimation sample consisting of 10,735 contracts and an evaluation sample consisting of 2,519 contracts.

Table 1 offers a summary of the option data. Panel A displays the data for the estimation period whereas Panel B contains data for the evaluation period. The observations are divided into six subgroups according to moneyness of the put options and four subgroups according to the number of days left to expiration. Moneyness is defined as the ratio of the exercise price to the stock price, where the average of the closing bid and ask quotes is used. A put is defined to be at-the-money when its moneyness is between 0.98 and 1.02. For the estimation period, around one third of the puts are out-of-the-money while over 50% of the observations belong to the in-the-money groups. Since options with a high probability of early exercise are of special interest, the moneyness groups are dispersed unevenly, with an additional group for very deep in-the-money options. The dispersion of the options into different subgroups according to moneyness is roughly the same for the options in the evaluation period.

Options with extreme moneyness levels are usually excluded from this kind of studies. However, as we are interested in examining if implied volatility functions are superior to at-the-money implied volatilities in valuing options with different moneyness levels, especially deep in-the-money options, we need to include also options that are rather far from being at-the-money. For at-the-money options, the at-the-money implied volatility should be the best volatility input for the pricing model. However, the question is whether the smile pattern in implied volatilities makes the valuation of in- or out-of-the-money options inaccurate using these volatilities. A wide range of moneyness levels is therefore included.

4. Methodology

To be able to investigate if the use of implied volatility functions is superior to using the implied volatility for the option closest to being at-the-money, this volatility is used as a benchmark. The implied volatility is for all options found using the average of the closing bid and ask prices for both the options and the underlying stocks. Since the options are American, the volatility is found by equating the Cox et al (1979) binomial model value and the market price of the option after allowing for early exercise. The remaining time to expiration is at all times divided into 100 time steps.

The same model is used to value put options using different specifications of implied volatility functions. The model values are compared to the benchmark values given the nearest-the-money implied volatility, and to market values respectively. Since the purpose is to analyse possible advantages with the use of an implied volatility function rather than at-the-money implied volatility, options are valued using the implied volatility of the same day rather than using the volatility from the day before.

Many previous studies use S&P 500 index option data since this is a widely traded option. The high trading frequency enhances the possibility of simultaneous prices. Since the relationship between option prices and prices of the underlying is used in the computation of the implied volatility, they should all be simultaneous.[5] The results could otherwise be misleading, and show an over- or underestimated volatility. The use of Swedish equity option data, where trading is not as frequent as in the US, could lead to difficulties with non-synchronised transaction prices since it is improbable that the prices of option and underlying stock are exactly simultaneous. Ideally, time stamped option- and stock prices should be used but, unfortunately, no such record of prices is available. In order to avoid too much time difference between the prices, the average of the daily closing bid/ask quotes is used for both options and underlying stocks. In addition, only the most frequently traded options are analysed. However, the bid/ask quotes are only valid for a certain number of options, and in addition, several transactions may be executed within the spread. It is also possible that market markers do not price illiquid options as thoroughly as the more frequently traded ones. Nevertheless, the choice of bid/ask quotes instead of transaction prices is likely to improve the analysis.

5. Patterns in implied volatility

In Table 2, the implied volatilities for both put and call options are averaged over the different time and moneyness groups described above. Panel A contains volatilities for the options in the six months estimation period. For put options, the results indicate a rather clear

U-shaped smile pattern, with the lowest average implied volatility found for the at-the-money options in all time groups except the longest time to expiration group. For call options, the pattern is similar to that for the puts although not as pronounced. Because moneyness is defined as the ratio of the exercise price to the stock price for both put and call options, the call options in the parentheses next to the deep out-of-the-money puts are deep in-the-money, and vice versa.

The average implied volatilities differ also for the different maturity groups. For both put and call options, the longer the time left to expiration, the lower the implied volatility tends to be in each moneyness group. In addition, the shorter the time left to expiration, the more pronounced the U-shape of the smile appears to be. This pattern is sometimes referred to as the options are dying smiling.

Differences in volatility level for the included underlying stocks may have influenced the results in Table 2. If e.g. options written on high volatility stocks have shorter time left to expiration than the other options this could explain the term structure effects mentioned above. Therefore, the pattern across moneyness and time to expiration is analysed also for relative volatilities, i.e. the implied volatility of each contract divided by the at-the-money implied volatility for that option series. Differences in the level of implied volatilities between time groups could of course not be detected using relative volatilities, but the smile patterns in the different groups are very similar to the results in Table 2.[6]

For comparison, the average implied volatilities for put options in the evaluation period are presented in Panel B of Table 2. The pattern is similar to that in Panel A in that the shorter time groups indicate a smile pattern between moneyness and implied volatility. However, for the group with the longest time left to expiration, the average implied volatility is increasing as the options get deeper in-the-money. This is the case also when relative volatilities are used. However, as can be seen in Table 1, this group does not contain many observations.

Because of put-call parity, European put and call options written on the same underlying asset and with identical exercise prices and expiration days should have the same implied

[5] For a discussion on the effect of using non-simultaneous prices see Harvey and Whaley (1991).
[6] The results for the relative volatilities are not presented, but are available upon request.

volatility. For American options, on the other hand, put-call parity does not hold because of the possibility of early exercise. Since corresponding put and call options are written on the same underlying stock, there is no clear reason why implied volatilities should differ depending on which of the two is used to find it. However, if the implied volatilities reflect the expected volatility, it may differ if traders with different beliefs choose to trade in different types of options (puts or calls).[7]

If the implied volatility of the put option in a put-call pair is compared to the corresponding call volatility, the difference is less than 1 percentage point for about a third of the options. However, according to the average values in Table 2, the implied volatilities for put options are higher than for corresponding call options. The greatest differences are found for deep in-the-money options. This might to some extent be explained by the higher value of early exercise premium embedded in the put option prices. Since there are no dividend payments during the analysed periods, the call options should not optimally be exercised early and the call prices therefore include no early exercise premium.

As mentioned above, Geske and Roll (1984) argues that the results of for example Rubinstein (1985) might be explained by the different probabilities of early exercise in different sub periods. The reason would be that possible early exercise has been taken into account inadequately by using Black’s approximation. In this study, a version of the binomial model that allows for early exercise is employed so this should not be the explanation to the detected smile pattern in implied volatilities. However, if option writers were concerned about the risk of being exercised against when writing an option, they would perhaps demand a comparatively higher price for the options during periods when early exercise is more likely. This would in turn yield a higher implied volatility.

To test whether the difference between implied volatilities for put and call options could be explained by the higher probability of early exercise for put options, the volatility difference is regressed on the empirically estimated early exercise premium (EEP) in the put option price. The early exercise premium for each option is estimated as the deviation of the put option market price from the put price according to the European put-call parity relationship. Due to the absence of dividend payments during the sample period, the call options can be

[7] See e.g. Easley, O’Hara and Srinivas (1998).

seen as European options. It is therefore assumed that the deviation from European put-call parity is caused by the early exercise premium in the American put option price.[8] Given this specification, the regression results are:

$$s_{impput} - s_{impcall} = -0.0052 + 0.0265\,EEP \qquad R^2 = 0.4061$$
$$(0.000) \quad (0.000)$$

where p-values using White (1980) heteroskedasticity-consistent standard errors are in parentheses. The results indicate that the difference between the implied volatilities for corresponding put and call options is positively related to the early exercise premium in the put option price. The possibility of early exercise can make it hard to find implied volatilities for deep in-the-money options, and the use of volatility functions could be especially fruitful for these options. Since the call options optimally should not be exercised early, there should not be much problem in determining the implied volatilities for the calls. The remainder of the study therefore concentrates on put options only.

6. Possible specifications of the volatility function

To estimate the shape of the implied volatility function, the data is fitted to several different specifications. In this study, options written on different underlying stocks are included into the sample, and to be able to handle different levels of volatilities for these stocks, the focus will be on relative volatilities. This means that when fitting the models, the explanatory variables are regressed on the relative volatilities rather the actual implied volatilities. It is not obvious how the volatility functions should be modelled, and since the smile patterns is different for different markets, the shape of possible volatility functions also differ. According to the results in Table 2, the time to expiration and the moneyness of the options are very important parameters, but exactly how they should be incorporated could be discussed. Most of the specifications used in this study are suggested in either Dumas et al. (1998) or Peña et al. (1999). As pointed out in the latter study, the specifications might be simplified by including the natural logarithm of the moneyness ratio, rather than the ratio itself, in the function. For an option that is exactly at-the-money, the regression intercept using this specification will represent the at-the-money implied volatility since the natural

[8] For a more elaborate discussion about this method, see Zivney (1991) or Engström and Nordén (2000).

logarithm of the moneyness ratio will be zero. Both the specifications with moneyness as one of the explanatory variables and those with the natural logarithm are tested, in- and out-of-sample, but the version that includes the natural logarithm of the moneyness ratio yield better results overall. Only these specifications are therefore employed in the following.

As a starting point, the following specifications are chosen:

Model 1: $s = s_{ATM}$

Model 2: $$\frac{s}{s_{ATM}} = a_0 + b_1 \ln\frac{X}{S} + e$$

Model 3: $$\frac{s}{s_{ATM}} = a_0 + b_1 \ln\frac{X}{S} + b_2\left(\ln\frac{X}{S}\right)^2 + e$$

Model 4: $$\frac{s}{s_{ATM}} = a_0 + b_1 \ln\frac{X}{S} + b_2\left(\ln\frac{X}{S}\right)^2 + b_3(T-t) + b_4 \ln\frac{X}{S}(T-t) + e$$

Model 5: $$\frac{s}{s_{ATM}} = a_0 + b_1 \ln\frac{X}{S} + b_2\left(\ln\frac{X}{S}\right)^2 + b_3(T-t) + b_4(T-t)^2 + b_5 \ln\frac{X}{S}(T-t) + e$$

Model 1 is the benchmark, the implied volatility of the at-the-money option. In Model 2, a linear relationship, i.e. a “sneer”, between volatility and moneyness is assumed. This specification is not consistent with the patterns in Table 2, but to rule out the possibility that a sneer would be a better description of the implied volatilities across moneyness than a smile, also this model is tested. One drawback with specifications where the volatility is linearly related to the moneyness is that at extreme moneyness values, the volatility may become negative. To prevent negative volatilities, a minimum value could be imposed. However, this is not a problem in this study. In Table 2, the pattern of implied volatility across moneyness shows a great deal of curvature. In Model 3, this curvature is accounted for by including also a quadratic moneyness term. Table 2 also indicates that the shape of the smile depends on the time left to expiration. Model 4 and 5, try to capture these time effects by including also time to expiration parameters, linear and quadratic, and cross terms with both time and moneyness. Several previous studies have found that the smile pattern may be asymmetric, and some additional models are therefore tested:

Model 6: $$\frac{s}{s_{ATM}} = a_0 + b_1 U + b_2 D^2 + e$$

Model 7: $$\frac{s}{s_{ATM}} = a_0 + b_1 U + b_2 \left(\ln \frac{X}{S} \right)^2 + e$$

Model 8: $$\frac{s}{s_{ATM}} = a_0 + b_1 U^2 + b_2 D + e$$

In these specifications $U = (U_1, \ldots\ldots, U_n)$ and $D = (D_1, \ldots\ldots, D_n)$, where,

$$U_i = \ln \frac{X_i}{S_i} \quad \text{if} \quad \ln \frac{X_i}{S_i} < 0 \text{ ,} \quad \text{and} \quad U_i = 0 \quad \text{if} \quad \ln \frac{X_i}{S_i} \geq 0$$

$$D_i = \ln \frac{X_i}{S_i} \quad \text{if} \quad \ln \frac{X_i}{S_i} \geq 0 \text{ ,} \quad \text{and} \quad D_i = 0 \quad \text{if} \quad \ln \frac{X_i}{S_i} < 1 .$$

Model 6, for example, assumes that the implied volatility is linear in moneyness up to the at-the-money mark. For moneyness levels above that, the volatility function gets some curvature. Also for the asymmetric models some time dependence might be included. However, to reduce the number of considered models, linear and quadratic time to expiration parameters are added to the asymmetric model with the best in-sample fit only. The results of fitting the relative volatilities to the different models are presented in Table 3. According to the table, Model 6 is the asymmetric model with the highest R^2 and the model specification is therefore given as:

Model 9: $$\frac{s}{s_{ATM}} = a_0 + b_1 U + b_2 D^2 + b_3 (T - t) + b_4 (T - t)^2 + e$$

According to Table 3, all coefficients are highly statistically significant. However, according to the coefficients of determination Model 2, which assumes a “sneer” pattern, is not a good model of the pattern of volatilities across moneyness compared to the “smile” pattern assumed in Model 3. Moreover, the three models that contain time parameters all have better in-sample fit than the models without it. However, Dumas et al. (1998) find that more parsimonious models perform better in out-of-sample tests. Especially, time parameters are found to be over-fitting the model. All eight models are therefore used in the out-of-sample tests.

Since relative volatilities are used in the fitting procedure, there is no obvious reason why the observed smile effects should be different for different underlying stocks.[9] However, as a check, the models are fitted for each of the underlying stock as well. The coefficients of determination for the different models are in Table 4 displayed for each underlying stock. Although the in-sample fit for the models when fitted like this is better for most of the stocks, the patterns are similar to the results of Table 3. Again, the models that include time parameters are superior to the others. However, for three of the stocks, the coefficients of determination are lower overall than for the other stocks (Ericsson, SEB and Skandia). An examination of the moneyness levels of the options written on different stocks results in no apparent correlation between the moneyness range or average moneyness in the group and model R^2. Duque and Lopes (1999) find that the smile gets more pronounced when the volatility increases. To examine if this could be the reason for the inferior fit for three of the underlying stocks, the average volatility and the average at-the-money volatility is computed for the options written on each of the stocks. However, as in the case with the moneyness levels, no relation can be found.

7. Out-of-sample tests

More than a good in-sample fit is required from an implied volatility function. Of greater interest is the goodness-of-fit in an out-of-sample period. To test the out-of-sample fit, it is tested whether or not the different volatility functions could improve the valuation of puts using the binomial model during the one-month evaluation period. The volatility to use for each option is determined by multiplying the pricing parameters by the appropriate model coefficients. Since the volatility models are specified in terms of relative volatilities, the resulting values are multiplied by the implied volatility for the option closest to being at-the-money in the option series the option belong to. Binomial model values are computed using the volatility given by the eight suggested volatility functions as well as the at-the-money implied volatility. These model values are compared to the actual bid and ask market prices

[9] However, as mentioned above, Dennis and Mayhew (2001) argues that the beta of the underlying stock influence the shape of volatility smile. This could make some model to work better for certain stocks.

of the options. Remember though that it is not the performance per se that is interesting, but the performance of the models relative the benchmark.[10]

In Table 5, four measures of the goodness-of-fit are presented for each volatility function. In Panel A, the root mean squared errors (RMSE) for the difference between the model values and the average of the market bid and ask prices are presented. Not very surprisingly, models 2 and 8 show the highest RMSE. On the other hand, five of the models actually yield option values with smaller RMSE than the benchmark at-the-money volatility. They are therefore considered better in terms of RMSE. In Panel B, instead the mean absolute errors (MAE) are displayed. According to this criterion, no model performs better than Model 1, but Model 6 is fairly close with a MAE that is only SEK 0.01 higher than for Model 1. In Panel C, the percentage of model values within the market bid/ask spread is presented for each model. No model produces a higher percentage than Model 1, although models 3, 6 and 7 are the closest with around 75% of the values within the spread. The results in Panels A, B and C suggest that the results for Model 1 could be driven by comparatively few but rather large deviations between model value and bid/ask midpoint. To investigate this, the squared deviations between model value and market midpoint is compared for Model 1 and the model in question for each contract day.[11] Panel D shows the percentage of contract days that each model outperforms Model 1 in terms of daily squared deviation. Model 6 is the "best" model according to this criterion, with a lower daily squared error than Model 1 during nearly 50% of the contract days.[12] However, this implies that Model 1 is the superior model during the remaining 50% of the contract days.

It was clear from the beginning that Model 1 would work well for at-the-money options. However, it is more interesting to compare the performance of Model 1 to other models when it comes to in- and out-of-the-money options. The RMSE and the percentage of model values within the bid/ask spread are therefore computed for each of the time/moneyness groups in Table 1. To avoid to cumbersome tables, only the results for four of the models are compared to Model 1. The chosen models are 3, 5, 6, and 9, and they are chosen because they are considered to be the "best" in their group of models, both in- and out-of-sample.

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[10] Otherwise, it would rather be a joint test of the volatility function and the binomial option pricing model.
[11] A contract day is a day when an option with a certain exercise price and expiration day is traded.
[12] The results would of course be the same if instead the daily absolute errors were compared.

In Table 6, the RMSE for each group and model is presented, whereas the results for the values within the spread are in Table 7. The two measures of the goodness-of-fit lead to quite different results in that the performance of Model 1 in terms of values within the spread is much better than in terms of RMSE, compared to the other models. This indicates that a choice has to be made about what kind of goodness-of-fit that is considered most important. Is it most important that the model produces values that most of the times fall within the market bid/ask spread, or is it to avoid large deviations from the market midpoint.[13]

For models 3 and 6, i.e. the smile and the asymmetric smile models, the overall performance is rather good. For at-the-money options, the models perform almost as well as Model 1, whereas they usually perform better than Model 1 for deep in-the-money options. Model 6 appears to produce slightly more accurate option values than model 3, indicating that the smile in Swedish equity options is a bit asymmetric. The time dependent models, i.e. models 5 and 9, perform comparatively badly overall, but for deep in- and also to some extent deep out-of-the-money options, these models work well compared to the other models. For at-the-money options, the performance is considerably worse than for the other models. The rather good performance of these models in table 5 could probably be explained by the comparatively few observations of at-the-money options, were they perform badly.[14]

Of greater interest than the individual performance of each model is whether the use of volatility functions, rather than at-the-money implied volatility, significantly increases the valuation performance. In terms of observations within the spread, there does not seem to be any major advantage with the use of volatility functions. Although some model produces higher percentages of values within the spread for certain options, the actual number of observations within the spread is not that much higher. If attention instead is focused on RMSE, the advantage of volatility functions is more pronounced. For deep in-the-money options, using one of the time dependent models significantly reduces the RMSE. The greatest reduction in RMSE, over SEK 1, is achieved if the volatility for deep in-the-money options with 31 to 90 days left to expiration is estimated using Model 5 rather than Model 1.

[13] By using RMSE, large deviations are punished more severely than smaller ones.

[14] Panel B of Table 2 indicate a pattern in implied volatilities that are rather a “reversed sneer” than a smile for the group of options with a remaining time to expiration in excess of 151 days. It is therefore tested whether Model 2b would be better in this time group than the four tested models. However, this is not the case.

When estimating the different volatility functions, there is the question about how long the estimation window should be. Dumas et al. (1998), among others, notice that the functions are not stable over time. Hence, a too long estimation period would not be meaningful. Instead, a rather short estimation period is called for. However, by using rather few observations, sampling variation may have been introduced. And if this sampling variation rather than parameter instability is the cause of a poor fit, a somewhat longer estimation period could lead to an improvement. The estimation period in this study is six months, which is a rather long period. To test the effects of reducing this period to one month, the models are re-estimated using data for just December 1 trough December 31, 1995. However, the data is fitted only to the alternative specifications of the models. The evaluation period is still January 1 through February 1, 1996.

The coefficients and R^2 for the fitted models are presented in Table 8. When the estimation period is reduced, the in-sample fit is significantly improved for all models. However, the coefficient for squared time to expiration in Model 5 is not significantly different from zero, whereas it is significant only at the 5-percent level for Model 9. Moreover, when it comes to the out-of-sample fit the results for the shorter estimation period is inferior to the results of the longer period. The goodness-of-fit for the shorter estimation period can be found in Table 9. The RMSE in Panel A is considerably increased compared to the RMSE in Table 5. In addition, no model produces an RMSE that is lower than the benchmark. Also when the other criteria are considered, Model 1 is the superior model. However, Model 1 is outperformed during up to 42% of the contract days by other models.

In all tests the entire sample of observations is used, regardless of underlying stock. Perhaps this has caused a greater bias than could be expected. To examine this possibility, the goodness-of-fit for the out-of-sample period is tested also when the volatilities in the evaluation period are computed using the model coefficients for each of the underlying stock. The goodness-of-fit for the four chosen models are presented in Table 10. As can be expected, the out-of-sample fit is better than in Table 5 although just slightly. However, for Model 9, the fit is slightly worse than when the entire sample is used in the fitting of the models. Hence, the similarities indicate that the results are not greatly biased by the use of data for all underlying stocks together.

8. Concluding remarks

This study investigates smile patterns in implied volatilities for Swedish equity options. A rather U-shaped smile is found, indicating that the implied volatility for the option closest to being at-the-money will not accurately value options that are far away from the at-the-money point. With the at-the-money implied volatility used as a benchmark, the performance of eight different specifications of deterministic volatility functions are tested.

The volatilities according to the volatility functions are used to value options in an out-of-sample evaluation period. The performance of the volatility functions depends on the time to expiration and the moneyness of the valued options. The results indicate that time parameters should be included into the function if the valued options are deep in-the-money. For options of comparatively short maturity the function should concentrate on the smile, i.e. have some kind of quadratic moneyness term. The overall performance of functions modelling a smile or an asymmetric smile with no time dependence is rather good. For at-the-money options, the models perform almost as well as the benchmark, whereas they usually perform better than the benchmark for deep in-the-money options. However, the asymmetric smile model appears to produce slightly more accurate option values than the smile model, indicating that the smile in Swedish equity options is a bit asymmetric.

For certain options, the use of implied volatility functions may lead to an economically significant improvement. For deep in-the-money options, using one of the time dependent models rather than the benchmark model may reduce the RMSE with more than SEK 1. However, no single model works very well for options of all moneyness levels. Since the volatility functions that perform well for deep in-the-money options perform very badly for at-the-money options, it is hard to say that the use of a function rather than the at-the-money volatility leads to an improvement. In a coming version of the paper, it will be tested whether a model that includes the time parameters differently for different moneyness levels using dummy variables could improve the performance.

When the estimation period is reduced to one month, the in-sample fit improves significantly whereas the out-of-sample fit is inferior to the results of the longer estimation period. Overall, Model 1 is the superior model, but it is outperformed in terms of squared deviation during up to 42% of the contract days. However, the results of this reduced estimation period may have been influenced by the choice of time period (December). It should also be tested how the results would be affected if the functions estimated during one month are used to value options during the following month, on a roll-over schedule. This will be tested in the coming version of the paper.

References:

Amin K. and Jarrow R. (1992) Pricing Options on Risky Assets in a Stochastic Interest Rate Economy. *Mathematical Finance 2*, 217-237.

Bakshi G., Cao C. & Chen Z. (1997) Empirical Performance of Alternative Option Pricing Models. *Journal of Finance* 52, 2003-2049.

Ball C. and Roma A. (1994) Stochastic Volatility Option Pricing. *Journal of Financial and Quantitative Analysis 29*, 589-607.

Bates D. (1996) Testing Option Pricing Models. In Maddala G. & Rao C., eds: Handbook of Statistics, vol 15: Statistical Methods in Finance, 567-611. North-Holland, Amsterdam.

Beckers S. (1981) Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability. *Journal of Banking and Finance 5*, 363-382.

Black F. (1975) Fact and Fantasy in the Use of Options. *Financial Analysts Journal 31,* 36-72.

Black F. (1976) Studies of Stock Price Volatility Changes. *Proceedings of the 1976 Meeting of the American Statistical Association,* 177-181.

Black F. and Scholes M. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy 81*, 637-657.

Corrado C. and Su T. (1996) Skewness and Kurtosis in S&P 500 Index Returns Implied by Option Prices. *The Journal of Financial Research 19*, 175-192.

Cox J., Ross S. and Rubinstein M. (1979) Option Pricing: A Simplified Approach. *Journal of Financial Economics 13*, 145-166.

Dennis P. and Mayhew S. (2001) Implied Volatility Skews: Evidence from Options on Individual Equities. *Working paper, University of Virginia/ University of Georgia.*

Derman E. (1999) Regimes of Volatility: Some Observations on the Variation of S&P 500 Implied Volatilities. *Goldman Sachs, Quantitative Strategies Research Notes.*

Dumas B., Fleming J. and Whaley R. (1998) Implied Volatility Functions: Empirical Tests. *Journal of Finance 53*, 2059-2106.

Duque J. and Paxson D. (1994) Implied Volatility and Dynamic Hedging. *The Review of Futures Markets 13*, 381-421.

Duque J. and Teixeira Lopes P. (1999) Maturity and volatility Effects on Smiles. Or Dying Smiling? *Paper presented at EFA 1999, Helsinki*.

Easley D., O'Hara M. and Srinivas P. (1998) Option Volume and Stock Prices: Evidence on Where Informed Traders Trade. *Journal of Finance 53*, 431-465.

Engström M. and Nordén L. (2000) The Early Exercise Premium in American Put Option prices. *Journal of Multinational Financial Management 10,* 461-479.

Geske R. and Roll R. (1984) On Valuing American Call Options with the Black-Scholes European Formula. *Journal of Finance 39,* 443-455.

Hafner R. and Wallmeier M. (2000) The Dynamics of DAX Implied Volatilities. Working paper, University of Augsburg.

Harvey C. and Whaley R. (1991) S&P 100 Index Option Volatility. *Journal of Finance 46,* 1551-1561.

Hull J. (1999) Options, Futures and Other Derivatives (fourth edition). Prentice-Hall.

Hull J. and White A. (1988) The Use of the Control Variate Technique in Option Pricing. *Journal of Financial and Quantitative Analysis 23*, 237-251.

Hull J. and White A. (1999) The Pricing of Options on Assets with Stochastic Volatilities. *Journal of Finance 42*, 281-300.

Johnson H. and Shanno D. (1987) Option Pricing when the Variance is Changing. *Journal of Financial and Quantitative Analysis 22*, 143-151.

MacBeth J. and Merville L. (1979) An Empirical Examination of the Black-Scholes Call Option Pricing Model. *Journal of Finance 34*, 1173-1186.

MacMillan L. (1986) Analytic Approximation for the American Put Option. *Advances in Futures and Options Research 1*, 119-139.

Mayhew S. (1995) Implied Volatility. *Financial Analysts Journal, July-August*, 8-20.

Merton R. (1976) Option Pricing When the Underlying Stock Returns are Discontinuous. *Journal of Financial Economics 4*, 125-144.

Peña I., Rubio G. and Serna G. (1999) Why do we Smile? On The Determinants of the Implied Volatility Function. *Journal of Banking and Finance 23*, 1151-1179.

Peña I., Rubio G. and Serna G. (2001) Smiles, Bid-Ask Spreads and Option Pricing. *European Financial Management*, forthcoming.

Roll R. (1977) An Analytical Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends. *Journal of Financial Economics 5*, 251-258.

Rubinstein M. (1985) Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978. *Journal of Finance 40*, 455-480.

Rubinstein M. (1994) Implied Binomial Trees. *Journal of Finance 49*, 771-818.

Scott L. (1987) Option Pricing when the Variance Changes Randomly: Theory, Estimation, and an Application. *Journal of Financial and Quantitative Analysis 22*, 419-438.

White H. (1980) A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica 48*, 817-838.

Zivney T. (1991) The Value of Early Exercise in Option Prices: An Empirical Investigation. *Journal of Financial and Quantitative Analysis 26*, 129-138.

Table 1: The dispersion of observations with respect to moneyness and time to expiration

Panel A: The number of observations in the estimation period

	Time (days left) to Expiration $(T-t)$				
Moneyness (X/S)	7 - 30	31 - 90	91 - 150	151 - 182	Total
< 0.90	102 (00.95)	522 (04.86)	736 (06.86)	147 (01.37)	1,507 (14.04)
0.90 - 0.98	337 (03.14)	776 (07.23)	818 (07.62)	394 (03.67)	2,325 (21.66)
0.98 - 1.02	165 (01.54)	433 (04.03)	520 (04.84)	282 (02.63)	1,400 (13.04)
1.02 - 1.10	301 (02.80)	918 (08.55)	1,070 (09.97)	499 (04.65)	2,788 (25.97)
1.10 - 1.20	201 (01.87)	606 (05.65)	672 (06.26)	201 (01.87)	1,680 (15.65)
> 1.20	69 (00.64)	552 (05.14)	333 (03.10)	81 (00.75)	1,035 (09.64)
Total	1,175 (10.95)	3,807 (35.46)	4,419 (38.65)	1,604 (14.94)	10,735 (100.00)

Panel B: The number of observations in the evaluation period

	Time (days left) to Expiration $(T-t)$				
Moneyness (X/S)	7 - 30	31 - 90	91 - 150	151 - 182	Total
< 0.90	70 (02.78)	106 (04.21)	123 (04.88)	4 (00.16)	303 (12.02)
0.90 - 0.98	157 (06.23)	169 (06.71)	272 (10.80)	33 (01.31)	631 (25.05)
0.98 - 1.02	74 (02.94)	93 (03.69)	152 (06.03)	32 (01.27)	351 (13.93)
1.02 - 1.10	148 (05.88)	176 (06.99)	272 (10.80)	60 (02.38)	656 (26.04)
1.10 - 1.20	96 (03.81)	126 (05.00)	209 (08.30)	10 (00.40)	441 (17.51)
> 1.20	27 (01.07)	53 (02.10)	57 (02.26)	0 (00.00)	137 (05.44)
Total	572 (22.71)	723 (28.70)	1,085 (43.07)	139 (05.52)	2,519 (100.00)

Panel A displays the number of observations that are included in the first part of the sample (July 1 through December 31, 1995), according to the moneyness and the time left to expiration of the put options. In parentheses the numbers are expressed as percentages of the total number of observations. Panel B contains the corresponding information for the observations that are included in the second part of the sample (January 1 through February 1, 1996).

Table 2: Average implied volatility with respect to moneyness and time to expiration

Panel A: Average implied volatilities for puts (calls) in the estimation period

	Time (days left) to Expiration $(T-t)$				
Moneyness (X/S)	7 - 30	31 - 90	91 - 150	151 - 182	Total
< 0.90	44.02 (38.46)	32.96 (30.43)	29.90 (27.70)	28.63 (25.79)	31.79 (29.55)
0.90 - 0.98	32.31 (27.93)	27.47 (25.67)	25.22 (24.31)	25.22 (23.63)	27.00 (25.16)
0.98 - 1.02	28.05 (25.66)	26.22 (25.38)	24.90 (24.39)	25.70 (24.56)	25.85 (24.88)
1.02 - 1.10	32.48 (29.42)	26.24 (25.91)	25.16 (24.94)	25.91 (25.17)	26.44 (25.78)
1.10 - 1.20	49.40 (41.89)	34.02 (30.13)	30.24 (28.07)	29.27 (28.63)	33.78 (30.53)
> 1.20	75.33 (55.67)	60.69 (37.61)	49.25 (32.95)	37.41 (31.56)	56.16 (36.84)
Total	38.22 (32.87)	33.64 (28.69)	29.90 (27.70)	26.96 (25.49)	31.25 (27.76)

Panel B: Average implied volatilities for puts in the evaluation period

	Time (days left) to Expiration $(T-t)$				
Moneyness (X/S)	7 - 30	31 - 90	91 - 150	151 - 182	Total
< 0.90	44.12	33.60	32.09	25.82	35.31
0.90 - 0.98	37.66	29.14	30.54	31.27	31.98
0.98 - 1.02	34.94	28.24	31.09	33.38	31.34
1.02 - 1.10	36.80	28.84	30.94	35.37	32.12
1.10 - 1.20	53.17	37.74	35.00	36.76	39.84
> 1.20	76.96	36.99	48.73	-	61.55
Total	42.44	34.01	32.71	33.76	35.37

The table presents the average implied volatility for subgroups classified on the basis of moneyness (defined as the ratio of the exercise price to the average closing stock price) and time to expiration. In Panel A, the volatilities for both puts and calls (calls in parentheses) are based on data from the first part of the sample (July 1 through December 31, 1995). Panel B depicts the average implied volatilities for put options during the second part of the sample (January 1 through February 1, 1996).

Table 3. Coefficients for different specifications of the volatility function

Coefficients	Alternative specifications of the volatility function							
	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Constant	1.1320*	1.0048*	1.1502*	1.2445*	0.9982*	1.0418*	0.9305*	1.2473*
$(\ln X/S)$	1.3185*	0.8295*	1.8535*	1.8384*				
$(\ln X/S)^2$		9.0209*	8.6194*	8.8336*		11.3236*		
$(T-t)$			-0.5490*	-1.5631*				-1.6047*
$(T-t)^2$				1.9680*				1.9552*
$(\ln X/S)*(T-t)$			-4.0716*	-4.0206*				
U					-1.0305*	1.4458*		-1.1017*
U^2							6.3976*	
D							3.3027*	
D^2					12.5881*			12.5907*
R^2	0.1847	0.4992	0.5677	0.5748	0.5032	0.4860	0.4645	0.5533

The table shows results of fitting relative put volatilities in the estimation period to different volatility functions. X is the exercise price, S is the underlying stock price, $(T-t)$ is the time to expiration and

$U_i = \ln\frac{X_i}{S_i}$ if $\ln\frac{X_i}{S_i} < 0$, $U_i = 0$ if $\ln\frac{X_i}{S_i} \geq 0$ and $D_i = \ln\frac{X_i}{S_i}$ if $\ln\frac{X_i}{S_i} \geq 0$, $D_i = 0$ if $\ln\frac{X_i}{S_i} < 0$.

White (1980) heteroskedasticity-consistent standard errors are used, and coefficients marked with * have p-values < 0.0001.

Table 4. Coefficients of determination for different volatility functions for each underlying stock

Underlying stock	Model R^2 for the alternative specifications							
	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Astra	0.2616	0.4196	0.5102	0.5477	0.4591	0.4265	0.3987	0.5707
Electrolux	0.3283	0.6318	0.7524	0.7726	0.6437	0.6354	0.5267	0.7266
Ericsson	0.0020	0.3044	0.3720	0.4134	0.3427	0.3039	0.2365	0.4412
Investor	0.3693	0.7730	0.8223	0.8352	0.7894	0.7751	0.6814	0.8414
MoDo	0.5943	0.7617	0.8685	0.8808	0.7628	0.7614	0.6960	0.8278
SCA	0.4766	0.7254	0.7645	0.7767	0.7242	0.7247	0.6485	0.7622
SEB	0.0134	0.4966	0.5610	0.5622	0.4504	0.4969	0.4781	0.5053
Skandia	0.0031	0.2126	0.3547	0.4134	0.2117	0.2129	0.1883	0.3960
Trelleborg	0.2948	0.5300	0.6178	0.6194	0.5346	0.5226	0.4958	0.6043
Volvo	0.2702	0.7121	0.7468	0.7667	0.7616	0.7257	0.5592	0.8034
Average	0.2614	0.5567	0.6370	0.6588	0.5680	0.5585	0.4909	0.6479
Minimum	0.0020	0.2126	0.3547	0.4134	0.2117	0.2129	0.1883	0.3960
Maximum	0.5943	0.7730	0.8685	0.8808	0.7894	0.7751	0.6960	0.8414

The table shows the in-sample fit, in terms of R^2 for the different specifications of the volatility functions for each underlying stock. Also minimum, maximum and average values of R^2 for the models are presented.

Table 5. Goodness-of-fit for the different volatility functions

Panel A: RMSE for the different specifications

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
0.9537	1.5052	0.9487	1.0297	0.9260	0.8685	0.9009	1.2520	0.9263

Panel B: MAE for the different specifications

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
0.5294	1.1220	0.5960	0.7028	0.6761	0.5409	0.6017	0.8546	0.6493

Panel C: Percentage of model values within the market bid/ask spread

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
83.37	41.92	73.92	66.18	65.58	76.42	74.32	58.59	67.69

Panel D: Percentage of contract days when model outperforms Model 1 in terms of daily squared errors

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
-	25.09	43.23	41.41	39.22	49.46	42.79	31.64	39.90

The table presents different measures of the goodness-of-fit of the examined volatility functions when used in an out-of-sample test to value options according to the binomial model. In Panels A and B, the root mean squared errors and the mean absolute errors for the different model values from the midpoint market value of the options are displayed. Panel C presents the percentage of values, according to each model, that are within the market bid/ask spread of the options. In Panel D, finally, the results of a daily comparison of squared deviations from the market midpoint between Model 1 (where the implied volatility of the option closest to being at-the-money is used) and the model in question are displayed.

Table 6: RMSE for the difference between model values and actual midpoint prices displayed with respect to moneyness and time to expiration

Maturity	Moneyness	Model 1	Model 3	Model 5	Model 6	Model 9
	< 0.90	0.4484	0.4277	0.4225	0.4143	0.5218
	0.90 - 0.98	0.4702	0.4721	0.6280	0.4552	0.9623
7 - 30	0.98 - 1.02	0.0798	0.0965	0.9602	0.0890	1.0087
	1.02 - 1.10	0.6727	0.6529	1.2310	0.6422	1.0004
	1.10 - 1.20	1.2697	1.1070	1.0020	1.1435	1.0285
	> 1.20	2.1857	2.0626	1.8168	2.0762	1.9601
	< 0.90	0.3846	0.3190	0.3144	0.3553	0.3539
	0.90 - 0.98	0.3625	0.3968	0.4368	0.3099	0.4636
31 - 90	0.98 - 1.02	0.1094	0.1485	0.4353	0.1203	0.4619
	1.02 - 1.10	0.6728	0.7350	0.9616	0.6452	0.7776
	1.10 - 1.20	1.7104	1.4372	1.3619	1.3673	1.3153
	> 1.20	2.8813	2.1130	1.8543	2.0985	1.9905
	< 0.90	0.4591	0.3759	0.3882	0.6244	0.4320
	0.90 - 0.98	0.3725	0.3846	0.6452	0.6690	0.4292
91 - 150	0.98 - 1.02	0.1345	0.1782	0.6174	0.1490	0.6960
	1.02 - 1.10	0.4722	0.9103	0.6484	0.5693	0.6930
	1.10 - 1.20	1.1753	1.6183	1.0607	1.2180	0.9046
	> 1.20	2.7980	2.3619	1.8555	2.2742	1.9685
	< 0.90	0.5591	0.8718	1.2608	1.9061	1.1145
	0.90 - 0.98	0.4657	0.3496	0.5497	1.1354	0.5797
151 - 182	0.98 - 1.02	0.1938	0.2138	1.2803	0.2457	1.6106
	1.02 - 1.10	1.1728	1.0676	1.6491	0.6732	1.7897
	1.10 - 1.20	0.9897	2.4848	0.3683	1.6276	0.6202
	> 1.20	-	-	-	-	-
Overall		0.9537	0.9487	0.9260	0.8685	0.9263

The table displays the RMSE for the differences between the model option values, according to the different volatility functions, and the averages of the closing bid and ask option prices. The observations are displayed with respect to moneyness and time to expiration.

Table 7: The number (percentage) of model values within the market bid/ask spread, displayed with respect to moneyness and time to expiration

Maturity	Moneyness	Model 1	Model 3	Model 5	Model 6	Model 9
	< 0.90	48 (68.57)	49 (70.00)	55 (78.57)	55 (78.57)	56 (80.00)
	0.90 - 0.98	96 (61.15)	93 (59.24)	98 (62.42)	117 (74.52)	70 (44.59)
7 - 30	0.98 - 1.02	74 (100.00)	73 (98.65)	21 (28.38)	72 (97.30)	17 (22.97)
	1.02 - 1.10	141 (95.27)	131 (88.51)	53 (35.81)	139 (93.92)	72 (48.65)
	1.10 - 1.20	84 (87.50)	89 (92.71)	81 (84.38)	89 (92.71)	86 (89.58)
	> 1.20	17 (62.96)	19 (70.37)	20 (74.07)	18 (66.67)	21 (77.78)
	< 0.90	76 (71.70)	84 (79.25)	84 (79.25)	71 (66.98)	72 (67.92)
	0.90 - 0.98	122 (72.19)	119 (70.41)	116 (68.64)	137 (81.07)	92 (54.44)
31 - 90	0.98 - 1.02	93 (100.00)	92 (98.92)	61 (65.59)	91 (97.85)	57 (61.29)
	1.02 - 1.10	167 (94.89)	137 (77.84)	87 (49.43)	164 (93.18)	123 (69.87)
	1.10 - 1.20	100 (79.37)	96 (76.19)	93 (73.81)	105 (83.33)	106 (84.13)
	> 1.20	32 (60.38)	28 (52.83)	38 (71.70)	29 (54.72)	33 (62.26)
	< 0.90	94 (76.42)	89 (72.35)	92 (74.80)	72 (58.54)	95 (77.24)
	0.90 - 0.98	230 (84.56)	227 (83.46)	157 (57.72)	149 (54.80)	215 (79.04)
91 - 150	0.98 - 1.02	152 (100.00)	149 (98.03)	108 (71.05)	150 (98.68)	100 (65.79)
	1.02 - 1.10	246 (90.44)	163 (59.93)	213 (78.31)	220 (80.88)	211 (77.57)
	1.10 - 1.20	179 (85.65)	79 (37.80)	156 (74.64)	112 (53.59)	171 (81.82)
	> 1.20	43 (75.44)	32 (56.14)	48 (84.21)	37 (64.91)	46 (80.70)
	< 0.90	3 (75.00)	1 (25.00)	0 (00.00)	0 (00.00)	0 (00.00)
	0.90 - 0.98	29 (87.88)	29 (87.88)	30 (90.91)	14 (42.42)	29 (87.88)
151 - 182	0.98 - 1.02	32 (100.00)	32 (100.00)	13 (40.63)	32 (100.00)	6 (18.75)
	1.02 - 1.10	36 (60.00)	47 (78.33)	18 (30.00)	47 (78.33)	18 (30.00)
	1.10 - 1.20	7 (70.00)	3 (30.00)	9 (90.00)	4 (40.00)	8 (80.00)
	> 1.20	-	-	-	-	-
Overall		2,100 (83.37)	1,862 (73.92)	1,652 (65.58)	1,925 (76.42)	1,705 (67.69)

The table displays the number of option values, given the different volatility functions, which are within the market bid/ask spread. The observations are displayed with respect to moneyness and time to expiration. In parentheses, the numbers of values within the spread are expressed as percentages of the total number of observations in each time/moneyness group during the evaluation period (January 1 through February 1, 1996).

Table 8. Coefficients for different specifications of the volatility function given a one-month estimation period

Coefficients	Volatility specifications							
	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Constant	1.1090*	0.9579*	1.0713*	1.0919*	0.9682*	1.0141*	0.8416*	1.1755*
$(\ln X / S)$	3.1990*	1.1168*	2.3491*	2.3285*				
$(\ln X / S)^2$		15.2957*	13.7722*	13.8220*		18.7863*		
$(T - t)$			-0.3918*	-0.6136Ψ				-1.1480*
$(T - t)^2$				0.4299ω				1.1176Ψ
$(\ln X / S) * (T - t)$			-5.2875*	-5.1898*				
U					-1.3645*	2.2486*		-1.3236*
U^2							9.9052*	
D							5.1667*	
D^2					19.9236*			19.3125*
R^2	0.4425	0.7158	0.7466	0.7467	0.7326	0.7160	0.6448	0.7479

The table shows results of fitting relative put volatilities during the period December 1 through December 31, 1995 to different volatility functions. X is the exercise price, S is the stock price, $(T - t)$ is the time to expiration and $U_i = \ln \frac{X_i}{S_i}$ if $\ln \frac{X_i}{S_i} < 0$, $U_i = 0$ if $\ln \frac{X_i}{S_i} \geq 0$ and $D_i = \ln \frac{X_i}{S_i}$ if $\ln \frac{X_i}{S_i} \geq 0$, $D_i = 0$ if $\ln \frac{X_i}{S_i} < 0$.

White (1980) heteroskedasticity-consistent standard errors are used, and coefficients marked with * have p-values < 0.0001, whereas those marked with Ψ or ω have p-values < 0.05 and > 0.10 respectively.

Table 9. Goodness-of-fit for the different volatility functions with estimation over one month

Panel A: RMSE for the different specifications

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
0.9537	2.2826	1.4408	1.2897	1.2621	1.3067	1.3477	2.0359	1.2906

Panel B: MAE for the different specifications

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
0.5294	1.6482	0.8578	0.8350	0.8299	0.7205	0.7732	1.3790	0.7992

Panel C: Percentage of model values within the market bid/ask spread

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
83.37	27.31	63.60	59.90	59.15	71.81	67.80	37.83	63.12

Panel D: Percentage of contract days when model outperforms Model 1 in terms of daily squared errors

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
-	14.37	33.03	29.69	30.81	42.00	34.14	21.95	36.52

The table presents different measures of the goodness-of-fit of the examined volatility functions when used in an out-of-sample test to value options according to the binomial model. The models are fitted during the period December 1 through December 31, 1995. In Panels A and B, the root mean squared errors and the mean absolute errors for the different model values from the midpoint market value of the options are displayed. Panel C presents the percentage of values, according to each model, that are within the market bid/ask spread of the options. In Panel D, finally, the results of a daily comparison of squared deviations from the market midpoint between Model 1 (where the implied volatility of the option closest to being at-the-money is used) and the model in question are displayed.

Table 10. Goodness-of-fit for some of the different volatility functions with estimation for each underlying stock

Panel A: RMSE for different models				
Model 1	Model 3	Model 5	Model 6	Model 9
0.9537	0.8591	0.8669	0.8456	0.9754

Panel B: MAE for different models				
Model 1	Model 3	Model 5	Model 6	Model 9
0.5294	0.5408	0.6312	0.5367	0.6909

Panel C: Percentage of values within the spread				
Model 1	Model 3	Model 5	Model 6	Model 9
83.37	77.45	68.52	76.78	65.30

Panel D: Percentage of days when better than Model 1				
Model 1	Model 3	Model 5	Model 6	Model 9
-	45.89	40.33	46.61	38.82

The table presents different measures of the goodness-of-fit of the examined volatility functions when used in an out-of-sample test to value options according to the binomial model. The models are fitted during the entire estimation period using different coefficients for different underlying stocks. In Panels A and B, the root mean squared errors and the mean absolute errors for the different model values from the midpoint market value of the options are displayed. Panel C presents the percentage of values, according to each model, that are within the market bid/ask spread of the options. In Panel D, finally, the results of a daily comparison of squared deviations from the market midpoint between Model 1 (where the implied volatility of the option closest to being at-the-money is used) and the model in question are displayed.