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Formulae

Foundation Examination

Fixed Income Analysis and Valuation

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Economics

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1. Fixed Income Valuation and Analysis

1.1 Time Value of Money

1.1.1 Present and Future Value

Simple discounting / compounding

$$Present\ value = \frac{Future\ value}{(1 + Interest\ rate\ p.a.)^{number\ of\ years}}$$

$$Future\ value = (Present\ value) \cdot (1 + Interest\ rate\ p.a.)^{number\ of\ years}$$

Continuous discounting / compounding

$$Present\ value = \frac{Future\ value}{e^{number\ of\ years \cdot continuous\ interest\ rate\ p.a.}}$$

$$Future\ value = (Present\ value) \cdot e^{number\ of\ years \cdot continuous\ interest\ rate\ p.a.}$$

1.1.2 Annuities

The present value of an annuity is given by

$$Present\ value = \sum_{t=1}^{n} \frac{CF}{(1+R)^t} = \frac{CF}{R} \cdot \left(1 - \frac{1}{(1+R)^n}\right)$$

where

CF	constant Cash flow
R	discount rate, assumed to be constant over time
n	number of cash flows

The future value of an annuity is given by

$$Future\ value = CF \cdot \left(\frac{(1+R)^n - 1}{R}\right)$$

1.2 Bond Yield Measures

Bond yields are calculated implicitly as an internal rate of return (IRR) using the following equations.

1.2.1 Current Yield

$$Current\ yield = \frac{Annual\ coupon}{Price}$$

1.2.2 Yield to Maturity

The bond price as a function of the yield to maturity is given by

$$P_0 = \sum_{i=1}^{N} \frac{CF_i}{(1+Y)^{t_i}} = \frac{CF_1}{(1+Y)^{t_1}} + \frac{CF_2}{(1+Y)^{t_2}} + \ldots + \frac{CF_N}{(1+Y)^{t_N}}$$

where

Y	yield to maturity
P_0	current paid bond price (including accrued interest)
CF_i	cash flow (coupon) received at time t_i
CF_N	cash flow (coupon plus principal) received at repayment date t_N
N	number of cash flows

Between two coupon dates, for a bond paying coupons annually, the bond price is given by

$$P_{cum,f} = P_{ex,f} + f \cdot C = (1+Y)^f \left[\frac{CF_1}{(1+Y)^1} + \frac{CF_2}{(1+Y)^2} + \ldots + \frac{CF_N}{(1+Y)^N} \right]$$

where

Y	yield to maturity
f	time since last coupon date in years
CF_i	cash flow (coupon) received at time t_i
CF_N	final cash flow (coupon plus principal)
$P_{cum,f}$	current paid bond price (including accrued interest)
N	number of cash flows

1.2.3 Yield to Call

$$P_0 = \sum_{i=1}^{N} \frac{CF_i}{(1+Y_c)^{t_i}} = \frac{CF_1}{(1+Y_c)^{t_1}} + \frac{CF_2}{(1+Y_c)^{t_2}} + \ldots + \frac{CF_N}{(1+Y_c)^{t_N}}$$

where

Y_c	yield to call
P_0	current paid bond price (including accrued interest)
CF_i	cash flow (coupon) received at time t_i
CF_N	cash flow (coupon plus principal) received at call date t_N
N	number of cash flows until call date

1.3 Term Structure of Interest Rates

1.3.1 Relation between Spot Rate and Forward Rate

$$(1+R_{0,t}) = \left[(1+R_{0,1})\cdot(1+F_{1,2})\cdot(1+F_{2,3})\ldots(1+F_{t-1,t})\right]^{\frac{1}{t}}$$

where

$F_{t-1,t}$	forward rate p.a. from $t-1$ to t
$R_{0,t}$	spot rate p.a. from 0 to t

$$(1+R_{0,t_1})^{t_1} \cdot (1+F_{t_1,t_2})^{t_2-t_1} = (1+R_{0,t_2})^{t_2} \Leftrightarrow F_{t_1,t_2} = \left(\frac{(1+R_{0,t_2})^{t_2}}{(1+R_{0,t_1})^{t_1}}\right)^{\frac{1}{t_2-t_1}} - 1$$

where

F_{t_1,t_2}	forward rate p.a. from t_1 to t_2
R_{0,t_1}	spot rate p.a. from 0 to t_1
R_{0,t_2}	spot rate p.a. from 0 to t_2

1.3.2 Theories of Term Structures

Unbiased expectations hypothesis

$$F_{t_1,t_2} = E(\widetilde{R}_{t_1,t_2})$$

Liquidity preference theory

$$F_{t_1,t_2} = E(\widetilde{R}_{t_1,t_2}) + L_{t_1,t_2}, L_{t_1,t_2} > 0$$

Market segmentation theory

$$F_{t_1,t_2} = E(\widetilde{R}_{t_1,t_2}) + \Pi_{t_1,t_2}, \Pi_{t_1,t_2} \gtrless 0$$

where

F_{t_1,t_2}	forward rate from t_1 to t_2
$\widetilde{R}_{t_1,t_2}$	random spot rate from t_1 to t_2
L_{t_1,t_2}	liquidity premium for the time t_1 to t_2
Π_{t_1,t_2}	risk premium for the time t_1 to t_2

1.4 Bond Price Analysis

1.4.1 Yield Spread Analysis

Relative Yield Spread

$$Relative\ yield\ spread = \frac{Yield\ bond\ A - Yield\ bond\ B}{Yield\ bond\ B}$$

Yield Ratio

$$Yield\ ratio = \frac{Yield\ bond\ A}{Yield\ bond\ B}$$

1.4.2 Bond Valuation

Valuation of Zero-Coupon Bonds

$$P_{0,t} = \frac{CF_t}{(1+R_t)^t}$$

where

P_0	current bond price
CF_t	cash flow (coupon plus principal) received at repayment date t_N
R_t	spot rate from 0 to t

Valuation of Coupon-Bearing Bonds

$$P_0 = \sum_{i=1}^{N} \frac{CF_i}{(1+R_i)^{t_i}} = \frac{CF_1}{(1+R_1)^{t_1}} + \frac{CF_2}{(1+R_2)^{t_2}} + ... + \frac{CF_N}{(1+R_N)^{t_N}}$$

where

P_0	current bond price
CF_i	cash flow (coupon) received at time t_i
CF_N	cash flow (coupon plus principal) received at repayment date t_N
N	number of cash flows
R_i	spot rate from 0 to t_i

Price with accrued interest of a bond paying yearly coupons

$$P_{cum,f} = P_{ex,f} + f \cdot C = \sum_{i=1}^{N} \frac{CF_i}{\left(1+R_{t_i}\right)^{t_i - f}}$$

where

$P_{cum,f}$	price of the bond including accrued interest
$P_{ex,f}$	quoted price of the bond
f	time since the last coupon date in fractions of a year
CF_i	cash flow at time t_i
R_{t_i}	spot rate from f to t_i
C	coupon

Valuation of **Perpetual Bonds**

$$P_0 = \frac{CF}{R}$$

where

P_0	current price of the perpetual bond
CF	perpetual cash flow (coupon)
R	discount rate, assumed to be constant over time

1.5 Risk Measurement

1.5.1 Macaulay's Duration

$$D = \sum_{i=1}^{N} \frac{t_i \cdot PV(CF_i)}{P} = \frac{\sum_{i=1}^{N} \frac{t_i \cdot CF_i}{(1+Y)^{t_i}}}{\sum_{i=1}^{N} \frac{CF_i}{(1+Y)^{t_i}}}$$

$$= \frac{\frac{t_1 \cdot CF_1}{(1+Y)^{t_1}} + \frac{t_2 \cdot CF_2}{(1+Y)^{t_2}} + \ldots + \frac{t_N \cdot CF_N}{(1+Y)^{t_N}}}{\frac{CF_1}{(1+Y)^{t_1}} + \frac{CF_2}{(1+Y)^{t_2}} + \ldots + \frac{CF_N}{(1+Y)^{t_N}}}$$

where

D	Macaulay's duration
P	current paid bond price (including accrued interest)
Y	bond yield (assumed to be the same for all maturities)
CF_i	cash flow (coupon) received at time t_i
CF_N	cash flow (coupon plus principal) received at repayment date t_N
N	number of cash flows

1.5.2 Modified and Price Duration

$$D^{mod} = -\frac{1}{P}\frac{\partial P}{\partial Y} = \frac{D}{1+Y}$$

$$D^{P} = -\frac{\partial P}{\partial Y} = D^{mod} \cdot P = \frac{D}{1+Y} P$$

where

D^{mod}	modified duration
D^{P}	price or dollar duration
D	Macaulay's duration
Y	bond yield (assumed to be constant for all maturities)

1.5.3 Price Change of a Bond

Price change approximated with duration

$$\Delta P \cong \frac{-D}{(1+Y)} \cdot P \cdot \Delta Y = -D^{mod} \cdot P \cdot \Delta Y = -D^{P} \cdot \Delta Y$$

$$\frac{\Delta P}{P} \cong \frac{-D}{(1+Y)} \cdot \Delta Y = -D^{mod} \cdot \Delta Y = \frac{-D^{P}}{P} \cdot \Delta Y$$

1.5.4 Portfolio Duration

Portfolio Duration

$$D_V = \sum_{i=1}^{N} x_i \cdot D_i$$

where

D_V	portfolio duration
N	number of bonds in the portfolio
x_i	proportion of wealth invested in bond i
D_i	duration of bond i
V	value of Portfolio
Y	bond yield (assumed to be constant for all maturities)

1.6 Credit Risk

1.6.1 Ratio Analysis

Profitability ratios

$$ROE\ (Return\ on\ Equity) = \frac{Net\ income}{Net\ worth} =$$
$$\frac{Net\ income}{Pre\text{-}tax\ profits} \cdot \frac{Pre\text{-}tax\ profits}{EBIT} \cdot \frac{EBIT}{Sales} \cdot \frac{Sales}{Assets} \cdot \frac{Assets}{Equity}$$

$$ROA\ (Return\ on\ Assets) = \frac{EBIT}{Total\ assets}$$

$$Profit\ margin = \frac{Net\ income}{Net\ sales}$$

Liquidity Ratios

$$Current\ ratio = \frac{Current\ assets}{Current\ liabilities}$$

$$Quick\ ratio = \frac{Current\ assets\ \text{-}\ inventories}{Current\ liabilities}$$

Solvency Ratios

$$\text{Debt ratio (Leverage)} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Interest Coverage ratio} = \frac{\text{Pre - tax income plus interest (EBIT)}}{\text{Interest expenses}}$$

$$\text{Fixed-charge Coverage ratio} = \frac{\text{Pre - tax income plus interest (EBIT)} + \text{lease payments}}{\text{Interest} + \text{lease expenses}}$$

Activity Ratios

$$\text{Average collection period} = \frac{\text{Receivables}}{\text{Sales per day}}$$

$$\text{Fixed asset turnover} = \frac{\text{Sales}}{\text{Fixed assets}}$$

$$\text{Inventory turnover ratio} = \frac{\text{Cost of sales}}{\text{Average inventory}}$$

1.7 Convertible Bonds

1.7.1 Investment Characteristics

$$\text{Conversion ratio} = \text{Number of shares if one bond is converted}$$

$$\text{Conversion price} = \frac{\text{Face value of the convertible bond}}{\text{Number of shares per bond (if there is a conversion)}}$$

$$\text{Conversion value} = \text{Conversion ratio} \cdot \text{Market price of stock}$$

$$\text{Conversion premium (in \%)} = \frac{\text{Market price of bond - Conversion value}}{\text{Conversion value}}$$

1.7.2 Investment Strategies

Payback Analysis

$$PP = \frac{(MP - CV)/CV}{(CY - DY)} = \frac{Conversion\ premium}{(CY - DY)}$$

where

PP	payback period in years
MP	market price of the of the security
CV	conversion value of the convertible
CY	current yield of the convertible = (coupon/MP)
DY	dividend yield on the common stock = dividend amount / stock price

Net Present Value Analysis

$$NPV = \frac{CP - FV}{(1 + K_{nc})^n} - \sum \frac{FV \cdot (K_{nc} - K_c)}{(1 + K_{nc})^t}$$

where

CP	call price
FV	face value
K_{nc}	yield on non-convertible security of identical characteristics
K_c	yield on the convertible security
n	years before the convertible is called

1.8 Callable Bonds

1.8.1 Price of a callable bond

Callable bond price = Call-free equivalent bond price – Call option price

1.9 Fixed Income Portfolio Management Strategies

1.9.1 Passive Management

Immunisation (multiperiod, surplus)

$$A = L$$

$$D_A = D_L$$

$$A \cdot D_A = L \cdot D_L$$

where

A	present value of the portfolio
L	present value of the debt
D_A	duration of the portfolio
D_L	duration of the debt

1.9.2 Computing the Hedge Ratio: The Modified Duration Method

$$HR = \rho_{\Delta S,\Delta F} \cdot \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = \frac{S_t \cdot MD_S}{F_{t,T} \cdot MD_F}$$

$$N_F = -\frac{N_S \cdot S_t \cdot MD_S}{k \cdot F_{t,T} \cdot MD_F} = -\frac{N_S \cdot S_t \cdot MD_S}{k \cdot S_{CTD,t} \cdot MD_F} \cdot CF_{CTD,t}$$

where

HR	hedge ratio
$\rho_{\Delta S,\Delta F}$	coefficient of correlation between ΔS and ΔF
$\sigma_{\Delta S}$	standard deviation of ΔS
$\sigma_{\Delta F}$	standard deviation of ΔF
S_t	spot price at time t
$F_{t,T}$	futures price at time t with maturity T
MD_S	modified duration of the asset being hedged
MD_F	modified duration of the futures (i.e of the CTD)
N_F	number of futures contracts
N_S	number of the spot asset to be hedged
k	contract size
$S_{CTD,t}$	spot price of the CTD
$CF_{CTD,t}$	conversion factor of the CTD

2. Economics

2.1 Macroeconomics

2.1.1 Measuring national income and prices

GNP

$$Y = C + I + G + \left(X - M\right) + NIRA$$

where

Y	GNP
C	private consumption
I	investment
G	government expenditure
X	exports
M	imports
$NIRA$	net income received from abroad
$X - M + NIRA$	current account balance

National saving and current account balance

$$\begin{aligned} CA &= S^P + S^G - I \\ &= S^P - BD - I \end{aligned}$$

where

CA	current account balance
S^P	private saving
S^G	government saving
$S^P + S^G$	national saving (S)
BD	budget deficit
I	investment

Price index: GDP (implicit price) deflator and Consumer Price Index (CPI)

$$GDP\ deflator_t = \frac{Nominal\ GDP_t}{Real\ GDP_t} \cdot 100 = \frac{\sum_{it} p_{it} \cdot q_{it}}{\sum_i p_i^* \cdot q_{it}} \cdot 100$$

$$CPI_t = \frac{\sum_i p_{it} \cdot q_i^*}{\sum_i p_i^* \cdot q_i^*} \cdot 100$$

where

p_{it}	price of final good or service *i* in year *t*
q_{it}	quantity of final good or service *i* in year *t*
p_i^*	price of final good or service *i* in the base year
q_i^*	quantity of final good or service *i* in the representative basket

Inflation rate

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where

P_t	(index) price level at time *t*
P_{t-1}	(index) price level at time *t*-1
π_t	inflation rate over period *t*-1 to *t*

Ex-post Fisher parity

$$r_t \approx i_t - \pi_t$$

where

r_t	real interest rate for the period (*t*-1, *t*)
i_t	nominal interest rate for the period (*t*-1, *t*)

2.1.2 Equilibrium in the real market

Consumption function

$$C^D = c_0 + MPC(Y - T)$$

where

C^D	desired consumption
c_0	constant intercept term
MPC	marginal propensity to consume
$Y - T$	disposable income

Desired investment

$$I^D = Y - C^D - G$$

where

I^D	desired investment

Budget surplus

$$BS = T - (G + TR + NINT)$$

where

BS	budget surplus
T	taxation
TR	transfer payments
$NINT$	net interest payments on public debt

Government-purchases multiplier

$$Y = \frac{c_0}{1 - PMC} - \frac{PMC}{1 - PMC} \cdot T + \frac{1}{1 - PMC} \cdot I + \frac{1}{1 - PMC} \cdot G$$

where

$\frac{1}{1 - PMC}$	government-purchases multiplier

Aggregate demand

$$AD = Y = C^D + I^D + G$$

where

AD	aggregate demand
Y	aggregate income

2.1.3 Equilibrium in the money market

Money demand function

$$\frac{MD}{P} = b_0 + b_1 \cdot Y - b_2 \cdot i$$

where

MD	nominal money demand
P	general price level
Y	real income (output)
i	nominal interest rate
b_0	constant parameter
b_1, b_2	positive parameters

2.1.4 Aggregate supply and determination of price of goods/service

Quantity theory of money (absolute form)

$$M \cdot V = P \cdot Y$$

where

M	quantity of money
V	velocity, a measure of turnover of money stock in a year
P	general price level
Y	real income (output)

2.2 Macro Dynamics

2.2.1 Inflation

Expectations-augmented Phillips curve

$$\pi_t = \pi_t^e + \alpha - \beta(u_t - u_t^*)$$

where

π_t	inflation rate
π_t^e	expected inflation rate for time *t*
α	constant parameter
β	constant positive parameter
$u_t - u_t^*$	cyclical unemployment (or "Keynesian" unemployment) at time *t*

2.3 International Economy and foreign exchange market

2.3.1 Open macro economics

Balance of payments accounting

$$BP = CA + KA + \Delta RA$$

where

BP	balance of payments
CA	current account
KA	capital account
ΔRA	official reserve account

2.3.2 Foreign exchange rate

Real exchange rate

$$S_{real} = \frac{S \cdot P^F}{P}$$

where

S	nominal spot exchange rate (in American terms)
P^F	foreign general price level in foreign currency
P	domestic general price level in domestic currency