BOND FUTURES

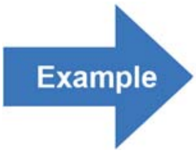
1. Terminology

A future is a contract to either sell or buy a certain underlying on a specified future date at a fixed rate. It is traded on the exchange. For the long-term, usually the underlyings are one (or more) specific government bonds.

Since different futures on the different markets have different names (EUR-Bund future, US treasury bond future, etc.) we will use bund future as a synonym for a future on a medium- / long-term bond.

Underlying

The underlying of a bond future is a synthetic bond with a defined term and defined coupon. The advantage of this synthetic bond over an actual bond is that the futures price can be better compared over time.



The underlying of a EUR-Bund future is a synthetic Bund with a 10-year term and a 6 % coupon. The T-bond (note) futures‘ underlying specification is 30 (10) years and 6 % coupon.

Contract size

The contract size is determined individually by the futures exchange. In case of a Euro-Bund future the contract size is EUR 100,000.

Table: Contract sizes and Conventions

Currency	Exchange	Future	Contract size	Underlying	Deliverable bonds (TOM in years) *)
EUR	EUREX	Bund-Future	100,000	Bund, 10y. 6 %	8.5 – 10.5
EUR	EUREX	Schatz-Future	100,000	Bund, 5y., 6 %	3.5 – 5
EUR	EUREX	Long-Gilt Future	100,000	Bund, 2y., 6 %	1,75 – 2,25
EUR	LIFFE	BOBL-Future	100,000	Bund, 10y., 6 %	8,5 – 10,5
GBP	LIFFE	Bund-Future	100,000	Long Gilt, 7 %	8,75 –13
JPY	TSE	JGB - Future	100,000	JGB, 20y., 6 %	15 – 21
JPY	TSE	JGB - Future	100,000	JGB, 10y. , 6	7 – 11
JPY	TSE	JGB - Future	100,000	JGB, 5 y., 6 %	4 - 5,25
CHF	EUREX	CONF – Future	100,000	Swiss Gvt. Bond, 10y., 6 %	8 – 13
USD	CBOT	10-y T-Note	100,000	T-note, 10 y.,6 %	6,5 – 10
USD	CBOT	5-y T-Note	100,000	T-note, 5 y.,6 %	1,75 – 5,25
USD	CBOT	2-y T-Note	200,000	T-note, 2 y., 6 %	4,25 – 5,15
USD	CBOT	T-Bond Future	100,000	T-bond, 30 y., 6 %	min. 15

*) TOM = term to maturity

Futures purchase

The buyer of a Bund future is obliged to buy the underlying bond at a fixed price on an agreed date. Because the prices of bonds rise when interest rates fall, a purchased future can be used to speculate on falling interest rates.

Futures sell

The seller of a bund future is obliged to deliver the underlying bond at a fixed price on an agreed date. Because the prices of bonds fall when interest rates rise, a sold future can be used to speculate on rising interest rates or to secure existing short positions against rising interest rates.

Tick

As with MM – Futures, a tick is the minimum price movement of a futures contract. In contrast to Money Market Futures where a tick is typically one hundredth of 1 % or at least in decimals, long-term futures sometimes move in 1/32 of 1 % (i.e. 0,0003125 or 3,125 BP),

e.g. T-bond futures. The tick size is typically defined according to the quoting conventions of the underlying bond. For example, EUR-Bunds are quoted in decimals on 1 BP, thus the tick value of the Bund-Future is 1 BP.
A tick has always an exactly defined value in relation to the contract, the tick value is the product of the contract value times the basis points of a tick (=tick size).

Example

The tick value of a EUR – Bund Future and a 10-y T-note Future respectively are:

EUR-Bund Future: 100,000 x 0.0001 = EUR 10
10-year T-note Future: 100,000 x 0.00015625 = USD 15.625

Tick table:

Currency	Exchange	Future	Tick size	Tick value
EURO	EUREX	Bund-Future	1 BP	EUR 10
EURO	EUREX	BOBL-Future	1 BP	EUR 10
EURO	EUREX	Schatz-Future	1 BP	EUR 10
EURO	LIFFE	Bund-Future	1 BP	EUR 10
GBP	LIFFE	Long-Gilt Future	1 BP	GBP 10
JPY	TSE	JGB - Futures	1 BP	JPY 10,000
CHF	EUREX	CONF – Future	1 BP	CHF 10
USD	CBOT	10-y T-Note Future	1 / 64 BP	USD 15.625
USD	CBOT	5-y T-Note Future	1 / 64 BP	USD 15.625
USD	CBOT	2-y T-Note Future	1 / 128 BP	USD 15.625
USD	CBOT	T-Bond Future	1 / 32 BP	USD 31.25

Exchange Delivery Settlement Price (EDSP)

Usually, the EDSP is a volume-weighted average of a certain number of prices that have been ultimately dealt at the end of the trading day.

The EDSP of a Bund-Future is the volume-weighted average of the latest 10 trading prices quoted during the last 30 minutes of the trading day. If the number of trades in the last minute of the trading day exceeds the number of 10, the EDSP is calculated as weighted average of all deals undertaken during the last minute.

Delivery dates and last trading day

In contrast to MM-Futures the delivery of bond futures is not standardised across the markets.
The delivery months of bond futures are March, June, September and December (such as with MM-Futures). For the delivery day, futures exchanges set the following rules:

Currency	Exchange	Future	Delivery day
EUR	EUREX	Bund-Future	10th day in the delivery month
EUR	EUREX	BOBL-Future	10th day in the delivery month
EUR	EUREX	Schatz-Future	10th day in the delivery month
EUR	LIFFE	Bund-Future	10th day in the delivery month
GBP	LIFFE	Long-Gilt Future	Any business day in delivery month (at seller's choice)
JPY	TSE	JGB - Futures	20th day in the delivery month *)
CHF	EUREX	CONF – Future	10th day in the delivery month
USD	CBOT	10-y T-Note Future	Last business day of the month *)
USD	CBOT	5-y T-Note Future	Last business day of the month *)
USD	CBOT	2-y T-Note Future	Third business day following the last trading day +)
USD	CBOT	T-Bond Future	Last business day of the month *)

*) The last trading day is 7 days before the last delivery day
+) The last trading day is the earlier of the second business day prior to the issue day of the 2-year note auctioned in the current month or the last business day of the calendar month

If not mentioned otherwise, the last trading day is two days prior to delivery date. If the last trading day is a holiday the following business day is the last trading day.

Delivery

Contrary to MM-Futures, bond futures are delivered physically if they have not been closed out prior to delivery date. The delivery of the futures contract must tackle the problem that the underlying bond is a synthetic instrument. Therefore, the seller can deliver from a basket of bonds. The settlement price is determined by means of a conversion factor (or price factor) that makes the price of the synthetic bond comparable to the price of the deliverable bond.

The conversion factor is calculated on the basis of the clean price of the bond. The present value of the deliverable bond is divided by the present value of the synthetic bond (= 100). The present value of the deliverable bond is calculated with a yield equal to the coupon of the synthetic bond, e.g. 6 % for the EUR – Bund Future. The price is determined with the classic bond formula, assuming a flat yield curve.

$$C = \frac{PV_D}{100}$$

C = Conversion factor

PV_D = Present value of the deliverable bond if the yield = coupon of synthetic bond

If the conversion factor is determined, the price of the deliverable bond for a yield equal to the coupon of the synthetic bond is related to the par price of the synthetic bond (= 100). Therefore, the

- **Conversion factor is bigger 1** if the coupon of the deliverable bond is higher than the coupon of the synthetic bond
- **Conversion factor is smaller 1** if the coupon of the deliverable bond is lower than the coupon of the synthetic bond

The conversion factor is mainly used in order to calculate the cash amount payable on the delivery day by the buyer of the future to the seller. The cash amount is determined on basis of the trading unit and calculated with the following formula:

Calculation of cash:

$$P = (EDSP/\ 100 \times C \times V) + AI$$

P = Cash amount payable for the delivered bond volume

EDSP = Exchange Delivery Settlement Price

C = Conversion factor

V = Contract size

AI = Accrued Interest

Example

The March Bund-Futures contract expires at 107.72. You can choose between the following two bonds for settling the future

Bond A	**Bond B**
Term to maturity: 10 years	Term to maturity: 10 years
Coupon: 5.375 %	Coupon: 7.000 %
Price: 102.90	Price: 115.44
Conversion factor: 0.9539995	Conversion factor: 1.0736009
Coupon days: 0	Coupon days: 0

Calculation Bond A

You deliver a notional of EUR 100,000 of the Bund and receive

P = 1.0772 x 0.9539995 x 100,000

P = 102,764 EUR

You need EUR 102,900 in order to purchase EUR 100,000 notional of Bunds. Thus, you make a loss of EUR 136 (= 102,764 – 102,900)

Calculation Bond B

P = 1.0772 x 1.0736009 x 100,000

P = 115,648 EUR

You need EUR 115,440 in order to purchase EUR 100,000 notional of Bunds. Thus, you make a profit of EUR 208 (= 115,648 – 115,440)

Consequently, you will deliver Bond B to the buyer.

Note: The example shows profit or loss on the delivery day, which can result by the choice of the deliverable bond from basket. The profit/ loss determined in the example gives no information about the total position because all previous profits/ losses have been settled through the margin account.

Cheapest-To-Deliver

The cheapest-to-deliver is that bond of the basket of deliverable bonds that has the lowest cost for the seller (in our example Bond B). As a rule of thumb, the cheapest-to-deliver can be determined by dividing the spot price of the bond by the conversion factor and choosing the bond with the smallest ratio.

$$CTD = \min \frac{Spot}{C}$$

CTD = Cheapest-to-deliver

Spot = Spot price of the bond

C = Conversion factor

The "CTD-ratio **estimates** the no arbitrage futures price for a bond with a deliverable grade. A "correct" futures price is a price where the cash settlement of the future and the repurchase of the required bond notional at the current market price produce neither a profit nor a loss. It is only an estimation because the accrued interest and the funding costs are neglected.

Example

Delivery date, March Bund-Future; Price: 107.72

Bond A	Bond B
Term to maturity: 10 years	Term to maturity: 10 years
Coupon: 5.375 %	Coupon: 7.000 %
Price: 102.90	Price: 115.44
Conversion factor: 0.9539995	Conversion factor: 1.0736009
Coupon days: 0	Coupon days: 0

Bond A:

$$CTD = \frac{Spot}{K}$$

$$CTD = \frac{102.90}{0.9539995}$$

CTD = 107.86

Bond B:

$$CTD = \frac{Spot}{K}$$

$$CTD = \frac{115.44}{1.0736009}$$

CTD = 107.53

Bond B is the cheapest-to-deliver because the theoretical futures price is 19 basis point below the current futures market price. (= 107.72 – 107.53)

Example

Check:

Suppose that the March-Future is quoted at 107.53 and you deliver Bond B. You receive EUR 115,444, i.e. exactly the amount that you need to repurchase Bond B in the market.

In our example we could determine an exact futures price because we do not need to consider accrued interest and funding costs.

Quotation / Pricing

The pricing of bond futures is based on the no arbitrage assumption. The seller of the future must buy the bond and fund the purchase in the money market. Since the seller will always choose the cheapest bond for delivery the futures price is based on the **cheapest-to-deliver**. The funding costs increase the futures price. The coupons outstanding until the delivery day of the future reduce the futures price because this share of the coupon is an income for the seller. If there is a coupon date during the futures term the revenues from reinvesting the coupons are deducted from the futures price.

Therefore, the price of a bond future is influenced by

- the current CTD bond price
- the accrued interest
- the remaining coupon days until the futures delivery date
- the funding costs of the bond purchase
- possible coupon payments
- possible reinvestment revenues

The theoretical futures price excluding interim coupon payments on the bond is

$$FP = \frac{CP_{CTD} + FC_{CTD} - E_{CTD}}{C_{CTD}}$$

FP	=	Futures price
CPCTD	=	Clean price of cheapest-to-deliver
FCCTD	=	Funding costs of cheapest-to-deliver
ECTD	=	Coupon from cheapest-to-deliver from trading day till futures delivery day
CCTD	=	Conversion factor of cheapest-to-deliver

The funding costs of the cheapest-to-deliver are calculated from the Dirty Price of the bond. The funding rate is supposed to be the repo rate.

Example

You should calculate the futures price for a future with a remaining term of 150 days. There is no coupon date until delivery day and the cheapest-to-deliver is currently traded at a

Clean price:	Euro 104
with	
Accrued interest:	Euro 3
Coupon until delivery date:	Euro 2.25
Coupon:	5.25 %
Conversion factor:	0.948594
Funding rate:	4 % p.a.

The funding costs are: 107 x 0.04 x 150 / 360 = 1.78

The futures price for a term of 150 days is:

$$\frac{104 + 1.78 - 2.25}{0.948594} = 109.14$$

The example shows that prices of bond futures can exceed 100 (contrary to money market futures).

Note: If you calculate the theoretical futures price on delivery day and if the delivery day is a coupon day (i.e. accrued interest = 0), the pricing formula is reduced to the CTD-ratio" formula.

2. Application

Hedging a fixed rate bond portfolio against rising rates

By selling futures a bond portfolio or a single bond can be hedged against an expected rate hike. This can make sense especially

- if the bond market is not very liquid and thus, wide bid offer spreads have to be paid
- if the hedge is supposed to be only for a short period
- if there is no way to go short the bond

As a bond portfolio mostly consists of a mixture of bonds with different terms and different coupons, this hedge cannot be perfect. By using risk measuring concepts like duration or „Present Value of a Basis Point" a relatively exact hedge volume can be determined and thus, the remaining risk can be reduced.

Hedging a planned fixed rate bond issue against rising rates

A company (or bank) which already knows the issue’s timing can hedge the risk of possibly rising rates by selling futures. If rates really rise, on the one hand the bond has to be issued with a higher coupon resp. a lower issue price, on the other hand the futures position will create a profit (or vice versa).
Intraday-hedging of interest rate risks
Long-term futures are commonly used in order to hedge open interest rate risks of other instruments (e.g. IRS) during the term. E.g. a market maker who has bought a 10-years IRS (fixed rate payer swap) can hedge his interest rate risk by buying futures for the meantime.

Consequently, as being market maker the trader can change his IRS quotation according to his position. If he is now fixed rate receiver in an IRS he was hedged against interest rate changes to a large extent in the meantime. Now, he only has to sell the future and the two IRS compensate each other in the position.

This is especially interesting because

- the bid offer spread for futures is smaller than for swaps
- the liquidity for futures is higher
- there is no credit risk when trading futures
- for futures no credit lines are necessary

Note: In practice, however, there is still a remaining risk for all shown examples which is called basis risk. This means that even when trying to calculate the exact hedge ratio, the futures result will not completely compensate the profits or losses of the spot position. This is possible as in the futures market specific contracts are traded whereas in the spot market terms, coupons and ratings will vary. There is a spread risk among the different products.