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Comments on any of the papers in this Forum are welcomed. Please submit them to Art Assantes, editor. They will be published in a future issue of the *Pension Section News*.

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The Pension
Forum

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Introduction to Yield Curve Forum

by Kenneth A. Kent, FSA, FCA

The concept of using a corporate bond yield curve for valuating pension plan liabilities exploded on the scene when the federal administration proposed using the yield curve as a means to replace the 30-year Treasury rate.

For many of us, our familiarities with the yield curve are associated with learning about forward rates and spot rates during our period of exam taking, but we have not worked with valuation systems that call for its direct application. And, it is clear the administration's approach of applying a corporate yield curve to a simple cash flow stream is more easily said than done when working with complex, multi-decrement models common among pension valuation systems. Nor may they be fully considering the implications on interest-sensitive benefits and rights common in today's pension plans.

What is of concern to members of the Society of Actuaries Pension Section Council is that actuaries have an understanding of the development of a corporate bond yield curve, including:

- The market basket of securities used to develop the curve,
- The quality of these securities,
- How the curve responds to changes in short- and long-term rates and
- How callable bonds are used and their impact if they are excluded from the curve structure.

Another important question is, if the corporate bond yield curve is the benchmark for liability calculations for pension funded status, can a similar portfolio of securities be purchased to hedge against interest rate risk?

As a first step in addressing the lack of literature that addresses this index as a viable market index to measure pension plan solvency, the Pension Section Council called for a paper to present a review and analysis of the yield curve and the securities used to develop the curve. We are pleased to present the results of our search with the first paper in this forum, "Understanding the Corporate Bond Yield Curve," by Holger Höfling, Rüdiger Kiesel and Gunter Löffler.

To accompany this centerpiece, we also include two papers that address some of the complex applications of a yield curve to benefits and valuations. Dick Wendt's paper, "Valuation of Pension Obligations with Lump Sums," provides insight into how to apply the curve in the valuation of lump sums. The yield curve could also offer some use as a tool in the valuation of expense under FAS 87 and FAS 106, as presented in the third paper prepared by Ron Iverson, Heidi Rackley, Steve Alpert and Ethan Kra titled, "Durational (Select & Ultimate) Discount Rates for FAS 87 &106 Valuations."

The appropriate application of the corporate bond yield curve may be debated for some time, even if it is ultimately deemed an appropriate means of measuring plan solvency. Regardless of debate, analysis of the yield curve offers insight into the valuation of liabilities as they relate to market interest rates and, accordingly, will continue to be of interest in relation to actuarial valuations of pension liability.

Please note that while we did not solicit discussants for this forum prior to publication, anyone wishing to discuss this forum further may send comments to the *Pension Section News* for publication. On behalf of the members of the Pension Section Council, I would like to thank all of the contributing authors for their participation in this forum.

Understanding the Corporate Bond Yield Curve

by Holger Höfling, University of Ulm
Rüdiger Kiesel, University of Ulm and LSE
Gunther Löffler, University of Ulm

Abstract

This paper discusses the construction of the corporate bond yield curve and possible applications to pension valuation. The first part addresses the mathematical theory and tools needed to extract the yield curve from corporate bond data and the issues that arise in contrast to the Treasury yield curve. Next, specific problems concerning the construction of the curve are explored, for example, the number of bonds available, the quality at different durations, etc. In the last part questions arising when applying the yield curve to pension valuation are discussed.

1. Overview: Issues in Constructing the Corporate Bond Yield Curve

The discontinuation of the 30-year Treasury rate made it necessary to introduce a new interest rate for valuing liabilities of pension funds. Current legislation proposes using a yield curve estimated from corporate bond data for this purpose.

In the literature several models are discussed that can be used to estimate yield curves: government and corporate. These methods are well documented and easy to use, provided that the necessary data are available.[1] The main differences in the estimation procedure between corporate and government bond yield curves are due to issues regarding the availability and quality of the data sets used for estimation purposes.

1.1 Government Bond Yield Curve

The U.S. government bond market is very liquid, resulting in price quotes that are accurate and up to date. The number of available issues is high and dense with respect to time and maturity, so that for every time to maturity, there is a bond that matures very close to it. Especially important is that for government bonds there is almost no default risk and all bonds are from one issuer so that uncertainty about date and amount of payments is reduced. Also, most government bonds do not have any special provisions such as call features, making it possible to use only bonds without options for the estimation process. In summary: since the quality of data sets available for U.S. government bonds is very high, yield curve estimation can be done with high accuracy.

1.2 Construction of a Corporate Bond Yield Curve

The situation is different for corporate bonds. Compared to the U.S. government yield curve, several important new factors that influence the corporate bond yield curve arise. The most important of these factors is the default risk, that is, the risk that the issuing company is not able (or not willing) to fulfill their obligations to pay coupons and/or pay back the face value of the bond. Since corporate bond holders face this default risk (which may be significant in the case of low-rated issuers), they ask for a premium to compensate for bearing it. Further issues are a lower liquidity of the market so that an investor has the risk of not being able to sell at every point

[1] A short introduction to these models can be found in the Appendix.

in time, different tax regulations for government and corporate bonds, and a larger share of bonds with call provisions or other special features. As corporate bonds face more risk factors, investors will ask for a premium for taking on these risks. Section 2 will discuss empirical results of the importance of these risks factors and the size of their contribution to the difference between the corporate and government bond yield curve (the credit spread). It will also briefly discuss bonds that include options, especially call options.

For constructing the yield curve, the data have to be sorted by the default risk of the company and whether the bonds include special options. Data on how many bonds are available for the different default risk classes — with and without options — and how the time to maturity is distributed are given in Section 3. After that, the quality of the yield curve, its behavior with respect to the interest rate environment, and the slope of the curve are discussed. The section ends with a discussion on bond indices and how they could be used to make valuation easier for small pension funds, as well as a description on alternative data sources for construction of a yield curve.

1.3 Application to Pension Funds

Section 4 discusses how the corporate bond yield curve might be applied in pension valuation and estimates what the effects would be in contrast to current legislation. Particular issues are interest-sensitive payment forms, cash-balance plans, and embedded options. It also discusses theoretical issues in smoothing rates of a corporate bond yield curve.

2. Credit Spread

When comparing a corporate and a government bond yield curve, one notices that the corporate bond yield curve is significantly higher than the government bond yield. The difference between these two curves is called the credit spread. The interest rate on government bonds is seen as the riskless interest rate, so that the credit spread can be viewed as the premium that investors ask for because of additional risks of corporate bonds. This section will first discuss the size of the credit spread, then give possible explanations and evaluation their importance.

2.1 Size of the Credit Spread

When estimating the yield curve for corporate and Treasury bonds, the yields on corporate bonds are significantly higher. One of the main reasons for this observed credit spread are credit risk and a risk premium that investors ask for taking on this risk. Here credit risk refers to the possibility that a company might default or that the rating of the company worsens, so that the bond loses value. Further factors responsible for the spread are options included in the bond, decreased (as compared to Treasury bonds) liquidity, and taxes.

The relative importance of these factors is the topic of ongoing research. To study the impact of credit risk one may sort the bonds by ratings according to one of the major rating companies such as Standard & Poor's or Moody's. However, ratings for companies in different sectors are not comparable, so one also has to distinguish by the economic sector of a company, for example financial or industrial. The impact of other factors may be taken into account by using only noncallable bonds, by eliminating all bonds with an issue size below a certain amount and by eliminating bonds with less than one year to maturity. Also, one should consider only bonds that are included in a major bond index to make trader quotes more reliable. Table 1 is taken from Elton et al. (2001) to illustrate the size of the credit spread and how it varies with rating class, time to maturity, and industry sector.

Table 1

Credit Spread by Time to Maturity and Rating Class

Maturity	Treasury Spot Rates	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
2	5.265	0.467	0.582	0.857	0.392	0.536	1.022
3	5.616	0.501	0.640	0.899	0.396	0.580	1.070
4	5.916	0.511	0.676	0.928	0.406	0.606	1.072
5	6.150	0.512	0.701	0.948	0.415	0.623	1.062
6	6.326	0.511	0.718	0.962	0.423	0.634	1.049
7	6.461	0.510	0.731	0.973	0.429	0.642	1.039
8	6.565	0.508	0.740	0.981	0.434	0.649	1.030
9	6.647	0.507	0.748	0.987	0.438	0.653	1.022
10	6.713	0.506	0.754	0.993	0.441	0.657	1.016

The data are average spot rates from 1992 to 1996, computed using bond data from the Fixed Income Database at the University of Houston. For more information on how the estimates were obtained see Elton et al. (2001).

While looking at the spreads, one notices how differently the credit spread changes with the rating category for financial and industrial bonds. For rating categories AA and A, the spread for industrial bonds is smaller; however, for BBB the spread for financials is smaller. One explanation for this behavior is different probabilities for credit rating changes. Nickell, Perraudin, and Varotto (1998) show in their analysis that the probability for downgrades for AAA, AA, and A is higher for financial than for industrial bonds, and the probability for upgrades is about the same. For BBBs it is the other way around: the probability for upgrades is higher, and the downgrade probability is almost identical. Another consideration is the low number of bonds issued by BBB financial companies (see Table 4). This also has been noted in Nickell, Perraudin, and Varotto (1998) and attributed to problems of running a bank if market confidence in the institution is low. Regulatory requirements might be another reason for the different credit rating transition probabilities.

2.2 Callability

Like Treasury bonds, corporate bonds are frequently callable, and this option in the bond changes the price. As this option is an additional risk for the bond holder, he or she wants a yield premium. Therefore, before constructing a yield curve the "option to call" has to be priced, and the yield curve should be adjusted accordingly. However, trying to account for the call option price is difficult and could introduce significant additional noise to the price of the bond. For recent advances into pricing of callable debt and some empirical analysis, see, for example, Berndt (2003), King (2002), and Duffee (1998).

Duffee (1998) states that callable bonds are much more sensitive to interest rate movements than are noncallable bonds. In his study he found that the credit spread on corporate bonds decreases with an increasing interest rate. As a call option on debt has a lower value for the issuer with rising interest rates, the spread on callable bonds decreases faster for increasing interest rates than it does for noncallable bonds.

King (2002) empirically analyzes the behavior of call option values in relation to the height and slope of the yield curve, interest rate volatility, industry group, rating of issuer, callable period or not, and call strategy of the issuer. All of these factors have a significant influence on the price of the call option. The height and slope of the yield curve especially are important. For example, when the short-term interest rate is about 3-4%, the yield curve has a steep slope, and interest rate volatility is high, the value of the call option is about 7.15% of the par value of the bond if the bond is in its callable period. On the other hand, for high short-term interest rates (>7%), flat yield curves, and low interest rate volatility, the value of the call option is about zero. However, call values are generally low if the remaining call protection period is longer than one year. This shows that to use callable bonds for yield curve estimation, some care and an estimate of the yield curve is needed. Several models are available that could be used to compute theoretical call option values. Because of these problems, callable bonds are usually excluded from estimation of yield curves in the scientific literature. The problems in valuing put options are similar, although the question has not been addressed as much as for call options. As the value of convertible bonds depends on the value of the underlying stock, these bonds are usually excluded as well. In our data set, about one-third of the bonds were callable. More information can be found below.

2.3 Default Risk

Corporations face a significantly higher default risk than do governments, which introduces a significant default risk for corporate bonds and has the effect that they are traded with a price discount in comparison to otherwise equivalent Treasury bonds. This discount varies from corporation to corporation because of different default risks. For estimating the yield curve, companies should have a comparable risk class. One way to do this is to use rating classes as an indicator for default risk. Then the estimation of the yield curve is made using only bonds of companies with a certain rating category, say, AA or A, provided by one of the major rating companies.

When valuing liabilities, one should also account for default risk. When investing the amount equivalent to discounted liabilities, one probably would not be able to service the liabilities because some bonds in the portfolio will default. In Elton et al. (2001) the authors suggest a simple method to estimate the rate spread due to default risk.[2]

However, as Elton et al. (2001) find, only about 10% of the credit spread for AA-rated companies is due to default risk, 20% for A-rated and 40% for BBB-rated bonds. The credit spread due to default risk assuming risk neutrality and no tax effects as estimated by Elton et al. (2001) can be found in Table 2. The credit spreads after adjusting for this default risk are shown in Table 3, and a graphical comparison of spot rates for Treasury and A-rated industrial bonds is shown in Figure 1. These findings are qualitatively confirmed by the more extensive analysis in Huang and Huang (2003). They calibrate most commonly used and analytically tractable structural credit risk models to historical data and calculate the percentage of credit spread due to credit risk (including a premium for taking on this risk) explained by the

[2] Let a be the constant fraction of the face value of the bond paid upon default. Let P_t be the probability of default in period t conditional on no default earlier. Let C be the constant coupon of the bond with face value 1, and r^C_{tt+1} and r^G_{tt+1} denote the corporate and government forward rate from period t to $t+1$, where the government rate is known. Then the rate spread due to default risk can be obtained recursively using the equations

$$\left[C + V_{t+1T}\right]e^{-r^C_{tt+1}} = V_{tT} = \left[C(1-P_{t+1}) + aP_{t+1} + (1-P_{t+1})V_{t+1T}\right]e^{-r^G_{tt+1}}$$

where V_{tT} denotes the value of the bond at time t, maturing at T. The probabilities and the recovery fraction are taken from transition matrices and recovery fraction tables provided by the major rating companies. The government rate is obtained by yield curve estimation using government bond data. Of course, there are several other ways to account for this default risk. Another possibility would be to use a structural or reduced form model. For an extensive empirical analysis using structural models see Huang and Huang (2003). For an introduction to structural and reduced form models see Bingham and Kiesel (2004).

Table 2

Amount of Credit Spread Due to Default Risk Assuming Risk Neutrality and No Tax Effects

Maturity	AA	A	BBB	Maturity	AA	A	BBB
1	0.000	0.043	0.110	2	0.004	0.053	0.145
3	0.008	0.063	0.181	4	0.012	0.074	0.217
5	0.017	0.084	0.252	6	0.023	0.095	0.286
7	0.028	0.106	0.319	8	0.034	0.117	0.351
9	0.041	0.128	0.380	10	0.048	0.140	0.409

Table 3

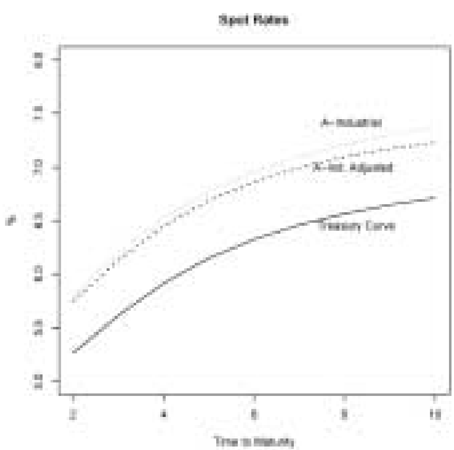
Credit Spread after Adjusting for Default Risk

Maturity	Treasury Spot Rate	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
2	5.265	0.463	0.529	0.712	0.386	0.483	0.877
3	5.616	0.493	0.577	0.718	0.388	0.517	0.889
4	5.916	0.499	0.602	0.711	0.394	0.532	0.855
5	6.150	0.495	0.617	0.696	0.398	0.539	0.807
6	6.326	0.488	0.623	0.676	0.400	0.539	0.763
7	6.461	0.482	0.625	0.654	0.401	0.536	0.720
8	6.565	0.474	0.623	0.630	0.400	0.532	0.679
9	6.647	0.466	0.620	0.607	0.397	0.525	0.642
10	6.713	0.458	0.614	0.584	0.393	0.517	0.607

models. They find that for bonds with an AAA, AA, or A rating and 10 years to maturity, the credit spread implied by these models is only about 20% of the actual observed credit spread. For BBB-rated bonds, this percentage increases to 30%. The percentage explained increases for lower-rated companies and decreases for bonds with shorter maturities.

Kiesel, Perraudin, and Taylor (2003) point out that especially for an investment-grade bond portfolio, changing credit quality (due to rating changes) has a significant impact on the observed credit spreads. This shows that even if the yields are adjusted for default risk, which should be done for valuing liabilities, the majority of the credit spread remains.

Figure 1—Spot Rates for Treasury Bonds and A-Rated Industrials, with and without Default Risk Adjustment



2.4 Liquidity

Two sources of pricing errors are related to bond liquidity. The first is a price discount for corporate bonds that do not have much liquidity. For bond holders, lack of liquidity introduces the risk that they might not be able to sell the bond at the time they want to. For this risk they expect a premium, which leads to a discount on the price. The second kind is called a stale price error. The bond prices usually are averages of dealer quotes. For bonds with a low liquidity, dealers might not update their quotes regularly because there is not much business to attract. This can result in prices that no longer reflect the market prices. Recently several papers have addressed these problems.

Diaz and Skinner (2001) regress yield errors of their analysis on proxies for liquidity like issue size and relative age of the bond. They show that the issue size is not a significant proxy for liquidity, which they attribute to the fact that they used only bonds included in the Lehman Brothers indices. These bonds must have a minimum issue size of 50 Mill.$ (starting 1989). The relative age of the bond, however, is clearly significant for the pricing error. For bonds that are close to maturity, the credit spread increases by several basis points (2-5 basis points per standard deviation change in relative age).

Another analysis was conducted by Perraudin and Taylor (2002). They find that liquidity plays an important role in determining credit spreads and that a substantial portion of the credit spread (up to 30 basis points, which is 30% to over 50% depending on rating class) for investment-grade bonds can be attributed to it. However, it is hard to find exact figures on how much of the credit spread can be attributed to liquidity. Often papers state just that the liquidity proxies are significant, but not the size of their effect.

2.5 Taxes

Another source for credit spreads is taxes. Treasury bonds are subject only to federal taxes, while corporate bonds are also subject to state taxes. As an investor earns only interest reduced by taxes, the lower earnings on otherwise equivalent bonds due to taxes lead to a premium for corporate bonds. Detailed information on how the authors adjust for taxes can be found in Elton et al. (2001), Delianedis and Geske (2001), Perraudin and Taylor (2002), or McCulloch (1971). We will just present some of the results of Elton et al. (2001).

In Elton et al. (2001) the authors assume the investor's federal tax rate to be t_g = 35% and a state tax rate of t_s = 7.5%. This leads to a marginal state tax rate of $t_s(1 - t_g)$ = 4.875%. This marginal tax rate minimizes the squared pricing error. With these figures they estimate that the credit spread due to taxes is about 35 basis points for all maturities, which would explain about 55% of the credit spread for AA-rated bonds, 35% for A-rated, and almost 25% for BBB-rated.

This shows that taxes account for a large part of the corporate-Treasury spread — especially for high-quality bonds. However, not all of the credit spread is explained by default risk, taxes, or liquidity.

2.6 Risk Premium

The risk premium is the part of the credit spread due to systematic market risk that cannot be diversified. Widely used measures for this risk in the equity market are the Fama-French factors introduced in Fama and French (1993). Elton et al. (2001) show that a large part of the spread after accounting for default risk and taxes can be explained by the Fama-French factors (between two-thirds and 85%). Similar results are presented in Perraudin and Taylor (2002) that also attribute a large part of the remaining pricing error to systematic risk proxies, after adjusting for default risk, liquidity, and taxes.

3. Issues in the Construction of the Corporate Bond Yield Curve

3.1 Depth

The data used for this section were obtained from Datastream[3] and analyzed using S-Plus.[4] We obtained data from 10,610 corporate bonds, traded in the United States and denominated in U.S. dollars, 3,162 being in the financial sector and 7,448 in the industrial sector. Government and mortgage bonds were excluded. After sorting out bonds that were callable, convertible, putable, or had sinking funds, no recent price, no current rating, no investment-grade rating, variable interest rates, or an amount outstanding less than 100 Mill.$ there were 4,026 bonds were left, 1,403 from the financial sector and 2,623 from the industrial sector. In terms of the amount of bonds issued, the data set included bonds with a face value of 4,399,979 Mill.$, 1,186,625 Mill.$ from the financial and 3,213,355 Mill.$ from the industrial sector. After the sorting process, bonds with a face value of 1,388,856 Mill.$ were left, 525,652 Mill.$ in financial and 863,204 Mill.$ in industrial-sector bonds.

The number of bonds for different rating categories and times to maturity is reported in Tables 4 and 5 and compared in Figure 2. The number of bonds outstanding in terms of face value for different rating categories and times to maturity is reported in Tables 6 and 7.

Table 4
Number of Issues per Rating Class and Time to Maturity by Industry Sector

Rating Class	Time to Maturity [0,1)	[1,2)	[2,3)	[3,5)	[5,7)	[7,10)	[10,15)	[15,20)	[20,25)	[25,30)	[30,inf)
All Bonds											
AAA	19	22	13	33	13	15	13	14	9	3	2
AA	30	40	29	61	21	28	13	11	14	14	5
A	117	173	164	308	158	277	98	64	97	104	42
BBB	123	184	165	337	229	425	131	82	149	142	35
Financial Sector											
AAA	16	16	10	24	10	5	4	3	3	1	2
AA	18	26	16	39	14	17	8	3	5	10	2
A	61	80	94	165	63	97	41	9	26	19	8
BBB	42	48	37	104	66	106	25	15	18	25	2
Industrial Sector											
AAA	3	6	3	9	3	10	9	11	6	2	0
AA	12	14	13	22	7	11	5	8	9	4	3
A	56	93	70	143	95	180	57	55	71	85	34
BBB	81	136	128	233	163	319	106	67	131	117	33

Table 5
Number of Issues by Rating Category

Bond Type	BBB	A	AA	AAA
All bonds	2002	1602	266	156
Financial	488	663	158	94
Industrial	1514	939	108	62

[3] A product of Thomson Financial

[4] A product of Insightful.

Figure 2—Overview of Issues per Rating Class and Time to Maturity by Industry Sector

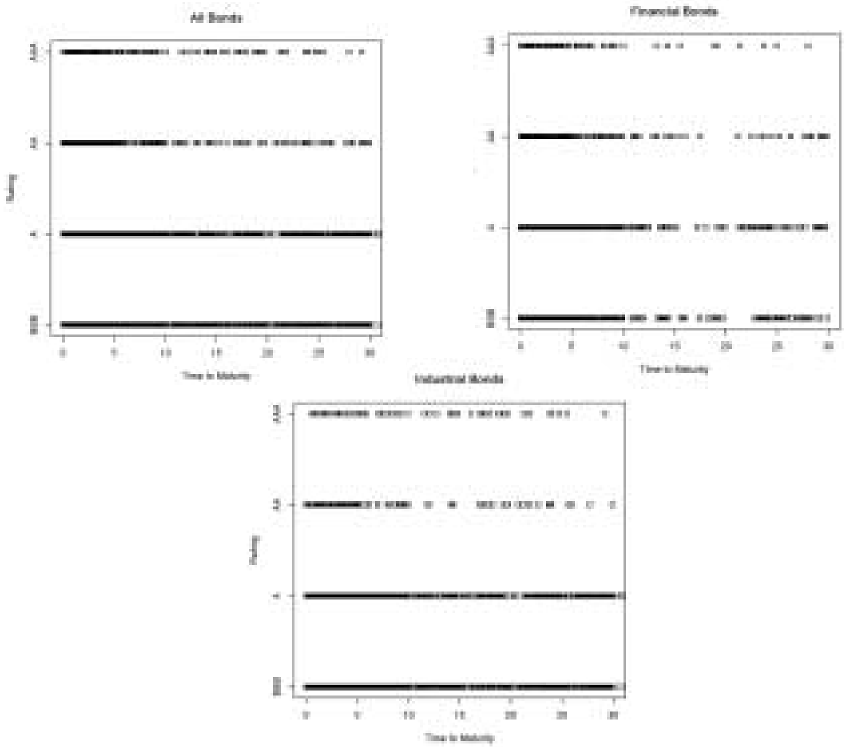


Table 6
Amount of Face Value of Issues per Rating class and Time to Maturity by Industry Sector in Mill.$

Rating Class	[0,1)	[1,2)	[2,3)	[3,5)	[5,7)	[7,10)	[10,15)	[15,20)	[20,25)	[25,30)	[30,inf)
						Time to Maturity					
						All Bonds					
AAA	8,470	9,692	6,361	14,280	4,819	12,915	3,321	3,357	2,693	1,550	475
AA	10,127	11,267	11,436	27,437	6,929	12,228	4,625	3,248	3,931	5,550	1,236
A	34,249	50,309	56,719	104,688	61,386	118,787	33,642	14,827	25,834	44,109	10,718
BBB	29,455	50,390	58,847	106,426	77,404	170,501	34,217	21,839	39,439	70,659	8,463
						Financial Sector					
AAA	7,195	7,212	5,561	11,530	4,333	4,700	600	834	400	750	475
AA	5,475	6,300	6,400	17,488	3,975	7,150	2,650	650	1,050	3,850	350
A	18,503	25,398	33,020	65,481	30,270	48,236	18,603	2,405	9,120	10,725	2,140
BBB	9,147	13,321	14,911	32,860	22,082	38,091	5,345	3,791	4,525	18,150	600
						Industrial Sector					
AAA	1,275	2,480	800	2,750	486	8,215	2,721	2,523	2,293	800	0
AA	4,562	4,967	5,036	9,949	2,954	5,078	1,975	2,598	2,881	1,700	886
A	15,746	24,911	23,699	39,207	31,116	70,551	15,038	12,422	16,714	33,384	8,578
BBB	20,308	37,069	43,937	73,567	55,322	132,410	28,872	18,048	34,914	52,509	7,863

In addition, one could also consider using callable bonds in the construction of a corporate bond yield curve. Altogether we had 3,644 callable bonds with a face value of 1,139,444 Mill.$, 621 (face value 215,928 Mill.$) financial and 3,023 (face value 923,516 Mill.$) industrial bonds. In Tables 8 and 9, we show these figures split up by their rating. However, after sorting the bonds using the criteria mentioned above, only 351 (121,350 Mill.$) bonds are left, 158 (59,197 Mill.$) financial and 193 (62,154 Mill.$) industrial-sector bonds. As these figures are rather low, we do not report them in more detail. The main reason for this low number (apart from unavailable ratings) is that most callable bonds have a speculative grade rating. One reason for this could be that in the currently rather low interest rate environment, mainly issuers with a low rating can hope for lower interest rates on their bonds (by getting a better rating), and therefore mainly they have a strong interest in being able to call the bonds.

It should be noted that we had only rating data from Standard & Poor's available, so these numbers are a lower boundary for the numbers and capitalization of bonds available (2,645 bonds with a face value of 1,049,224 Mill.$ did not have a current Standard & Poor's rating).

To estimate a yield curve, one has to use bonds with ratings of AA or lower because there are not very many bonds in the AAA rating category, especially at the long end of the yield curve. Mixing rating categories is not a feasible option, as this will lead to large pricing errors and make it harder to adjust for default risk (see Elton et al. 2001). In addition, mixing the AAA with the AA rating cat-

Table 7
Amount of Face Value by Rating Category in Mill.$

Bond Type	BBB	A	AA	AAA
All bonds	667,640	555,267	98,014	67,935
Financial	162,822	263,901	55,338	43,591
Industrial	504,819	291,366	42,676	24,344

Table 8
Number of Callable Bonds by Rating Category

Bond Type	NR	D	C	CC	CCC	B	BB	BBB	A	AA	AAA
All Bonds	1412	39	6	28	267	921	330	302	279	29	31
Financial	220	3	1	2	16	73	40	107	133	14	12
Industrial	1192	36	5	26	251	848	290	195	146	15	19

Table 9
Amount of Face Value of Callable Bonds by Rating Category in Mill.$

Bond Type	NR	D	C	CC	CCC	B	BB	BBB	A	AA	AAA
All Bonds	385,301	9,416	1,835	5,875	82,689	262,564	104,364	118,024	142,472	16,480	10,425
Financial	68,530	601	198	317	5,854	20,148	9,103	29,239	69,422	7,710	4,806
Industrial	316,276	8,815	1,637	5,558	76,835	242,416	95,261	88,785	73,050	8,770	5,619

egory would affect only the short end of the AA yield curve but hardly the long end, as almost no AAA bonds have a long time to maturity.

Of course, one faces the problem of the weight of one company in the estimation of the yield curve. The weight of a single company should not be too large because the company's problems would influence the yield curve estimation too much. In Tables 10 and 11,[5] we see that in the case of AAA-rated bonds, the largest company accounts for 13% of the number of bonds used. If we are looking just at the financial sector, it is even more — 20%. For AA-rated bonds, the biggest issuer accounts for about 6%, 3% for A-rated, and 1% for BBBs. In terms of the amount of outstanding bonds, the picture is even worse: 19% for AAA-rated bonds, 9% for AAs, 4% for As and 2% for BBBs. The weight of the biggest issuer is very high for AAAs, still high for AAs, and becomes negligible for lower-rated bonds. When looking at just one sector, the situation becomes worse in general.

3.2 Quality of the Curve at Different Durations

One way of describing the quality of the curve is to measure how well discounting future payoffs using the yield curve recovers the actual market price of the bond, or, what is the size of the difference between the market yield and the model yield on the bond.

Table 10
Biggest Five Issuers per Rating Category by Sector

BBB	A	AA	AAA
All Bonds			
21	38	16	19
19	30	13	14
19	27	12	7
16	23	10	6
16	22	10	6
Financial Sector			
13	38	16	19
12	30	12	14
11	27	9	6
11	20	8	6
9	15	8	5
Industrial Sector			
20	23	13	7
19	22	10	7
19	19	7	6
16	14	7	6
16	13	6	5

Table 11
Biggest Five Issuers (in Terms of Face Value)
per Rating Category by Sector

BBB	A	AA	AAA
All Bonds			
13,879	19,545	7,838	13,065
13,475	12,385	6,500	6,600
8,441	10,815	5,050	5,000
7,990	10,495	4,900	3,400
6,400	9,700	4,550	3,100
Financial Sector			
13,475	19,545	6,500	13,065
6,729	12,385	5,050	6,600
5,498	10,815	3,100	2,275
5,200	9,700	2,800	2,085
4,885	9,150	2,450	2,040
Industrial Sector			
8,441	10,495	7,838	5,000
7,990	8,050	4,900	3,400
7,150	7,600	4,550	3,100
6,400	7,100	2,750	2,625
6,100	5,000	2,200	2,167

[5] Inconsistencies between the tables for all bonds and the tables for financial and industrial bonds occur because Datastream classified some bonds of one issuer as financial and other bonds of the same issuer as industrial.

It is hard to find exact numbers on the pricing error of corporate bonds for different durations. Pricing errors for different durations are available only for government bonds. Overall pricing errors averaged over all durations are available for government and corporate bonds.

For government bonds we see in Bliss (1996) that the average absolute pricing error of a \$100 bond increases from \$0.01 for maturities up to 1 year to \$0.7 for maturities over 10 years. Bonds with long time to maturity are harder to price accurately than bonds that are close to maturity.

From Elton et al. (2001) we know that the average root-mean-squared error (RMSE) for treasuries is \$0.2 per \$100 face value. For corporate bonds, the RMSE is \$0.5 for AAs, \$0.9 for As, and \$1.2 for BBBs. The results presented in Diaz and Skinner (2001) for the errors in the yield of the bonds are similar. The RMSE for the yield on treasuries is 0.0002, compared with the RMSE for AA corporate bonds, which is 0.001, five times as big. Thus, pricing errors for corporate bonds are higher than the pricing errors for government bonds, which is not surprising as there are more unaccounted for sources of risk (e.g., different issuers, etc.). However, these pricing errors have to be interpreted with a bit of caution. Because of the mentioned unaccounted factors and pricing variability for bonds due to imperfect markets, no yield curve method gives zero error. Even for one method, increasing the number of bonds used for estimation does not necessarily lead to lower pricing errors. To some extent, these pricing errors just reflect the variability of bond prices in the market. With bond portfolios one has to consider how they behave in relation to each other. First, negative and positive pricing errors tend to cancel each other out. But, depending on the method used, it is possible that the errors have a structure, for example, the pricing errors for bonds with a long time to maturity all have the same sign, and the errors for short-term bonds have the opposite sign. This structural effect could also occur for bonds of one industry group or one issuer. It also depends on the estimation method and possibly other factors such as the current interest rate environment. In this case the value of liabilities could significantly deviate from the value of a bond portfolio with payouts that match the liabilities perfectly. Apart from this, one should note that the pricing error for different times to maturity will depend strongly on the minimization criterion in the yield curve estimation technique and what kinds of weights were used. In most studies the inverse of the duration of the bond was used as weight, where the error was measured as the difference of the actual and the model price. This means that less weight is put on long-term bonds.[6] Therefore, the pricing error for bonds with long times to maturity will be larger than the error for short-term bonds. Thus, when estimating the spot rate curve for pension valuation purposes, a weighting scheme that puts more emphasis on long-term bonds (e.g., constant weight for all bonds) might be better.

In addition to the pricing (or yield) error, one can also consider the stability of the estimate of the yield curve — in the sense of how much the curve would be affected by an additional error in one or several bonds. This also depends on the estimation method used, but as a general rule one can say that the more bonds are used in the estimation process, the more stable the curve will be.

3.3 Sensitivity Issues

An important question is how sensitive corporate bonds are to the interest rate environment. The corporate bond yield curve consists of the Treasury yield curve and the credit spread curve. So, depending on how much and in which direction credit spreads evolve, the corporate bond yield curve will move more or less closely with the Treasury yield curve. Duffee (1998) states that, with increasing interest rates, the spreads for

[6] Using this type of weighting gives approximately the same results as using the yield error without any weights.

noncallable AA bonds decrease by about 15% of the amount that the Treasury rates increase. Thus, the corporate bond yield curve moves in the same direction as the Treasury yield curve, but not quite as much. To illustrate the movement of corporate bond rates in comparison to Treasury rates, we compared the yields on the Lehman U.S. Corporate AA Long Term, Lehman U.S. Corporate AA Intermediate, and Lehman U.S. Corporate AA Aggregate Index to the yield on the 30-year Treasury bond. The data we used contained the yields at the end of every month, starting January 1980. Some summary data can be found in Table 12. In Figure 3 one can see the monthly changes in the yield on the Lehman U.S. Corporate AA Intermediate Index in comparison to the changes in the yield on 30-year Treasury bonds. Figure 4 shows the same for the long-term index. One can see that the long-term corporate bond rate changes more closely than the intermediate-term corporate bond rate. In Figure 5 we show the absolute value of the long-term corporate bond index yields in comparison to the Treasury bond yields. It shows a quite straight line, except at the lower end. These deviating observations are all very recent and thus probably due to problems with the 30-year Treasury rate.

Another issue is how interest rates react to the market environment. Elton et al. (2001) show that the risk premium is to a large percentage determined by the Fama-French factors, introduced in Fama and French (1993), which are

Figure 3—Monthly Yield Changes of Lehman U.S. Corporate Intermediate vs. 30-Year Treasury Rate

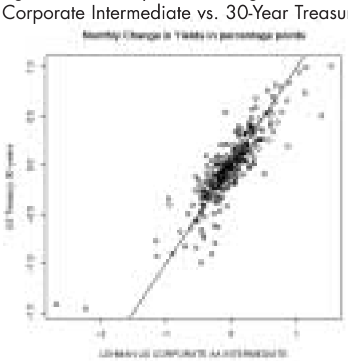


Figure 4—Monthly Yield Changes of Lehman U.S. Corporate Long Term vs. 30-Year Treasury Rate

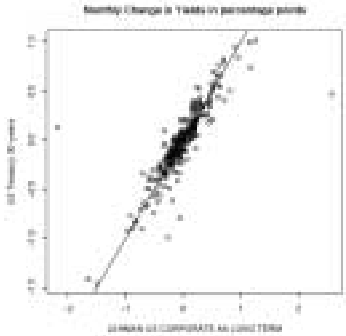


Figure 5—Yields of Lehman U.S. Corporate Long Term vs. 30-Year Treasury Rate

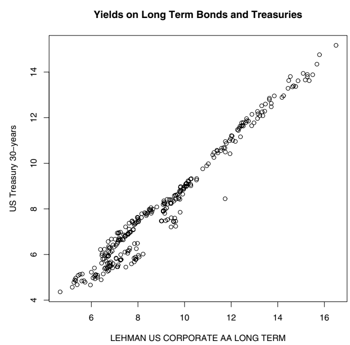


Figure 6—Slope of Government Bond Yield Curve vs. Slope of Corporate Bond Yield Curve

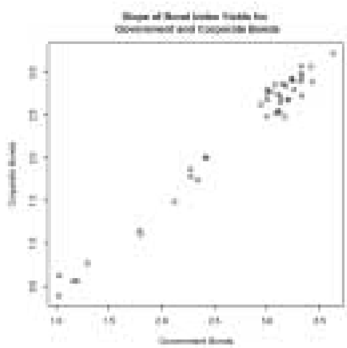


Table 12
Summary Information on Yields of Lehman U.S. Corporate Intermediate, Long Term, Aggregate, and U.S. 30 Year Bonds

	Intermediate	Long Term	Aggregate	Treasury
Mean	8.375	9.912	8.683	8.190
Max	16.610	16.500	16.540	15.171
Minimum	2.800	4.660	3.130	4.361
SD of ΔY (per year)	1.475	1.435	1.371	1.242
Mean duration	4.039	9.004	6.115	11.721

Note: SD of ΔY is the standard deviation of the annual change of the yeild.

three indices that describe systematic risk in the stock market. This means that if the systematic risk in the stock market measured by these factors increases, the risk premium and thus the corporate bond yield curve would increase.

3.4 Curve Behavior

As we have seen above, the Treasury yield curve and the corporate bond yield curve move very closely. This can also be seen in Figure 6. Here we used the yields on the Merrill Lynch AA-rated Corporate Bond Index with times to maturity of 1-3 years and 10-15 years and the Merrill Lynch Government Bond Index for 1-3 years and 10-15 years to maturity. We calculated the difference of the yields for the 10-15 year indices and the 1-3 year indices to get a proxy for the slope of the yield curves. We then plotted the proxies for the government slope against the proxy for the corporate slope. For the Treasury yield curve, data from Datastream indicate that during the last few years, yield curve inversions occurred only for a few months in 2000. The data presented above suggest that this is also true for the corporate bond yield curve. However, we did not compute the corporate bond yield curve explicitly for this time period, so we cannot verify this statement directly.

Another issue is how the yield spread depends on the time to maturity. From the spread data presented earlier, we can see that for AA-rated bonds, the spread increases with time to maturity. However, we can also observe that for other rating categories, the spread is upward sloping at the short end but downward sloping at the long end of the curve.

3.5 Determinants of Credit Spreads

As mentioned above, the most important factors explaining the credit spread are default risk, taxes, risk premium, and liquidity. The default risk of a company is measured using rating categories. The credit spread due to default risk is higher the lower the rating of the company is. Default risk is responsible for 10% up to 40% for AA- to BBB-rated companies. Another reason for the spread is the different tax regulations for corporate and Treasury bonds. For corporate bonds the investor has to pay taxes on the state level, from which Treasury bonds are exempt. Taxes explain 55%-25% of the spread for AA to BBB bonds. The rest of the spread can be attributed to risk premium and liquidity. Liquidity certainly plays a role, but the size of its effect is probably quite small — up to a few basis points. The remaining spread after adjusting for the other factors is explained in large part by proxies for systematic risk factors in the stock market. All in all, these four components explain most of the credit spread. However, one should note that the variables used above have only limited power to explain monthly changes in the spreads of corporate bonds. These changes are driven by another, still unknown factor. In Collin-Dufresne, Goldstein, and Martin (2001), the authors suggest that these changes are driven by local supply/demand shocks.

3.6 Bond Indices

When looking at bond indices, one should note especially two issues. First, if callable bonds are included in the index, the yield on this index will be higher than the yield for an otherwise equivalent index containing only noncallable bonds. Furthermore, the index will react more weakly to interest rate changes. When interest rates are increasing, the call option loses its value, and this reduces the additional spread due to the call option. Thus, the yield on the index will increase less than the yield on an index of noncallable bonds.

As an example with special application to the pension system, we discuss the Citigroup Pension Liability Index.[7] This index is created in two steps. First, a yield curve for AA-rated corporate bonds is derived. Then this yield curve is applied to a typical pension liability profile.

For the yield curve the Treasury yield curve is derived first. The reason is that the Treasury market has a very large amount of outstanding bonds and is very liquid. This allows one to get a stable estimate for the spot rate curve. Then the par yield curve (the coupon required for a bond with a certain maturity so that it trades at par) is calculated from these data (this is a unique transformation — no information is lost). The average spread of corporate AA bonds over Treasury bonds for a certain maturity is added to the Treasury par yield curve. This estimate of a AA corporate bond par yield curve is then transformed back into a spot rate curve (which gives the yield on a zero-coupon bond). To estimate the spreads on coupon bonds, they also use callable bonds, adjusted for the price of the call option. As they have a Treasury yield curve, there are various theoretical models available that can be used to adjust for the value of the call option. However, to reduce volatility of the call option price, they restrict themselves to callable bonds that have at least three years of call protection left and have a spread of at least 10 points between the earliest call price and its market price.

Afterwards, they apply this spot rate curve to the valuation of a typical pension liability portfolio and report the average duration and the average annualized yield on this portfolio. For pension funds whose liability profile is similar to the one used by Citigroup, this single annualized yield can be used as a discount factor and will give very similar results to what one would get if the liabilities of the plan were discounted using the whole spot rate curve. However, this is not the case for pension plans with a different liability profile. Such plans should use the whole spot rate curve for their liability valuation (which is being published as well). More information can be found in Bader (1994) and Bader and Ma (1995).

3.7 Credit Default Swap Spread Curves

Recently much research on the relationship between default-free interest rates, default-risky interest rates, and default swap rates has been done. See, for example, Hull, Predescu, and White (2003), Hull, Nelken, and White (2003), Houweling and Vorst (2001), Longstaff, Mithal, and Neis (2003), and Blanco, Brennan, and Marsh (2003). The main issues these papers address is whether the credit spread is close to the default swap rate and how default swap rates behave with respect to changes in credit ratings.

The main problem in determining the credit spread of a bond is the choice of the risk-free rate. Usually bond traders use the Treasury yield curve as the risk-free rate. However, a few problems exist with this rate. The main issues are the quite high spread of AAA bonds, different tax regimes, and different regulations for financial institutions that hold government and corporate bonds. Therefore, another popular choice for the risk-free rate is the swap zero curve, which is calculated from LIBOR deposit rates, Eurodollar futures, and swap rates. These rates are liquid with a low credit risk and are also usually higher than the corresponding Treasury rate.

The papers mentioned above use different choices for the risk-free rate. Longstaff, Mithal, and Neis (2003) use the Treasury rate. They find that the default swap rate is significantly lower than

[7] Formerly the Salomon Brothers Pension Liability Index

the rate implied by the credit spread (using a simple reduced form model). These findings are consistent with the analyses of Houweling and Vorst (2001) and Hull, Predescu, and White (2003). They state that using the swap rate as the risk-free rate reduces the difference between the default swap rate and the credit spread. More specifically the default-free interest rate implied by the default swap rate lies between the Treasury rate and the swap rate, about 83% of the way from the Treasury rate to the swap rate. This shows that the risk-free rate used by the market is the swap rate. The Treasury rate is perceived by the market to be too low.

Longstaff, Mithal, and Neis (2003) further analyzed the difference between the credit spread on the bond and the default swap rate for the same company. They found that it varies between 12 and 100 bps for different companies and can be explained at least in part by different liquidity for the companies. With respect to changes in credit ratings, Hull, Predescu, and White (2003) find that negative changes in the ratings can be predicted by rising default swap rates. These results are much weaker for positive changes in credit ratings.

3.8 Emerging Market Bonds

In addition to corporate bonds, emerging market bonds may be considered. However, analyzing the credit spreads observed in the market for these bonds proves even more involved than for Treasuries. The main reason for this increased complexity is the additional impact that macroeconomic factors (inflation, exchange rates, commodity prices, etc.) and political factors (stability of governments) has on the bond price movements.

4. Applications of a Corporate Bond Yield Curve in Pension Liability Management

4.1 How the Yield Curve is Applied in Pension Valuation

4.1.1 Economic Assumptions

The most important economic and actuarial assumptions in the context of pension valuation are the inflation rate, investment return rate, discount rate, and compensation scale. Here we discuss the economic assumptions with regard to the use of the corporate bond yield curve to obtain the discount rate. During our discussion we do not distinguish between the investment return rate and the discount rate.

When valuing pension liabilities using a yield curve, different discount rates for each future cash-flow date are used. The discount appropriate for a point in time n years from now is $e^{-n \cdot R(0,n)} = e^{-\int_0^n f(0,t)dt}$, where $R(0,n)$ is the n-year spot rate taken from the yield curve. This means that in the period from the beginning of year n to the end of year n, the forward rate $R(0, n-1, n)$ is used for discounting (see the Appendix for the definition of the forward rate). To have consistency within the set of different assumptions, it seems necessary to use variable inflation rates. As we are using a yield curve for discounting, we have to deal with an implicit inflation component. The inflation rate for a specific year should be below the discount rate (investment return rate) effective for the same time period ($R(0, n-1, n)$ for time period $[n-1, n]$) to avoid negative real returns. So selecting inflation rates based on historical data unrelated to the yield curve model will be problematic. Instead, to ensure consistency, one should estimate the inflation component from government or high-quality corporate bonds. Especially in situations where the yield curve has a high slope, it might be necessary to select different inflation rates for different points in time. One possibility is to use the corporate bond forward rate minus a historically justified premium for default risk (if not already adjusted for that), risk premium, and real risk-free rate of return. Some care should be taken so that the resulting inflation rates do not become negative. One could address this by subtracting a certain percentage instead of a constant from the forward rate.

Constructing the compensation scale by combining inflation, productivity growth, and merit scale and using the above estimate for the inflation assumption would ensure consistency. See Actuarial Standards Board (1996) and Winklevoss (1993) for additional information.

4.1.2 Using the Yield Curve

When valuing payments, especially annuities, not only the age of the recipient but also the starting date is important. The valuation of a life annuity for a person aged x, starting n years from now, is then:

$$\ddot{a}_{x,n} = \sum_{t=n}^{\infty} {}_t p_x^{(m)} e^{-t \cdot R(0,t)},$$

where ${}_t p_x^{(m)}$ is the probability of survival for t years for a person aged x, and r_t is the t-year spot rate. Observe that by using the yield curve approach we replace all interest rates that used the 30-year Treasury rate until now with a spot rate from the yield curve and the appropriate time to maturity. If the yield curve is calculated only for maturities up to 30 years, cash flows more than 30 years in the future should be discounted using the last available rate. All other valuation formulas are adjusted similarly, exchanging the fixed interest rate against the appropriate t-year spot rate. However, some care should be taken, as some simplifications used in the derivation process in the case of a single interest rate might not be possible in a yield curve environment.

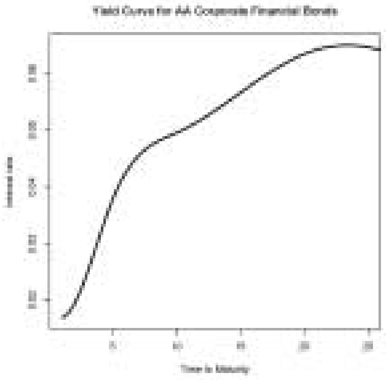
Based on the combined mortality RP-2000 tables with 50% males and 50% females, we calculated the value of a life annuity (no death benefits, no survivor benefit, one payment date per year at the end of the year), payable at age 65 for different current ages x. People older than 65 are considered to be retired already. For the interest rates, we used four scenarios.

The first scenario is the 30-year Treasury rate at 3 March 2004 of 4.98%. The second scenario is a corporate bond yield curve computed using AA-rated financial bonds (we used financial-sector bonds, as there were more available with a AA rating). The curve is shown in Figure 7. Here we held the interest rate constant at the 25-year level for all maturities longer than 25 years. The third scenario is the average yield on the Merrill Lynch U.S. Corporate Bond Index with AA ratings and time to maturity longer than 15 years. The average

Table 13
Life Annuity Values, Starting at Age 65, for Varying Interest Rate Scenarios

Age	(1)	(2) Scenario	(3)	(4)	Change 2-1	Change 3-1	Change 4-1
80	7.03	7.24	6.79	7.41	3.00%	-3.44%	5.41%
75	8.68	8.84	8.31	9.09	1.80	-4.26	4.72
70	10.34	10.36	9.81	10.71	0.18	-5.11	3.60
65	11.93	11.73	11.22	12.20	-1.66	-5.95	2.21
60	8.94	8.25	8.10	8.78	-7.71	-9.44	-1.79
50	5.26	3.89	4.42	4.35	-26.14	-16.05	-17.34
40	3.16	1.94	2.46	2.28	-38.63	-22.17	-27.73
30	1.91	1.01	1.38	1.25	-47.09	-27.84	-34.49
20	1.16	0.53	0.77	0.69	-54.38	-33.10	-40.62

Figure 7—Yield Curve Constructed Using AA-Rated Financial Bonds



redemption yield on this index is 5.737 as of 3 March 2004. The last scenario is the corporate bond yield curve with AA-rated financial bonds again, but this time a constant spread of 50 basis points was subtracted as a risk adjustment. The 50 basis points are about the average spread reported in Table 1, and therefore the last scenario is a proxy for the Treasury yield curve.

In Table 13, we see that when using the corporate bond yield curve instead of the 30-year Treasury rate, liabilities increase by 0–3 % for older individuals and decrease up to 50% for younger individuals. When using the corporate AA bond index, all liabilities would decrease, by 3–5% for older individuals and up to 33% for younger individuals. In the last scenario we changed Scenario 2 by subtracting a constant risk adjustment. Compared to the 30-year yield curve this leads to higher increases in liability for older people (3–6%) and lower decreases for younger people (up to 40%).

The reason for this behavior is that, for an upward sloping yield curve, the short end of the corporate bond yield curve can be lower than the 20-year Treasury rate. This will lead to higher values of short-term liabilities. This is offset by the higher yield on long-term corporate bonds, which leads to lower values for long-term liabilities. When introducing the yield curve, pension plans with long-term liabilities will benefit more than those with short-term benefits, provided the yield curve is not inverted.

All in all, we see that in the current situation, exchanging the 30-year Treasury rate for a AA corporate bond index would lead to lower liabilities for pension funds. For a corporate bond yield curve, the situation is not that clear. Without a risk adjustment, increases would be quite small and only for a small group of people, but when using a risk adjustment, the increases would occur for all current retirees, while almost all people younger than 65 still working would have lower liabilities. The situation of a specific pension fund depends then on the age structure of that fund.

Immunizing a pension fund against changes in the yield curve would require the fund to replicate the expected liabilities with their bond portfolio.[8] However, with corporate bonds, which are defaultable, matching payouts or only matching the duration of the portfolio and the liabilities presents another problem. The effective duration of a corporate bond is shorter than its actual duration because default can occur. The earlier payout of (part of) the principal leads to a reduced duration of the bond portfolio so that changing interest rates would affect the liabilities and the bond portfolio differently.

Some institutions have voiced concerns that using a corporate bond rate will decrease calculated pension liability values, therefore requiring less funding by the companies, and in effect eroding the financial basis of the pension system. However, when discussing whether liabilities will increase using the yield curve instead of the 30-year Treasury rate, it has to be noted that this very much depends on the exact yield curve used (type of bonds, risk adjustments) and the interest rate

[8] Other sources of risk such as uncertainty in the amont of future payments (e.g., due to salary increases or inflation adjustments) or the time point of the future payouts (e.g., uncertainty about the time of retirement and death of the plan participants) remain.

environment. Both scenarios with an increase and a decrease in liabilities are possible. Overall, it can be said that the higher the risk adjustment and the better the quality of the bonds, the lower the resulting interest rates will be and thus the higher the liability values. Furthermore, as we compare a long-term rate against a yield curve, the behavior of the yield curve at the short end is important. If the interest rate at the short end is higher than the 30-year Treasury rate (in a scenario of a flat or inverted yield curve), liabilities would decrease. In the other case, liabilities would tend to increase.

All in all, the question whether liabilities will increase or decrease cannot be answered in a general sense at this point, and even after the exact yield curve methodology is known, both situations may occur. From this point of view the mentioned concerns of eroding the funding basis very well may be justified. However, valuing future payments using a yield curve approach (depending on risk adjustments) most accurately measures how much money has to be put into a portfolio of high-quality corporate bonds to be able to meet future obligations. Of course, using a lower rate increases liabilities and therefore improves funding. An in-depth study using various modeling approaches and interest rate scenarios should shed some light on the question of which actions are most appropriate to sustain of increase funding levels (low interest rates, higher required funding ratios, etc.). Unfortunately, conducting such a study is beyond the scope of this paper.

4.2 Discussion of Issues in Valuation of Pension Plans

4.2.1 Interest-Sensitive Payment Forms

We now discuss the effect of valuation rates for calculating lump-sum payments and for the liabilities of an employee. The calculated liability reflects the amount of money set aside by the pension plan to fund the benefits of the employee. The use of a lower interest rate than the yield curve used in liability calculations would increase the lump-sum payment. This would lead to increased payouts, which would deplete the pension plan funds, making additional funding necessary. Apart from this, it would be an incentive for retirees to take the lump sum instead of the annuity — increasing their personal old-age funding risk. On the other hand, an interest rate that is higher than the yield curve would discriminate against those who choose the lump sum and reallocate pension money to the other pension plan participants. Therefore, when calculating lump-sum payments and valuing liabilities, it is important to use the same interest rates.

As the short end of the yield curve is more volatile than the long end, using the yield curve for lump-sum calculations will increase the volatility of the lump sum, making it harder for the employee to estimate what amount of money he or she can get upon retirement. This, however, just reflects the volatility in the plan assets (also in the case of bonds), which the pension plan has to face as well. Using a less volatile interest rate for these valuations would lead to the problems outlined above and in addition reallocate market risk to the other plan participants or the funding company.

To summarize, the only way to make the pension fund indifferent toward whether annuities or lump sums are paid is to compute lump sums using the liability discount rates. It should be noted that we have ignored mortality adverse selection in this discussion.

However, another issue has raised concerns among actuaries in the United States. New legislation contemplates requiring using today's yield curve for calculating future lump-sum values instead of the estimated future yield curve. We will show the effects of these two possibilities with a short example. Assume an employee aged x today ($t = 0$) would currently get \$1 per year upon retirement at the normal retirement age r. We want to value this liability using today's yield curve.

We get

$$
\begin{aligned}
L &= \sum_{k=r-x}^{\infty} {}_k p_x \delta(0,k) = \sum_{k=r-x}^{\infty} {}_k p_x \exp(-kR(0,k)) \\
&= {}_{r-x}p_x \exp(-(r-x)R(0,r-x)) \sum_{k=0}^{\infty} {}_k p_r \exp(-kR(0,r-x,r-x+k)) \\
&= {}_{r-x}p_x \exp(-(r-x)R(0,r-x)) L_{r-x},
\end{aligned}
$$

where the second equality holds because $(T-t)\,R(t,T) = (T_1-t)\,R(t,T_1) + (T-T_1)\,R(t,T_1,T)$, which can be shown by a brief calculation. Here L_{r-x} is the liability at $t = r-x$ years, if the employee is still living, and using the yield curve that the market expects in $r-x$ years when the employee reaches retirement age (the appropriate forward rate curve), that is, the lump sum the employee could get upon retirement. We see that today's liability L is the same as the liability in $r-x$ years, L_{r-x}, multiplied by the probability that the employee is still living $r-x$ years from now and discounted using the $r-x$ year spot rate. Therefore, the plan is indifferent between the annuity and the lump-sum payment. What happens if we use today's yield curve to compute the liability L_{r-x} instead of the appropriate forward rate? We then get

$$
\tilde{L}_{r-x} = \sum_{k=0}^{\infty} {}_k p_r \exp(-kR(0,k)).
$$

For an upward sloping yield curve, the liability when using today's yield curve instead of the future rates would be higher and lower for downward sloping yield curves. Using the yield curve from above, we get the changes in liability at time $t = 0$ shown in Table 14.

4.2.2 Cash Balance Plans

Cash balance plans have become more popular in recent years, so we want to discuss them a bit more closely. In a cash balance plan a fixed percentage of pay is credited on an account. Apart from this, interest on the account is also credited to the account. The interest rate can be a fixed rate, change from year to year, or be tied to an index. How is the actuarial liability affected by these choices? First, it is in general not equal to the account balance of the employee and in fact can vary very much, depending on the chosen actuarial funding method. Lowman (2000) showed in his study that the funding method can change the actuarial liability of an employee up to 40% (at a certain age of the employee; other assumptions are omitted here). Of course, the interest rate for the plan also has a large impact. However, no general method exists for choosing the interest rate so that actuarial liability is close to the account balance for every plan participant all the time. For simplicity, we will discuss this just for the traditional unit credit method. Other methods will require different solutions. In the case of the unit credit method, the account balance is projected to the normal retirement age using the plan's interest rate or an estimation in case it is still unknown. Then the resulting amount is discounted back using the spot rate curve. The case becomes

Table 14
Changes in Liability of Annuities if Using Spot Rate for Lump-Sum Valuation instead of Forward Rate
(Retirement Age 65)

Age	Forward Rate	Spot Rate	Change Spot-Forward
60	8.25	9.34	13.10%
50	3.89	4.67	20.24
40	1.94	2.14	10.62
30	1.01	1.12	10.62
20	0.53	0.58	10.62

particularly easy if the one-year spot rate of the curve is used as the account interest rate. The estimates for future interest rates would then be the appropriate forward rates, and the projection and discounting back would just set each other off. Of course, even then the actuarial liability still does not have to be equal to the account balance because of possible forfeitures (termination of employment before vesting, death). However, if the account balance is paid as a lump sum in case of death before retirement, it should be close, as no leveraging has been anticipated. When a fixed interest rate is being used, this largely depends on the current interest rate environment. If the account interest rate is below the spot rate, leveraging will lead to an actuarial liability that is below the account balance.

Another related issue occurs in the case where the employee wants to have pension benefits in the form of a lump sum earlier than the scheduled retirement age. Depending on the interest credit rate, the account balance projected to retirement age and discounted back using the spot rate curve can be higher or lower than the account balance. IRS Notice 96-8 deals with the important question under which circumstances the plan has to pay more than the account balance to an employee who receives a lump-sum payment. The minimum lump-sum computation rules of IRC Section 417(e) prescribe projecting the account balance to retirement age, converting it to an actuarially equivalent annuity, and discounting it back to the employment termination date using the 30-year Treasury rate. If this amount is greater than the account balance, the employee has to receive the higher amount. IRS Notice 96-8 also determines under which circumstances these calculations do not have to be performed. How these regulations change when a yield curve is used is still unclear. More information on cash balance plans can be found in Lowman (2000), Coleman (1998), and Kopp and Sher (1998).

4.2.3 Embedded Options in Plan Structure

Pension plans often include provisions for alternative retirement ages, additional benefits for early retirees, or shutdown benefits. When using the yield curve, the valuation formulas have to be adjusted appropriately to reflect different interest rates for payments at different points in time. However, stating whether liabilities will increase or decrease is not possible for the reasons mentioned above. Table 15 reports values of annuities for employees aged x, triggered at once, using the scenarios and assumptions mentioned above. The effect of changing from the Treasury rate against to the yield curve seems to have a significant impact, especially for young people. When using the yield curve with the risk adjustment, this effect is substantially reduced. However, without a detailed study, no firm statement can be made on the effects on embedded options of a change from a Treasury to a yield curve model.

4.2.4 Smoothing of the Yield Curve

Until now the discount rate used for measuring plan obligations has been a 4/3/2/1 weighted average, going back in time, of the Treasury rates of the last four years. This weighting is designed to reduce volatility in the market. It has been claimed (see, e.g., Ryan Labs, Inc. 2001) that no averaging should be used for valuation purposes. When hedging a risk such as future payments, one can transact only at market prices. Using weighted interest rates may lead to the situation that the actuarial liability and the cost for future

Table 15
Life Annuity Values, Payable at Different Ages for Varying Interest Rate Scenarios

Age x	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Change 2-1	Change 3-1	Change 4-1
40	17.57	15.87	15.90	16.83	-9.69%	-9.49%	-4.19%
50	15.89	14.78	14.58	15.57	-6.98	-8.25	-2.01
55	14.76	13.97	13.64	14.65	-5.34	-7.54	-0.69
60	13.42	12.95	12.52	13.52	-3.54	-6.76	0.74

payments on the market may deviate. Also, the reduction of volatility will depend on the particular asset class or payment value under consideration. While the overall liability of a pension may be more volatile using a yield curve, the difference between liabilities and asset values may (depending on the assets) become less volatile. For example, consider a fully funded pension plan with a portfolio of bonds that match future payments. Using an unsmoothed yield curve, the effect of changing interest rates is the same for the asset and the liability sides. When using smoothing, over- or underfunding can occur, although the portfolio still matches future payments because the liability side is affected differently than the asset side.

Another issue is the volatility of pension contributions to meet newly acquired obligations. The volatility just reflects the underlying economic conditions. Another, more transparent, way of stabilizing contributions is to adapt the funding measures used.

Even if smoothing of the yield curve should be adopted, some issues remain unclear. Smoothing might result in "strange" shapes of the yield curve. This can be undesirable, as one gets unrealistic scenarios. Apart from this, the way smoothing should be done and especially in which sense this smoothing method would be optimal remain to be discussed.

5. Conclusion and Issues for Further Research

The proposed termination of the 30-year Treasury bonds by the U.S. government make it necessary to consider different interest rate than the 30-year Treasury rate for pension valuation purposes. An investment-grade corporate bond yield curve measures today's market value of future payments quite accurately, the better the higher the rating of the bonds. The mathematical tools needed to extract the yield curve from market bond data are well developed and well tested. One issue when using a corporate bond yield curve instead of a Treasury yield curve is the additional risk imposed by changing credit spreads. While the determinants of credit spreads appear to be well understood, there is still the need for an investigation of the impact this additional source of volatility has for the actuarial issues. The main question to be addressed is how one should account for default risk and its risk premium. Various approaches have been suggested, but a detailed study comparing these suggestions on a theoretical level and with respect to their practicability remains to be done. In addition, the use of a yield curve requires information about the underlying financial market and the economic environment. To justify the various assumptions used, a careful empirical analysis of estimated parameter values has to be performed. As an example, consider the problem of forecasting several years into the future. One approach is to use long-term interest rates. How accurate these estimations are and whether they are consistent with other estimation techniques for the inflation rate still needs to be discussed. Whether the estimated inflation rate is consistent with the interest rates that are used on the financial market is not new — it just does not appear explicitly when using the 30-year Treasury rate. Information on future increases in salaries is another difficult issue. How to deal with varying informational quality of the economic assumptions is also in question. Because of these issues, reaching consistency between the various assumptions is harder when using a yield curve than it is for a single interest rate. Finally, whether the yield curve should be smoothed to reduce interest rate volatility or not still needs to be discussed.

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Appendix
Models for Yield Curve Estimation

A.1 Introduction to Interest Rates

The price of capital (e.g., the interest rate) depends on the time period for which it is borrowed and on the time when the contract is settled. To be able to talk about these prices, some terminology is needed. By p(t,T) we denote the price of a risk-free zero bond at time t that pays one unit of currency at time T. We will use the interest rates with continuous compounding, that is, a zero bond with interest rate r maturing at T will have the price p(t,T) = e–r(T–t). On a basic level, one distinguishes between the following interest rates:

1. The forward rate at time t for time period $[T1, T2]$ is defined as

$$R(t, T_1, T_2) = \frac{\log(p(t, T_1)) - \log(p(t, T_2))}{T_2 - T_1}.$$

2. The spot rate for the time period $[T_1, T_2]$ is defined as

$$R(T_1, T_2) = R(T_1, T_1, T_2).$$

3. The instantaneous forward rate is

$$f(t, T) = -\frac{\partial \log(p(t, T))}{\partial T}.$$

4. The instantaneous spot rate is

$$r(t) = f(t, t).$$

The forward rate is the interest rate at which parties at time t agree to exchange K units of currency at time T_1 and give back $Ke^{R(t,T1,T2)(T2-T1)}$ units at time T_2. This means that one can lock in an interest rate for a future time period today. The spot rate $R(t,T_1)$ is the interest rate (continuous compounding) at which one can borrow money today and has to pay it back at T_1. The instantaneous forward and spot rate are the corresponding interest rates at which one can borrow money for an infinitesimal short period of time. The spot rate $R(t,T)$ as a function of T is referred to the yield curve at time t. The best estimate at time t for the yield curve at a future point in time T_1 is the forward rate $R(t,T_1,T_2)$ as a function of T_2, because one could lock in the forward rate today (e.g., by buying forward contracts). Another possibility for describing the structure of interest rates is the discount rate function $\delta(t,T)$, which is defined by $\delta(t,T) = e^{-(T-t)R(t,T)} = p(t,T)$.

The whole structure of interest rates at a point in time t can be described by either $R(t,T)$, the instantaneous forward rate $f(t,T)$, or the discount rate function $\delta(t,T)$. All of these functions can be transformed into one another (given they are smooth enough, which we assume here) without loss of information because the following relations hold:

1. $R(t, T_1, T_2) = \int_{T_1}^{T_2} f(t, u)du$

and therefore $R(t, T) = \int_t^T f(t, u)du$

2. $f(t, T) = \frac{\partial R(t, T)}{\partial T}$

3. $R(t, T) = -\frac{\log(\delta(t, T))}{T - t}.$

Therefore when estimating the yield curve, one could estimate the instantaneous forward rate or the discount rate function without loss of information.

A.2 Constructing the Yield Curve

As already stated above, the yield curve at a certain point in time t is defined by $R(t,T)$ as a function of T. Of course, something like a yield curve is not traded directly on the market and therefore cannot be observed directly (only a few simple interest rates like the LIBOR can be observed directly on the market). We can measure only the effects it has on other assets such as bonds. Let us assume that we observe the prices of n bonds with comparable risk class and liquidity, which are non-callable and do not have any other special features.

The cash-flow times of the bonds are $\tau_1,\ldots,\tau_m$. The cash flows (coupons and face value at maturity) of the bonds are $C = \{c_{ij}\}$ $i = 1,\ldots,n$, $j = 1,\ldots,m$, where c_{ij} is the payment of bond i at time τ_j. The prices of the bonds are $P(t) = (p_1(t),\ldots,p_n(t))'$, where $p_i(t)$ is the price of the bond i at time t.

The relationship between interest rates and bond prices that we are going to exploit to compute the yield curve is

$$p_i(t) = \sum_{j=1}^{m} c_{ij} e^{-R(t,\tau_j)(\tau_j - t)} \quad (1)$$

However, if one tries to do this, several problems can occur. First, the information of n bonds is not enough to determine the yield curve uniquely. Second, a function so that Equation (1) is satisfied for all bonds does not have to exist or might not have the properties that one expects of the yield curve. Because of this, usually one only wants to find a yield curve that has certain properties and recovers the bond prices as well as possible.

A.2.1 Properties

There are a few properties that are desirable for a yield curve, but depending on the estimation method, not all of them will hold. We will discuss this later in more detail. The desirable properties for a yield curve are

- Continuity
- Existence of an instantaneous forward rate curve
- Non-negativity of the yield curve
- Non-negativity of the instantaneous forward rate curve.

These properties just reflect that the yield curve should not have any jumps (just because you lend someone money for an additional day, you will not expect to get a significantly higher interest rate), have a certain smoothness, and be non-negative (it does not make much sense that you have to pay someone so that you are able to lend them money). However, the set of functions that fulfill some or all of the above criteria still is very large. To be able to find a function numerically that fits the bond prices, the complexity of the problem has to be reduced. How this is achieved will be discussed in detail below.

A.2.2 Fitting Criteria

We have discussed how a yield curve should look. It still remains to discuss what it means that the yield curve should recover the prices "as well as possible." Here we will introduce the most common ways to define a measure for "goodness of fit" in a yield curve environment. First, one has to decide how to measure the error. There are two common methods to do this. The first is to define the error for bond i, ε_i, as the difference between the actual price of bond i, p_i, and the estimated price

$$\hat{p}_i = \sum_{j=1}^{m} c_{ij} e^{-\hat{R}(t,\tau_j)(\tau_j - t)}$$

where $\hat{R}(t,T)$ is the estimate for the spot rate curve at time t. The other commonly used method is to compute the actual yield on bond i, $\hat{y}_i$, and the estimated yield $\hat{y}_i$ (computed using the estimated price $\hat{p}_i$) and define the error as $\varepsilon_i = y_i - \hat{y}_i$[9]. A commonly used alteration is to weight the error of a bond by the inverse of its duration, thus putting more emphasis on short-term than on long-term bonds because one would expect to be able to measure the price of a short-term bond

9 The yield on a bond with price p and future cash flows c_j, j=1,...,m at time points τ_j, j=1,...,m, is defined as the single interest rate y so that the equation $p = \sum_{j=1}^{m} c_j e^{-y\tau_j}$ holds.

more accurately. One could also add a roughness penalty, which prefers smooth functions over rough functions, to the error function. In this way a smoother yield function may be preferred over a rougher one, even if the fit of the rough function would be better. One should note that, for different error functions, the obtained results will be different in general.

To decide which yield curve has the best fit, a criterion is needed that takes into account the pricing errors for all bonds. Therefore, one has to choose a function of the *n* pricing errors ε_i, $i = 1,\ldots,n$. The most common choice is the sum of the squared errors $\sum_{i=1}^{n} \varepsilon_i^2$, mostly because of its analytical simplicity. Another choice is the sum of absolute errors $\sum_{i=1}^{n} |\varepsilon_i|$. The main difference between these approaches is their analytical tractability and the way in which they penalize large versus small deviations; the squared error penalizes large errors much more strongly than the absolute error function.

A.2.3 Construction Methods

The various approaches that have been proposed to construct a yield curve from prices of (corporate) bonds (or financial assets with a time-dependent maturity) broadly can be described by three general methods:

- Spline-based method
- Nelson-Siegel approach
- Nonparametric methods.

The first two methods are parametric, that is, they try to fit a function that can be defined by a fixed number of parameters. The other methods are nonparametric, which means that they try to find the best among all possible functions, not just of a certain type with a fixed parameter set. As these nonparametric methods do not play a prominent role in yield curve construction, they will be discussed only briefly. We will describe each method and discuss its advantages and disadvantages. A comprehensive overview of the techniques discussed in this Appendix can be found in James and Webber (2000).

A.2.3.1 Spline Methods

Spline-based methods were introduced to the approximation problem for interest rate curves by McCulloch (1971).

In general, one can use a *k*th-order spline, which is a piecewise polynomial, defined on $[|\xi_0,\xi_p|]$ with polynomials of degree k, differentiable $k-1$ times everywhere. The points where adjacent polynomials meet are called knot points and will be denoted by $[|\xi_0,\ldots,\xi_p|]$ with $\xi_i < \xi_j$ for $i < j$. The spline crucially depends on these knot points. For fitting purposes, usually a cubic spline is used, as it already looks quite smooth.

When using a spline to estimate the yield curve, we can try to estimate the yield curve directly by fitting a spline to the yield curve, or we can fit the spline to any other function that uniquely determines the yield curve. The most common choice of functions to which the spline is fit are

- Spot rate curve $R(t,T)$
- $R(t,T) \cdot (T-t)$
- Instantaneous forward rate curve $f(t,T)$
- Discount rate curve $\delta(t,T)$.

Apart from the function to which one has to fit the spline, the choice of the error function is also important. In Steeley (1991) the author uses the sum of the squared price errors. Usually these are weighted by the inverse of the duration, as stated above. Fisher, Nychka, and Zervos (1995) and Waggoner (1997) added a roughness penalty. In this way the algorithm favors smooth yield curves over rough ones, even if the fit of the rough yield curve is better to a certain degree.

The main advantage of the spline method is that it is very adaptable, easy to implement, and numerically fast and stable. It reduces the problem of estimating a whole curve to the problem of estimating a finite set of parameters (the coefficients of the polynomials). In general it also gives a very smooth yield curve that is continuously differentiable, thus admitting the existence of a continuous forward curve.

One of its strengths is also one of its weaknesses. As the method is highly adaptable, it can overfit the data. Another problem is how to determine the number and position of the knot points. Steeley (1991) chooses these points so that an equal number of bonds lie in each interval, but the number of knots remains arbitrary. However, the adaptability depends on the knot points, and thus the method as a whole remains somewhat arbitrary. Fisher, Nychka, and Zervos (1995) address this problem by choosing the number and position of knot points to minimize a cross-validation criterion.[10] This helps, although not completely, as the cross-validation criterion could have been chosen differently. Last but not least, we also want to point out that the spline method does not guarantee that the spot rate and the forward rate remains non-negative. Problems with negative forward rates especially have been reported (see, e.g., Steeley 1991).

A more mathematical point of view. The general form of a cubic spline (which is most commonly used) is

$$s(\tau) = \sum_{i=0}^{3} a_i \tau^i + \sum_{i=1}^{p-1} b_i (\tau - \xi_i)_+^3,$$

where $(x)_+ := \max\{x, 0\}$. We see that $s(\tau)$ is a linear combination of the functions

$$\{\tau^i\}_{i=0,1,2,3}\ ,\ \{(\tau - \xi_i)_+^3\}_{i=1,\dots,p-1}$$

These functions are unbounded, which can lead to computational instability. Instead, one can use the B-Spline basis, in which functions are bounded and have compact support. Apart from this, the two methods are equivalent. The parameters for B-Splines are just a reparameterization of those presented above.

B-Splines. The *i*th cubic B-Spline is defined as

$$B_i(\tau) = \sum_{j=i}^{i+4} \left(\prod_{k=i, k \neq j}^{i+4} \frac{1}{\xi_k - \xi_j} \right) (\tau - \xi_j)_+^3$$

on $[\xi_i, \xi_{i+4}]$ and 0 otherwise.

The cubic spline has $p+3$ parameters. To have $p+3$ B-Splines we need to define additional knot points $\xi_{-3}, \xi_{-2}, \xi_{-1}, \xi_{p+1}, \xi_{p+2}, \xi_{p+3}$. Then a cubic spline can be written as a linear combination of the basis functions

$$s(\tau) = \sum_{i=-3}^{p-1} \lambda_i B_i(\tau).$$

The spline crucially depends on the number and position of the knot points. Here no exact rule exists. One can say only that if more knot points are used, the fit improves but the curve gets less smooth. In addition, too many knot points tend to overfit the data, that is, introduce spurious effects.

The approximation problem becomes particularly easy if we decide to approximate the discount function δ and minimize the sum of the squared price errors. Then we define

$B = \{b_{ij}\}$ $i = 1,\dots,m$, $j = -3,\dots,p-1$, with $b_{ij} = B_j(\tau_i)$

and write, $\hat{\delta} = (\delta(\tau_1),\dots,(\delta(\tau_m))' = B\lambda$ with $\lambda = (\lambda_{-3},\dots,\lambda_{p-1})'$.

The minimization problem becomes

$$\lambda^* = \arg\min_{\lambda} \{\varepsilon'\varepsilon | \varepsilon = P - CB\lambda\},$$

which can be solved using standard ordinary least-squares regression techniques. Setting D:= CB, the solution is given by

$$\lambda^* = (D'D)^{-1} D'P$$

One problem of this technique is that the fit on the short end of the curve is not satisfactory. As a

[10] The simplest form of cross-validation is to divide the observations (here the bonds) into two groups. One group (the training set) is used to find the pricing errors. For the testing set, the estimate of the yield curve for the training set is used to predict the bond prices of the testing set, and then the pricing errors are measured. This method gives a more accurate value for the pricing errors because overfitting of the data is not an issue. More sophisticated methods are K-fold cross-validation and Leave-one-out cross-validation.

remedy the additional constraint $\delta(0)=(B_{-3}(0),\ldots,B_{p-1}(0))\lambda=W'\lambda=1$, with W defined appropriately, is often imposed. The solution to the problem

$$\hat{\lambda}=\arg\min\{\varepsilon'\varepsilon|\varepsilon=P-CB\lambda, W'\lambda=\omega\}$$

is given by

$$\hat{\lambda}=\lambda^{*}+(D'D)^{-1}W(W'(D'D)^{-1}W)^{-1}(w-W'\lambda^{*})$$

Smoothing splines. Smoothing splines address the problem that the spline is no longer smooth enough when the number of knot points increases. Usually some sort of a roughness penalty is introduced. Among the various possible variants we concentrate on the following approach, which was introduced by Fisher, Nychka, and Zervos (1995) and extended by Waggoner (1997).
Instead of minimizing the squared error of the bond prices (or yields), the criterion becomes

$$\min_{\theta}(P-C\delta_{\theta})'(P-C\delta_{\theta})+\int_{0}^{T}\lambda(t)\frac{\partial^{2}h(t,\theta)}{(\partial t)^{2}}dt.$$

where the first part is the original criterion used before and the second part is the roughness penalty (h is the function that is approximated by the spline, and the second derivative of h is a measure for the curvature of h). Fisher, Nychka, and Zervos (1995) choose $\lambda(t)$ to be a constant that is determined by a technique called generalized cross-validation. They impose the same penalty for curvature on the entire yield curve. This usually leads to a worse fit at the short end of the curve because the yield curve usually has more curvature there. Waggoner (1997) addresses this problem by taking $\lambda(\tau)$ to be

$$\lambda(\tau)=\begin{cases}0.1, & 0\le\tau<1\\ 100, & 1\le\tau<10\\ 100{,}000, & \tau\ge 10.\end{cases}$$

Other choices for l are possible and are suggested in Bolder and Gusba (2002). Further criteria for smoothing splines are discussed in James and Webber (2000), and general information on splines can be found in Bolder and Gusba (2002).

[11] The name is proposed by Bolder and Gusba (2002).

A.2.3.2 Function-Based Models

A second important class are the function-based models,[11] which use functions defined on $[0,\infty)$.

MLES Model The Merrill-Lynch exponential spline (MLES) model, first proposed in Li et al. (2001), follows a somewhat hybrid approach between splines and more general functions. It uses a linear combination of exponential functions to approximate the discount function δ. The method is not a spline method in the strict sense because it fits a linear combination of functions on a single interval. The approximating function is

$$\hat{\delta}(\tau)=\sum_{i=1}^{N}\lambda_i\exp(-i\alpha\tau)$$

with the additional constraint

$$1=\hat{\delta}(0)=\sum_{i=1}^{N}\lambda_i$$

The parameters to be estimated are $(\alpha,\lambda_1,\ldots,\lambda_N)$. The number of functions N is judgmental and has to chosen by the user. In Bolder and Gusba (2002) $N = 9$ is suggested. They argue that using more than nine functions improves the fit only marginally while leading to computational instability. The parameter α can be interpreted as the long-term instantaneous forward rate.

Bolder and Gusba (2002) also proposed a variation on this concept. They use a Fourier-series basis instead of exponential functions:

$$\left\{1,\sin\left(\frac{n\tau}{10}\right),\cos\left(\frac{n\tau}{10}\right);\ n=1,2,3,4\right\},$$

where the stretch factor 1/10 is chosen ad hoc and can be adapted to the situation at hand. Other functions and additional restrictions are possible: see again Bolder and Gusba (2002) for a model in which the prices of certain benchmark bonds are fitted perfectly.

All in all, the advantages and disadvantages of this method are similar to the ones for splines. The problem with the knot points vanishes, but therefore the number of functions to use is judgmental.

Nelson-Siegel Curves. The idea of this approach is to fit the instantaneous forward curve by a function of several parameters. The most widely used function of this type is the Nelson-Siegel approach, which is

$$f(\tau)=\beta_0 + (\beta_1 + \beta_2\tau)e^{-k_1\tau}$$

with parameters $\theta=(\beta_0,\beta_1,\beta_2,k_0)$; it was originally described in Nelson and Siegel (1987). Afterwards several generalizations were proposed, adding more terms and more parameters to make the curve more flexible. These additional approaches were proposed in Svensson (1994), Wiseman (1994), and Bjork and Christensen (1997). Here is a short overview of these functional forms:

Nelson and Siegel:

$$f(\tau)=\beta_0 + (\beta_1 + \beta_2\tau)e^{-k_1\tau}$$

with parameters $\theta=(\beta_0,\beta_1,\beta_2,k_0)$. This curve was originally described in Nelson and Siegel (1987).

Svensson:

$$f(\tau)=\beta_0 + (\beta_1 + \beta_2\tau)e^{-k_0\tau} + \beta_3\tau e^{-k_1\tau}$$

with parameters $\theta=(\beta_0,\beta_1,\beta_2,\beta_3,k_0,k_1)$.
It was first proposed in Svensson (1994).

Wiseman:

$$f(\tau)=\sum_{j=0}^{n}\beta_j e^{-k_j\tau}$$

with parameters $\{(\beta_j, k_j)\}_{j=0,\dots,n}$ It was first presented in Wiseman (1994).

Bjork and Christensen:

$$f(\tau)=\sum_{j=0}^{n}p_j(\tau)e^{-k_j\tau}$$

with $p_j(\tau)$ polynomials and additional parameters $\theta=(k_0,\dots,k_n)$ It was proposed in Bjork and Christensen (1997). As stated above, we can use the forward rate curve to compute the spot and discount rate curve and then compute model prices for the bonds used. This can be done by first calculating the spot rate from the forward rate curve using

$$r(\tau \mid \theta)=\frac{1}{\tau}\int_0^{\tau} f(t \mid \theta)dt$$

and then the discount factor by

$$\delta(\tau \mid \theta)=e^{-r(\tau|\theta)\tau}.$$

For the original Nelson-Siegel version, the spot rate function is

$$r(\tau \mid \theta)=\beta_0-\frac{\beta_1}{k_1}e^{-k_1\tau}-\frac{1}{\tau}(e^{-k_1\tau}-1)\left(\frac{\beta_1}{k_1}+\frac{\beta_2}{k_1^2}\right).$$

For the other functional forms, this can be done similarly, although the functional forms of the spot rate are getting more complicated and numerically more problematic. The resulting nonlinear minimization problems can be solved using standard Gauss-Newton methods. However, some care is needed because convergence problems with these methods have been reported in, for example, Bolder and Gusba (2002).

The original Nelson-Siegel version especially has a very simple functional form that uses just four parameters and therefore does not need much data to get a unique result. These results are very good if data are scarce. Another advantage is that the forward rate is described directly, giving a smooth estimate and making it easier to ensure a forward rate curve that is positive everywhere.

There are two main disadvantages of this method. First, the already mentioned computational instability: it has been reported that parameters can change dramatically although the interest rate environment has hardly changed. Second, the simple Nelson-Siegel form especially is not very adaptable (only four parameters) and gives poor fits when the actual shape of the yield curve becomes unusual.

Another interesting aspect to note is the theoretical foundation of a certain type of these models. Standard stochastic interest models, such as the Vasicek or Cox-Ingersoll-Ross models (see Bingham and Kiesel 2004 for details), take an initial yield curve and describe how it evolves with time. When using an initial yield curve from the class of Nelson-Siegel functions, no interest rate model exists so that the yield curve stays within the Nelson-Siegel class at all times. However, the class of functions proposed by Bjork and Christensen is consistent with the Hull and White extended Vasicek term structure model, which is shown in Bjork and Christensen (1997).

A.2.3.3 Nonparameteric Methods

Parametric methods restrict the set of functions that they consider to a set that can be described by a finite number of parameters. This makes the optimization problem easier, but it also makes the method less flexible. A nonparametric method in general uses a set of functions that is so rich that it cannot be described by a finite number of parameters.

In general, one has a vector Y with the response variable (e.g., price of a bond) and a matrix X with the predictors (amount and time of coupon and face value payments). The relationship of these variables is described by

$$Y_i = f(X_{i1},\ldots,X_{im}) + \varepsilon_i,$$

where ε is an error term and f is a function of a certain type (e.g., twice differentiable, positive). We have to find a function f from the given set of functions that recovers the response Y as well as possible (e.g., error measured by quadratic distance). One standard method is to use kernel smoothers. Given a set of predictors x, they try to recover $f(x)$ by giving more weight to responses Y_i, for which the predictor variables X_i are closer to x. The difficulty is to determine how big the neighborhood should be for which one gives a "high" weight. If the neighborhood is too big, one smoothes very much and loses information. On the other hand, if the neighborhood is too small, one does not distinguish between noise and information. Specific applications to yield curve estimation are described below.

Kernel method. The main idea of this method was developed in Tanggard (1993). Extensions to the case of coupon bonds can be found in Linton et al. (1998) and Tanggard (1993); see also James and Webber (2000) for additional information.

The easiest way to describe this method is to consider only pure-discount bonds. Then the spot rate $R(t,T)$ at time t is a weighted average of the yields of the bonds, where the weight of bond i increases when the difference $T - \tau_i$ gets smaller, where τ_i is the maturity of bond i. With a coupon bond, the problem gets much more technical. Detailed descriptions of this method can be fund in the articles mentioned above.

This method is quite new and has not been tested very extensively. However, it seems to be very adaptable and yields good results (see Linton et al. 1998).

Fama-Bliss method. For the sake of completeness, we briefly want to describe this method, although it is not nonparametric. It was first published in Fama and Bliss (1987). The forward rate is computed stepwise to fit bond prices exactly. First, the forward rate up to the shortest maturity is computed, then the forward rate is extended step by step to fit the prices of bonds with longer maturities. Filters are used to eliminate bonds with extreme prices. The result is a piecewise linear forward rate from which the discount function can be computed. It fits the data very close but also tends to overfit, which can lead to spurious effects. Another problem is that the yield curve can show quite erratic behavior.

A.2.3.4 Comparison of the Models

One of the most important criteria is how well the model recovers the actual bond prices. Here we can distinguish between in-sample fit (how well the model recovers the prices of the bonds used to fit the model) and out-of-sample fit (how well it recovers the prices of other bonds that were not used to fit the model). The out-of-sample fit is much more reliable, because when fitting a model to data, the model may incorporate features that are not really characteristic of all data of this type but just of the specific sample that is used for estimation.

In terms of goodness of fit — in-sample as well as out-of-sample — the spline methods proposed in Fisher, Nychka and Zervos (1995) and Waggoner (1997) as well as the MLES model do quite well. The other models either do not show a good fit at all, which is true for the original Nelson-Siegel-type model, or they have a good in-sample but a bad out-of-sample fit, suggesting that the models overfit the data. This is the case for the McCulloch spline method (especially with a high number of knot points) and the Fama-Bliss method (not surprisingly, as it tries to fit most bond prices exactly).

Another issue is the stability of the yield curve. Often, especially for maturities where there are not very much data, different yield curves can have a similar fit. A model is said to be unstable if only small changes in the bond prices lead to large changes of the yield curve. The problem with this when applying such models to the valuation of future cash flows is obvious — large changes in the value could occur. The model proposed in Svensson (1994), which is a variant of the original Nelson-Siegel approach, poses the biggest problems here. All other methods do quite well, with the cubic spline approach suggested by Waggoner (1997) being the best.

Several of the methods presented here are actually used in practice. The model proposed by Svensson (1994) is used in central banks. A version of the model proposed by Fisher, Nychka, and Zervos (1995) and Waggoner (1997) is used by the Federal Reserve Bank. Bolder and Gusba (2002) recommend a version of the Fisher, Nychka, and Zervos model and the MLES model to the Bank of Canada.

A.3 Summary of Model Comparisons by Various Authors

A.3.1 Analysis by Bolder and Gusba

In Bolder and Gusba (2002) the authors compare the fit of eight different yield curve estimation techniques to daily Canadian government bonds from 1 April 2000 to 11 July 2002. In particular, they use four spline-based and four function-based models. The spline-based models were the McCulloch model and three variants of the model proposed by Fisher, Nychka, and Zervos (FNZ) and extended by Waggoner and Anderson and Sleath (2001). In these three models the splines are fit to the forward rate curve, $t \cdot r(t)$, and the discount function. The four function-based models are the standard MLES model, the version of the MLES with a Fourier basis, an MLES version that forces the discount function to fit the prices of certain benchmark bonds particularly well, and the model proposed by Svensson. They usually use weights that are the inverse of the modified duration.

In their analysis they compare the mean absolute error and the RMSE in-sample and out-of-sample for the models used (out-of-sample only for the best in-sample models). In their in-sample analysis they find that the three variants of the MLES model are particularly good. The spline methods do quite well too, except for the FNZ model that fits the forward rates. Also, the Svensson model does not have good fits. The authors put this down to computation instability and so-called catastrophic jumps, which refer to large changes in the parameters from day to day. In their out-of-sample analysis, they restrict themselves to the McCulloch model, the FNZ model fitting $t \cdot r(t)$, the standard MLES model and the MLES model with a Fourier basis. Here the McCulloch model is significantly worse than the others, indicating that this model tends to overfit the data. The MLES model with a Fourier basis also lags a bit behind the others. All in all, the authors find the standard MLES model and the FNZ models for $t \cdot r(t)$ to be the most appropriate. One should also note that the FNZ model for the forward rate doesn't do a good job, and the same is true for the Svensson model. Whether the poor result for the Svensson model also applies to the original Nelson-Siegel model is not clear.

A.3.2 Analysis by Anderson and Sleath

In Anderson and Sleath (2001) four yield curve estimation methods are tested and applied to daily U.K. government bond data from 1 May 1996 to 31 December 1998. The goal of this paper is to find an estimate for the forward rate curve. Therefore they use the function-based approach suggested by Svensson and the more parsimonious approach in the early work of Nelson and Siegel. They also use two spline-based methods, the original method of Fisher, Nychka, and Zervos and a variant of the extension proposed by Waggoner. Both spline methods fit the spline to the forward rate curve.

They find that in the out-of-sample goodness-of-fit test, the variation of the method by Waggoner is best, leading the Svensson, Nelson-Siegel, and original FNZ models. This result is consistent with the results of Bolder and Gusba. Anderson and Sleath also tested the stability of the different methods by adding additional noise to bond prices and observing changes in the yield curve. They obtain the result that the Svensson model is particularly unstable in its results. The Nelson-Siegel model is more stable, but its goodness-of-fit is worse. One reason for this could be the lower number of parameters used for the Nelson-Siegel model. The variant of Waggoner's approach is the most stable model.

A.3.3 Analysis by Bliss

Bliss implements and tests five yield curve estimation techniques on monthly U.S. government bond data from January 1970 to December 1995 (see Bliss 1996) and gives in-sample and out-of-sample results. The methods used are the Fama-Bliss method; the McCulloch spline method, which approximates the discount function with a cubic spline; the original FNZ method, which is applied to the estimation of the forward rate; a modified version of the Nelson-Siegel approach; and a smoothed version of the Fama-Bliss method, which approximates the obtained discount rates by the discount rate function used in the Nelson-Siegel method.

In his in-sample analysis he finds that pricing errors increase with time to maturity and become serious for all methods used. Apart from this, all methods except the FNZ method yield at least satisfactory results. The FNZ approach suffers from a poor fit especially at the short end of the yield curve. The main reason for this is that the penalty factor for the roughness of the curve in the original FNZ approach is the same for the entire yield curve. As there is usually more curvature at the short end, this uniform penalty leads to a poor fit. This issue is addressed in Waggoner (1997).

In the out-of-sample test, the methods used show only small differences except for the FNZ model, which still has the worst fit of all methods tested. When considering the hit ratio (percentage of model prices that lie within the bid-ask spread of the bond price), the picture looks only slightly different. The Fama-Bliss and McCulloch methods are still the best, but now the FNZ approach has a higher hit ratio than the extended Nelson-Siegel model.

A.3.4 Analysis by Others

Several other analyses have a different mix of implemented models or concentrate on other issues in the comparison between the models. See, for example, Dahlquist and Svensson (1996), Bekdache and Baum (1997), and Jeffrey, Linton, and Nguyen (2001).

Bekdache and Baum (1997) compare the ex ante price and yield prediction accuracy for six different spline-based models and the Nelson-Siegel approach (however, the Nelson-Siegel method is fitted to a different data set). They find that a spline method similar to the FNZ model for the forward rate has the best prediction accuracy for one to three months into thefuture. The Nelson-Siegel method becomes better for longer prediction periods and is the best for a three-to-five month prediction.

Jeffrey, Linton, and Nguyen (2001) compare the kernel smoothing method presented in Linton et al. (1998) to the McCulloch spline and the Fama-Bliss methods. They find that the kernel-smoothing method has a superior fit, especially for longer time to maturity.

Durational (Select and Ultimate) Discount Rates for FAS 87 and 106 Valuations

by Ron Iverson, Heidi Rackley, Steve Alpert, and Ethan Kra

Introduction

Employers are constantly looking for ways to increase revenue or decrease expense. One technique used to increase revenue or decrease expense involves changing the interest cost component of FAS 87 pension expense and FAS 106 other postretirement benefit cost. This technique is sometimes referred to as "select and ultimate" discount rates or "forward" discount rates. Using this technique the company applies a separate one-year discount rate to each year's cash flow in the valuation period and then determines interest cost using only the lower first-year discount rate. We believe that this technique might be being used inappropriately. In informal discussions the Financial Accounting Standards Board (FASB) staff also questioned the validity of this technique.

For example, assume that the employer chooses durational discount rates that rise from 3.00% in the first year to 8.00% after eight years. These durational discount rates may be expected to produce a PBO/APBO similar to that produced by a level 7.00% discount rate. Based on a $100 million PBO/APBO, calculating interest cost using the 3.00% first-year rate rather than the 7.00% level rate would reduce interest cost by $4.0 million.

In our opinion the use of durational discount rates is fully justified under FAS 87 and FAS 106. However, calculating interest cost based on the first-year discount rate is inappropriate for four reasons:

1. It is a bad assumption. To base the interest cost calculation on the first-year discount rate, one must assume that the noninflation components of interest rates remain constant with respect to maturity and that the Treasury yield curve is a good measure of inflationary expectations. In this paper we will show that real returns and risk premiums are not level by duration and that changes in the yield on Treasury securities are at least partially attributable to noninflation-related factors; therefore, the first-year interest cost method described above is inappropriate.

2. It is a bad method. When all actuarial assumptions are realized, a proper actuarial method should not generate any experience gain or loss. Under the durational discount rate method, if the interest rate environment is unchanged during the year, the expense will be understated, and the plan will generate a large experience loss.

3. It may violate Actuarial Standard of Practice No. 27, Selection of Economic Assumptions for Measuring Pension Obligations. ASOP 27 requires all economic assumptions to be consistent. As noted above, a select and ultimate discount rate infers that the inflation rate assumption also varies by duration. Therefore, under ASOP 27 all other economic assumptions (e.g., expected rate of investment return or salary increase assumption) should vary by duration. If this method is modified to incorporate properly consistent future-inflation changes in expected return, salary increases, etc., pension expense for funded plans would not change dramatically, and spurious experience losses would be avoided. (Note: if one has a durational inflation assumption and this method is applied correctly, unfunded pension plans and unfunded postretirement medical plans may realize an appropriate reduction in expense.)

4. It may violate Actuarial Standard of Practice No. 4, Measuring Pension Obligations. ASOP 4 requires complete disclosure of the net pension cost calculation and any long-term trend of costs resulting from the methods and assumptions used. Since future experience losses are expected, failure to disclose this issue would mislead the employer and plan's auditor and thus violate these standards. In addition, the resulting financial statement disclosure may mislead investors.

Using durational discount rates correctly requires a thorough understanding of how durational rates should be developed and what the underlying yield curve says (and does not say) about investors' inflation and interest rate expectations. This paper provides the background information on durational rates to educate actuaries, plan sponsors, and auditors on the theory of durational rates and the consequences of inappropriate usage. If the employer (with their auditor's approval) prescribes the use of durational discount rates, the actuary should document fully how the rates are used (including the details of the interest-cost calculation). If the first-year select rate is used to determine interest cost, the actuary should also disclose that future actuarial losses are expected, which will lead to an increasing cost pattern.

Do FAS 87 and FAS 106 Allow Use of Select and Ultimate Discount Rates?

Neither FAS 87 nor FAS 106 states explicitly whether durational (or spot) discount rates may be used. By inference, we believe that they can be used. Paragraph 44 in FAS 87 and paragraph 31 in FAS 106 both refer to assumed discount rates (plural). Paragraph 199 in FAS 87 even recommends the use of durational discount rates: "The disclosures required by this Statement regarding components of the projected benefit obligation will be more representatively faithful if individual discount rates applicable to various benefit deferral periods are selected." The answer to question 59 in A Guide to Implementation of Statement 87 on Employers' Accounting for Pensions, in response to a question whether discount rates can differ for VBO, ABO, and PBO, states: "The assumed discount rates for pension benefits that mature in a particular year should not differ." Based on these references, we believe that durational discount rates may be used. If separate discount rates are appropriate for each year's cash flow, then those discount rates can be transformed easily as described below into "select and ultimate" or "forward" rates.

How Would an Appropriate Set of Durational Discount Rates Be Developed?

A yield curve sets forth the relationship between bond yields and the period to maturity. Therefore, durational discount rates can be obtained from a yield curve either by using the actual spot rates or by developing the implied forward rate based on a straightforward mathematical formula. For FAS 87 and FAS 106 calculations, the underlying yield curve should reflect the yield to maturity (determined as of the measurement date) of high-quality corporate or government bonds with maturity periods typically ranging from six months to 30 years. A published yield curve, such as the Salomon Brothers' Pension Discount Curve (shown in Table 1 as of 30 April 2002), may be used for this purpose.

The yields shown in Table 1 are called spot rates (since they are hypothetical zero-coupon bonds). They represent the effective annual yield over the entire period to maturity. For example, the 5.5-year spot rate of 5.65% quantifies the present value of a zero-coupon \$1,000 Aa-rated corporate bond maturing in 5-1/2 years (i.e., \$1,000/ 1.05655^{55} = \$739). We will use S_t to denote the spot rate for a zero-coupon bond maturing at time t.

Forward rates discount the cash flow on a year-by-year basis. The forward rate for the first six months is set equal to the six-month spot rate.

Table 1
Salomon Brothers' Pension Discount Curve (30 April 2002)

Maturity	Yield	Maturity	Yield	Maturity	Yield
0.5	2.44%	10.5	6.83%	20.5	7.21%
1.5	3.37	11.5	6.93	21.5	7.24
2.5	4.23	12.5	7.01	22.5	7.27
3.5	4.86	13.5	7.10	23.5	7.29
4.5	5.32	14.5	7.18	24.5	7.30
5.5	5.65	15.5	7.24	25.5	7.31
6.5	5.92	16.5	7.27	26.5	7.31
7.5	6.18	17.5	7.25	27.5	7.31
8.5	6.43	18.5	7.23	28.5	7.30
9.5	6.66	19.5	7.21	29.5	7.33

For any subsequent period beginning at time t and ending at time $t+1$, the *forward rate,* denoted $f_{t+.5}$, is developed from spot rates using the following formula:

$$f_{t+.5} = [(1 + s_{t+1})^{t+1}/(1 + s_t)^t\ ] - 1.$$

For example, the forward rate f_5 for the period from time 4.5 to time 5.5 derived from the Salomon Brothers' Pension Discount Curve at 30 April 2002, would be

$$f_5\ = 7.15\% = (1.0565^{5.5}/1.0532^{4.5}) - 1.$$

Table 2 shows the complete set of forward rates developed from the Salomon Brothers' Pension Discount Curve in Table 1.

The present value of a \$1,000 cash flow at time 5.5, determined using these forward rates, is equal to the value of the \$1,000 bond maturing at time 5.5 determined as follows:

$$\$1{,}000/(1.0244^{0.5} \times 1.0384 \times 1.0553 \times 1.0645 \times 1.0695 \times 1.0715) = \$739.$$

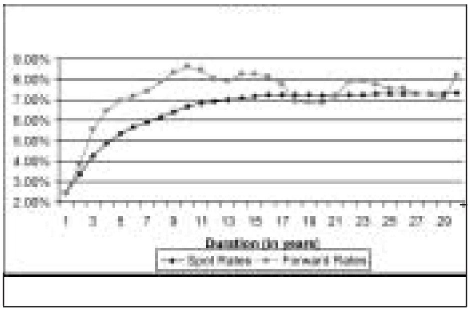
Table 2
Forward Rates Developed from Salomon Brothers' Pension Discount Curve (30 April 2002)

Period	Forward Rate	Period	Forward Rate	Period	Forward Rate
0–0.5	2.44%	9.5–10.5	8.46%	19.5–20.5	7.21%
0.5–1.5	3.84	10.5–11.5	7.99	20.5–21.5	7.86
1.5–2.5	5.53	11.5–12.5	7.93	21.5–22.5	7.92
2.5–3.5	6.45	12.5–13.5	8.23	22.5–23.5	7.74
3.5–4.5	6.95	13.5–14.5	8.27	23.5–24.5	7.54
4.5–5.5	7.15	14.5–15.5	8.11	24.5–25.5	7.56
5.5–6.5	7.42	15.5–16.5	7.74	25.5–26.5	7.31
6.5–7.5	7.89	16.5–17.5	6.92	26.5–27.5	7.31
7.5–8.5	8.32	17.5–18.5	6.88	27.5–28.5	7.03
8.5–9.5	8.64	18.5–19.5	6.84	28.5–29.5	8.19

Figure 1 compares the *spot* and *forward rates* in Tables 1 and 2. It is immediately apparent that forward rates are far more volatile than spot rates. From the formula used to develop the forward rates, we can demonstrate the following:

- Forward rates are higher than spot rates when spot rates are rising
- Forward rates equal spot rates when spot rates are flat
- Forward rates are less than spot rates when spot rates are declining.

Figure 1—Spot Rates vs. Forward Rates



This series of *forward rates* typically would be smoothed for use in FAS 87/106 calculations. For example, on the basis of Table 2, the employer might decide to use durational discount rates starting at 3.00% in the first year, increasing by 1.00% until reaching 6.00% in the fourth year, and increasing 0.50% until reaching 8.00% in the eighth year. The PBO/APBO would be calculated by projecting benefit and expense payments in each future year, then discounting them back, year by year, to the measurement date using the forward rates.

How Should Interest Cost Be Calculated When Durational Discount Rates Are Used?

Paragraph 22 of FAS 87 states: "The interest cost component recognized in a period shall be determined as the increase in the projected benefit obligation due to the passage of time. Measuring the projected benefit obligation as a present value requires accrual of interest cost at rates equal to the assumed discount rates." Therefore, the interest cost is part of the reconciliation of the PBO/APBO from the beginning to the end of the year, as shown in the financial statement disclosure. Although changes in the PBO/APBO due to changes in assumptions are added to the accumulated experience gains and losses, we believe that the interest cost methodology should not introduce additional gains or losses into the results when discount rates do not change.

To determine how interest cost should be calculated, we first must decide what interest rates we expect to use to calculate the PBO/ABPO at year-end. There are two possible alternatives:

- Method 1—The year-end durational rate for year *t* will equal the beginning-of-year durational rate for year *t + 1*. That is, short- and intermediate-term interest rates are expected to increase during the year, presumably due to an increase in inflation, and the yield curve is expected to become flatter over time.

- Method 2—The year-end durational rate for year t will equal the beginning-of-year durational rate for year *t*. That is, interest rates are not expected to change during the year.

Table 3 is an illustration of these two alternatives for beginning-of-year durational discount rates that increase from 3.00% in year 1 to 5.00% in year 2 and 7.00% in year 3 and later.

Table 3
Alternative Year-End Select and Ultimate Discount Rates

	Beginning of Year	End of Year	
		Method 1	Method 2
First year	3.00%	5.00%	3.00%
Second year	5.00	7.00	5.00
Third and later	7.00	7.00	7.00

If one expects to use Method 1 rates at year-end, then interest cost should be calculated by multiplying the PBO/APBO by the first-year rate—3.00%. But if one expects to use Method 2 rates at year-end, interest cost should be calculated by using the final rate applicable to each year's cash flow—3.00% for the first-year cash flow, 5.00% for the second-year cash flow, 7.00% for the third-year cash flow, etc. Generally Method 2 will result in an interest cost that is slightly higher than that produced by the equivalent level discount rate, while Method 1 produces a much lower interest cost.

Which method is the correct one to use? To answer this question, we need to understand why the durational discount rates are increasing, and what that increase says about investors' future interest rate expectations. And to understand that, we need a better understanding of the yield curve.

Components of the Aa Corporate Bond Yield Curve

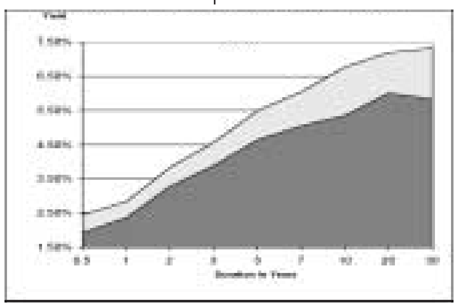
A yield curve developed from high-quality corporate bond yields can theoretically be broken down into two main components: U.S. Treasury yield curve and "risk premium" (also known as the "spread"). Although the risk premium is chiefly driven by investors' perceptions of default risk and the creditworthiness of the issuer, it also includes other secondary components, such as state tax premium, supply and demand, friction, or temporary market inefficiencies.

Default Risk (Also Known as Credit Risk) Premium

The lower the bond rating, the higher the default risk premium. This default risk premium depends on the investment time horizon: the longer the investment time horizon, the higher the default risk premium. This is as we would expect: the probability that a company, currently rated Aa, ultimately will default on its bonds should increase with the number of years to bond maturity.

Figure 2 demonstrates the increasing risk premium by length of investment. It shows the U.S. Treasury yield curve (based on zero-coupon bonds) and the risk premium components of the Salomon Brothers' Pension Discount Curve at 30 April 2002. At this date the U.S. Treasury zero-coupon spot curve increases from 1.91% for six-month investments to approximately 5.35% for 10-year investments, and the default risk premium increases from 0.53% for six-month investments to 1.43% for 10-year investments.

Figure 2—Salomon Brothers' Pension Discount Curve: 30 April 2002



The risk premium does not remain constant over time. In fact, depending on the economy, the risk premium can change rapidly, and the change is typically larger for longer periods to maturity. This is illustrated in Table 4, which shows the change in the Salomon Brothers' Pension Discount Curve risk premium between 31 October and 30 November 1998. There was a flight to quality (from corporate to Treasury bonds) at the end of October, which lessened in November. During this one-month period, the risk premium decreased 0.45% for 30-year investments.

Table 4

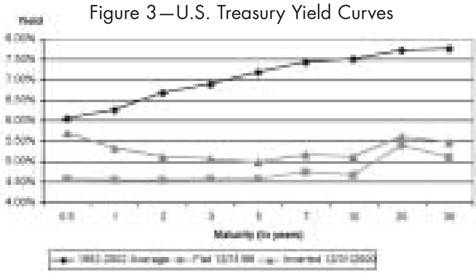
Change in Default Risk Premium from October to November 1998

	6 Months	1 Year	3 Years	5 Years	10 Years	20 Years	30 Years
31October 1998							
Salomon Aa	5.47%	5.43%	5.44%	5.52%	6.19%	7.04%	7.16%
Treasury yield	4.36	4.18	4.24	4.24	4.64	5.38	5.15
Difference	1.11	1.25	1.20	1.28	1.55	1.66	2.01
30 November 1998							
Salomon Aa	5.48	5.51	5.51	5.58	5.97	6.56	6.64
Treasury yield	4.58	4.53	4.54	4.51	4.74	5.33	5.08
Difference	0.90	1.08	0.97	1.07	1.23	1.23	1.56
Change in risk	-0.21	-0.17	-0.23	-0.21	-0.32	-0.43	-0.45

Yield Curve Based on Constant Maturity U.S. Treasury Bonds

As illustrated in Table 5, the U.S. Treasury yield curve can take on three different shapes: (1) normal: upward sloping, (2) inverted: downward sloping, or (3) flat. Generally, longer-duration Treasury bonds carry higher yields than shorter-duration Treasury bonds, resulting in a normal (or upward sloping) yield curve. As of 30 April 2002, the yield curve is normal but has a much steeper than average slope.

Figure 3—U.S. Treasury Yield Curves



The shape of the yield curve depends primarily on the economy. When the economy is expected to expand rapidly, the yield curve may increase steeply because investors expect inflation and interest rates to rise in the future. When the economy is entering a recession, the yield curve may be inverted because investors expect inflation and interest rates to fall. Regardless of whether the yield curve shape is normal or inverted, most of the change in interest rates is expected to occur in the first five years, with much more gradual changes between five and 30 years to maturity.

The difference in yields by maturity length reflects three factors:

1. Inflation risk: yield must reflect investors' future inflation expectations
2. Liquidity premium: investors demand a premium for holding longer-duration Treasuries, which fluctuate more in value as interest rates change
3. Market segmentation: large institutional investors' preference for specific maturities that match their needs. While this preference is generally temporary, there may be ongoing market segmentation effects for certain maturities. For example, institutional investors use long-duration Treasury bonds in various hedging strategies.

It is very difficult to identify the yield difference due to each separate factor. In fact, each individual investor assigns a different value to each factor through their choice of investment.

Table 5
20-Year History of U.S. Treasury Bond Yields

Calendar Year-End	6 Month	1 Year	2 Years	3 Years	5 Years	7 Years	10 Years	20 Years	30 Years	Shape
31.12.1982	8.42%	8.68%	9.48%	9.74%	10.09%	10.32%	10.36%	10.62%	10.43%	Normal
30.12.1983	9.73	10.08	10.85	11.13	11.57	11.77	11.82	11.98	11.87	Normal
31.12.1984	8.64	9.22	10.02	10.52	11.08	11.52	11.55	11.70	11.54	Normal
31.12.1985	7.44	7.60	7.98	8.22	8.49	8.86	9.00	9.50	9.27	Normal
31.12.1986	5.87	5.95	6.35	6.56	6.81	7.09	7.23	7.39	7.49	Normal
31.12.1987	6.47	7.10	7.77	8.04	8.33	8.67	8.83	n/a	8.95	Normal
30.12.1988	8.67	9.02	9.14	9.18	9.14	9.18	9.14	n/a	9.00	Flat
29.12.1989	7.87	7.76	7.87	7.87	7.86	7.97	7.93	n/a	7.98	Flat
31.12.1990	6.73	6.82	7.15	7.40	7.68	8.00	8.08	n/a	8.26	Normal
31.12.1991	4.00	4.12	4.77	5.11	5.93	6.38	6.71	n/a	7.41	Normal
31.12.1992	3.38	3.61	4.56	5.12	6.04	6.43	6.70	n/a	7.40	Normal
31.12.1993	3.30	3.63	4.25	4.58	5.21	5.53	5.83	6.48	6.35	Normal
30.12.1994	6.57	7.20	7.69	7.80	7.83	7.84	7.84	8.02	7.89	Normal
29.12.1995	5.17	5.18	5.18	5.25	5.38	5.49	5.58	6.01	5.96	Normal
31.12.1996	5.33	5.51	5.88	6.04	6.21	6.34	6.43	6.73	6.65	Normal
31.12.1997	5.45	5.51	5.66	5.68	5.71	5.77	5.75	6.02	5.93	Flat
31.12.1998	4.55	4.53	4.54	4.55	4.56	4.73	4.65	5.39	5.09	Flat
31.12.1999	5.74	5.98	6.24	6.29	6.36	6.55	6.45	6.83	6.48	Normal
31.12.2000	5.70	5.32	5.11	5.06	4.99	5.16	5.12	5.59	5.46	Inverted
31.12.2001	1.83	2.17	3.07	3.59	4.38	4.84	5.07	5.74	5.48	Normal
Average	6.04	6.25	6.68	6.89	7.18	7.42	7.50	7.71	7.74	
Median	5.81	5.97	6.30	6.43	6.59	6.82	6.97	n/a	7.45	

From Table 5 we conclude that the U.S. Treasury yield curve typically slopes upward. The difference between six-month and five-year yields averages 1.14%, with an additional 0.32% increase between five- and 10-year yields, and a 0.24% increase between 10- and 30-year yields, for a total 1.70% increase between six-month and 30-year yields.

If the U.S. Treasury yield curve is steeper than normal, it may be reasonable to attribute the yield increase in excess of 1.7% to investors' expectations of increasing inflation and interest rates. As of April 2002, the yield curve was sloped steeply upward, with a 3.7% increase between six-month and 30-year yields. This may indicate that investors expect inflation and interest rates to increase substantially (i.e., approximately 2%) in future years.

Now that we have the background information, we need to resolve the question of the correct interest cost calculation when using durational discount rates.

The Correct Interest Cost Calculation

If the durational difference in bond yields is due entirely to expected future changes in inflation and interest rates, then Method 1 is the appropriate interest cost calculation. If, as we showed above, risk premiums always vary by duration, Method 1 never can be correct.

If the durational difference in bond yields is due entirely to liquidity/default risk premium (i.e., inflation is not expected to change by duration), then Method 2 is the appropriate interest cost calculation. Although this is possible, it is unlikely that inflation is expected to remain constant over time. Therefore, Method 2 is also generally incorrect, and the correct calculation will be something between Method 1 and Method 2.

The U.S. Treasury yield curve should be used to estimate the expected change in inflation at the measurement date. If the durational rates based on the yield curve are normally sloped, there is little expected change in inflation. If the durational rates are steeply sloped, there is an expected increase in inflation. If the durational rates are flat or decreasing, there is an expected decrease in inflation. The magnitude of change in inflation is a subjective exercise. As noted above, the change in the U.S. Treasury yield in excess of 1.7% could be used to help estimate the expected change in inflation.

Based on the steeply sloped April 2002 U.S. Treasury yield curve, investors may expect either a significant increase in inflation or a steep rise in interest rates. Therefore, for 2002 pension expense, a combination of Method 1 and Method 2 may be an appropriate interest cost calculation method.

In a year when durational discount rates include a changing inflation element, the increase in the select and ultimate rates should be broken down into two components:

1. An inflation component determined by comparing the U.S. Treasury yield curve at the measurement date to the normal U.S. Treasury yield curve

2. A liquidity/default risk premium component determined as the difference between the select and ultimate rates derived from high-quality corporate bond yields and the inflation component.

Table 6 shows hypothetical select and ultimate discount rates with an increasing inflation component and the corresponding rates that would be used for the year-end PBO/APBO. If one selects these assumptions, interest cost should be determined using Method 2, reflecting each year's real risk-free return, liquidity/default risk premium component plus the first-year inflation component. In practice, the actuary would determine the expected year-end PBO/APBO using the year-end rates, then back into the interest cost necessary to reconcile the PBO/APBO from the beginning to the end of the year.

Table 6

Select and Ultimate Discount Rates with Inflation Component

	Increasing Inflation	Real Return	Liquidity/ Default	Beginning-of-Year Rates	Year-End Rates
First year	1.00%	1.50%	0.50%	3.00%	3.75%
Second year	1.75	1.50	0.75	4.00	4.75
Third year	2.50	1.50	1.00	5.00	5.75
Fourth year	3.25	1.50	1.25	6.00	6.25
Fifth year	3.50	1.50	1.50	6.50	6.50
Sixth year	3.50	1.50	2.00	7.00	7.00
Seventh year	3.50	1.50	2.50	7.50	7.50
Eighth and later	3.50	1.50	3.00	8.00	8.00

Note that very few actuaries use varying inflation components (other than varying medical trend rates). That is partially due to the difficulty in establishing the appropriate assumption. In addition, if a varying cost-of-living inflation component is used, it should also be included in all other inflation-affected assumptions (i.e., expected investment return, salary increase, IRS limits on compensation and benefits, Social Security assumptions, etc.), which adds to the complexity of its use.

Other Considerations in Using Durational Discount Rates

Are Durational Discount Rates a Method or an Assumption?

In our discussions with the FASB staff, they indicated that changing from a level discount rate to a select and ultimate discount rate might be construed to be a change in accounting method rather than a change in assumptions. Based on FAS 87 Q&A 57, we believe that the change to a durational discount rate would be a change in assumption rather than a change in method. Although we do not view the change as a change in accounting method, if the auditor/SEC views it as a change in accounting method, a cumulative true up of expense may be required in the year of change. If this is a change in assumption, no additional disclosure is needed. Since the PBO/APBO produced by a change from a level discount rate to a select and ultimate discount rate should be equivalent, this change should not affect the unrecognized experience gain and loss or the following-year amortization of gain and loss.

Expense Variation

Durational discount rates are more likely to change materially from one year to the next than are level discount rates. In some years, although the level rate would not change materially, the durational discount rates must be changed to match changes in the shape of the yield curve. Therefore, if one uses Method 1 to calculate interest cost, the interest cost can change significantly from one year to the next, depending on the economy and federal policy.

Correlation to Profits

Method 1 generally produces much lower pension expense when the economy is growing quickly (i.e., steep, upward-sloped yield curve) and produces much higher pension expense when the economy is entering a recession (i.e., inverted yield curve). For some plan sponsors, this could result in higher expense in years with lower profits.

ASOP 27 Consistency Requirements

If Method 1 is used appropriately (i.e., when the approach includes a component for increasing inflation rates), then the compensation scale, expected return on assets, health care cost trend rate, etc., must be modified to reflect the increasing inflation component of the durational discount rates. Section 3.10 of Actuarial Standard of Practice No. 27, Selection of Economic Assumptions for Measuring Pension Obligations, requires consistency in the selection of all economic assumptions. It states:

> For example, if the actuary has chosen to use select and ultimate inflation rates, the actuary should ordinarily choose select and ultimate investment return rates, discount rates, and compensation scales, and both the periods and levels of select and ultimate inflation rates should be consistent within each assumption.

Note that ASOP 27 applies not only to assumptions an actuary selects but also to advice the actuary gives regarding assumption selection. ASOP 27 has applied to economic-related assumptions for pension plans since 15 July 1997. Based on the second amendment to ASOP 6, ASOP 27 also applies to retiree group benefit obligations effective 1 January 2003.

For funded plans, this means the expected return on plan assets should be determined using the assumed real rate of return plus the first-year inflation rate. When a consistent assumption set is used for a well-funded plan, the reduction in expense due to the decrease in the interest cost will generally be offset by the consistent reduction in the expected return on plan assets. For unfunded or poorly funded pension plans or other postretirement benefit plans, using a consistent durational economic assumption set may reduce the expense.

ASOP 4 Disclosure Requirements

Since durational discount rates always include a liquidity/default risk premium component, the use of Method 1 is expected to result in continual liability experience losses. Given that the average difference between short- and long-term interest rates is close to 2% (1.4% U.S. Treasury liquidity premium and 0.6% high-quality corporate bond default risk premium), these experience losses will be very significant and will result in an increasing net pension cost pattern. Section 6.3(h) of ASOP No. 4, Measuring Pension Obligations, requires that "If the actuary expects that the long-term trend of costs resulting from the continued use of present assumptions and methods would result in a significantly increased or decreased cost basis this should also be communicated." Therefore, the actuary using Method 1 to compute interest cost should include such a statement in the communication of actuarial results.

Disclosure of Discount Rates and Interest Cost Method

Both FAS 87 and FAS 106 require only the disclosure of weighted-average discount rates. Therefore, the disclosure does not list whether select and ultimate rates were used. Also, the disclosure does not identify the method used to calculate interest cost. Therefore, stockholders and investors will not know how interest cost was calculated. In addition, since it is not disclosed, we do not know how many companies are using durational discount rates or how they are calculating interest cost.

Regardless of what the client discloses in their financial statements, we believe that the actuary's communication of FAS 87/106 results should include the following items:

- A complete description of the select and ultimate discount rates used
- A description of the method used to calculate interest cost and the implied year-end discount rates and
- If Method 1 is used, a statement that the current assumptions and methods are expected to result in actuarial losses and an increasing net pension cost pattern.

Conclusion

In most years interest rates vary by duration. Therefore, the use of durational discount rates may be appropriate. Our main concerns are the following:

- Assuming all durational interest rate differences are due to inflation is incorrect
- Assuming durational inflation requires consistent use in all assumptions
- Assuming durational inflation requires consistent calculation of both interest cost and expected investment return
- If durational discount rates are used, they should be used for all years consistently rather than used selectively
- If durational discount rates are used, they should be disclosed properly in the actuarial report and the plan sponsor's financial statements (since FAS 132 requires only disclosure of the weighted average rate, some additional disclosure is preferred). Calculation of interest cost should be disclosed completely in the actuarial report.

Before a plan sponsor and their auditor approve the use of this technique, they both should be informed fully on the issues covered in this paper.

Valuation of Pension Obligations with Lump Sums

by Richard Q. Wendt, FSA, CFA

Introduction

It is common for defined benefit pension plans to allow participants to choose between annuity payments and lump-sum payments at retirement. In traditional defined benefit plans, the plan provisions determine the annuity payment amount, and the lump-sum amount is based on a conversion from annuity payments to the lump sum. In account-based plans, the plan provisions determine the lump-sum amount, and the annuity payment amount is determined by conversion from the lump sum to annuity payments.

Because of the many factors that affect the amount and value of future lump sums, as well as the multiple valuation bases, actuaries have typically used "rules of thumb" to value plans with lump sums. Rule-of-thumb assumptions may not be sufficiently detailed in their development to assure plan sponsors and accountants that the plan benefits are correctly valued. In some cases there has been confusion about the economic consequences of the different types of lump sums. For example, some practitioners are under the misimpression that the presence of lump sums means that the plan obligation has a shorter duration than the obligation for a plan without lump sums. That is correct only in a limited number of situations.

This paper will discuss the valuation of lump sums for both funding and FAS 87 valuations, particularly the selection of appropriate discount rates. Because of the complexity of the requirements for setting discount rates for FAS 87, most of the discussion is with respect to FAS 87. Since the mathematical calculations required for exact calculation of lump-sum values may be incompatible with some actuarial valuation systems, the last part of the paper discusses some approaches for approximating the detailed calculations with simplified approaches.

This paper shows that reasonable approximations exist to the theoretically correct methodology for valuing interest-sensitive lump sums. One approach would be, first, to determine the discount rate by referencing the underlying annuity cash flow and then use an assumed lump-sum conversion rate equal to the discount rate less an appropriate spread. A second approach would be to apply a load factor to new benefits. A third approach would be to use an expected average lump-sum conversion rate based on implied future Treasury bond yields to project lump-sum cash flows.

The Appendix to this paper provides a detailed analysis of the underlying financial principles.

General Approach for Selecting Discount Rates for FAS 87

Actuaries use several approaches to select discount rates for FAS 87.[1] Two of the approaches have sufficient detail to provide explicit validation for the selection of the discount rate: the bond portfolio approach and the hypothetical yield curve approach.[2] This paper uses the hypothetical yield curve approach, but the principles also apply to the bond portfolio approach.

[1] The general FAS 87 standard for discount rates, as amplified by the SEC, is to use hypothetical spot rates for high-quality, noncallable, zero-coupon corporate bonds (top two rating categories).

[2] Under the bond portfolio approach, the actuary would construct a portfolio of bonds that match the expected benefit cash flow; under the hypothetical yield curve approach, the actuary would use bond yield data to construct a yield curve that can be used to discount benefit cash flow.

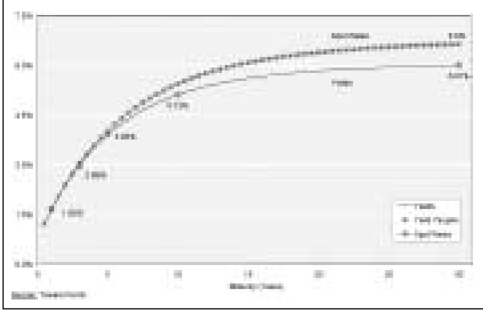
For traditional defined benefit plans with optional lump-sum benefits, typical benefit projections would indicate that the lump sums cause the liability duration to shorten and would imply that the discount rate should be lower than for a plan with strictly annuity payments. This is only half the story. This paper shows that there are two distinct types of lump-sum benefits: interest-sensitive and non-interest-sensitive payments. Traditional defined benefit plans typically pay interest-sensitive lump sums, while cash balance and other account-based plans typically pay non-interest-sensitive lump sums. This paper shows that the duration of interest-sensitive lump-sum benefits is the same as for the underlying annuity payments and illustrates the rationale for this concept.[3]

Actuaries who rely on professional judgment to choose discount rates often come close to the theoretically correct liability value, but may find it hard to explain the impact of lump-sum benefits on the results. On the other hand, actuaries who use more precise methodology may need to use advanced procedures to ensure proper valuation of interest-sensitive lump sums. This paper supports both practice methodologies.

The general concept used to value FAS 87 benefit obligations is to determine the "settlement" value of the benefit obligation. This is the value required to provide fully for expected future benefits, with no risk.

Under the hypothetical yield curve approach, the practitioner estimates a term structure of yields and corresponding spot rates for high-quality, noncallable corporate bonds. The actual construction of the hypothetical yield and spot-rate curves is beyond the scope of this paper.[4] This discussion follows the convention that spot rates beyond 30 years are assumed level. Figure 1 shows an estimated yield curve and corresponding spot-rate curve at December 31, 2003. Note that the spot-rate curve, which is composed of the rates that would apply to hypothetical zero-coupon bonds, is above the yield curve. This is typical for "normally shaped," i.e., upwardly sloped yield curves.

Figure 1—Corporate Team Structure
December 31, 2003



Once the spot rates are determined, the practitioner calculates the present value of benefits by applying the spot rates to the projected benefit cash flow. The discount factor for each year's benefit payments is the spot rate for that time horizon.[5] Solving for the internal rate of return that matches the present value to the cash flow provides the level discount rate for FAS 87 purposes.

Non-Interest-Sensitive Lump Sums

The application of the spot-rate curve to future lump sums is straightforward, as long as the lump sums do not vary with future interest rates. For example, account-based plans determine benefit payments by projecting a hypothetical account balance for each participant, based on expected service credits and interest credits; the benefit obligation is determined by discounting the lump

[3] Some plans may have benefits that are a combination of interest-sensitive and non-interest-sensitive features. For example, a plan that pays the larger of a cash balance account balance and the lump-sum equivalent of a final-pay benefit would be at least partially interest sensitive. Valuation of that type of benefit requires advanced analysis

[4] Interested readers may refer to the presentation Supporting an SEC Acceptable Discount Rate, by Larry Bader and Richard Wendt at the 1994 CCA meeting, for descriptions of possible methodologies for constructing yield curves.

[5] Although some may consider the use of spot rates to be a form of select and ultimate rates, as long as the spot rates are derived from the initial yield curve, their use is similar to the standard approach for pricing financial instruments.

sum by the spot rates. Since the lump-sum amount does not vary with future interest rates, this is considered a non-interest-sensitive benefit.

Some account-based plans allow the participant to choose between the lump sum and optional annuity payments, where interest rates in effect at the retirement date determine the annuity payment amounts. Since the discounted value of the annuity at retirement is equal to the lump sum, the obligation amount is determined in the same way as for the lump sum: by discounting the lump-sum cash flow by the spot rates.[6]

Another example of a non-interest-sensitive lump sum is the use of a plan-specified, fixed interest rate to convert an annuity benefit for a traditional defined benefit plan to a lump sum. Since the lump-sum benefit does not vary with future interest rates, the benefit obligation for the participants assumed to elect lump sums also is determined by discounting the lump-sum payments by the spot rates.

Non-interest-sensitive lump sums do shorten the duration of the plan obligation and normally imply a lower discount rate than for annuity payments.

Interest-Sensitive Lump Sums

The analysis of interest-sensitive lump sums is of more interest, since the results may strike the reader as counterintuitive.

The benefit formula for a traditional defined benefit plan defines the benefit in terms of annuity payments. If the participant is able to elect a lump-sum payment, then market rates as of the retirement date typically determine the conversion of the annuity benefit to a lump sum.

Since the amount of the future lump sum depends on interest rates at retirement, this is considered an interest-sensitive benefit. In most cases the plan-specified market rates are 30-year Treasury bond yields rather than corporate yields. The use of Treasury yields creates an interest subsidy, which we will ignore for the time being. For now, this discussion assumes that the future market yield used to calculate lump sums is the corporate yield curve, the same yield curve quality that is used to value today's obligation.

To illustrate the proper valuation of an interest-sensitive lump sum more easily, consider the example of a single participant in a highly simplified plan:

> Participant X is expected to retire in 10 years. The benefit from this particular plan is $10,000, payable as a single payment five years after retirement. At retirement X will have the option of choosing either the $10,000 payable five years later or a lump sum equal to the present value of the $10,000 discounted at then current rates.

The following relationships for participant X will be demonstrated:

- The present value of lump sums = the present value of annuities
- The duration of lump sums = the duration of annuities
- The present value of the lump sums depends only on the current yield curve and does not require the projection of future yields.

How should this benefit be valued? One approach would be to look at today's five-year spot rates and assume that the rate is unchanged at the future retirement date. If the current five-year spot rate were 4.0%, and the current 10-year spot were 5.5%, then the amount of the future lump sum would be[7]

6 This methodology assumes that the annuity payments are determined consistently with the spot-rate curve, i.e., the corporate yield curve. To the extent that the annuity payments are determined on a different basis, the practitioner should make an adjustment for the difference.

7 Spot rates in this example are the estimated spot rates at December 31, 2003.

$$\frac{\$10{,}000}{1.040^5}$$

and today's present value would be

$$\frac{\$10{,}000/1.040^5}{1.055^{10}}$$

which results in a present value of $4,812. The correct answer, however, is to use the 15-year spot rate (6.1% in this example), and the present value would be

$$\frac{\$10{,}000}{1.061^{15}}$$

which is $4,114. That is a reduction of 15% from the prior calculation.

Here is why the 15-year spot rate is appropriate: If the plan sponsor were to buy a 15-year zero-coupon bond today, with a $10,000 maturity value, the price would be $4,114, as determined by the 15-year spot rate.

The value of the zero-coupon bond after 15 years will be exactly $10,000, but the value of the bond at the expected retirement date will not be known until that date arrives. However, we do know that, at retirement, it will have five years remaining until maturity, and the future five-year spot rate will determine the value of the bond at that time. The future five-year spot rate will also determine the amount of the lump sum.

Whatever the value of the bond at retirement, it will be exactly equal to the value of $10,000 payable five years after retirement. Therefore, if the participant elects the lump sum, the zero-coupon bond will exactly cover the value at retirement; if the participant elects the future payment, then the maturing bond will pay the $10,000 five years after retirement. The 15-year zero-coupon bond exactly supports either outcome, and the plan sponsor should be indifferent to participant X's choice of annuity or lump sum.

Note that there is absolutely no risk, no matter what the value of the lump sum in 10 years. Consequently the 15-year zero-coupon bond fully settles the benefit obligation. *In other words, we have proven that the present value of both the benefit payment and lump sum can be determined by discounting the benefit payment by the 15-year spot rate.* Since an annuity is simply a series of single payments, it is clear that the same logic would apply to an annuity.

Basic Principle 1: The present value of a future benefit, which may be either a defined annuity or an equivalent lump sum, is equal to the present value of the annuity, as long as:

- No interest rate subsidies are included; i.e., the lump-sum values are determined from a future yield curve with the same credit quality as the yield curve that is used to set the discount rate, and
- Benefit subsidies (e.g., early retirement subsidies), if any, are equally applied to annuities and lump sums.

Furthermore, since the 15-year zero coupon bond fully settles the liability, regardless of any future interest rate changes, the duration of the benefit payment and the lump sum are both equal to the duration of the 15-year zero-coupon bond. In an annuity context, we know that the duration of an annuity is equal to a present-value weighted average of the duration of each annuity payment.[8] Therefore, the duration of the annuity equals the duration of the corresponding lump sum.

Finally, note that the current yield curve is the basis for all discounted values. It was not necessary to project future interest rates. This is the third of the three demonstrations promised earlier.

Basic Principle 2: The duration of the lump-sum benefit described above is equal to the duration of the benefit payable as an annuity.

[8] In this context "duration" refers to the modified Macaulay duration, which is a measure of sensitivity to interest rate changes. The Macaulay duration, or weighted average time to payment, would be higher for the annuity than for the lump sum.

Lump Sums with Interest Subsidy

In the "real world" most traditional defined benefit plans with interest-sensitive lump sums use 30-year Treasury bond yields to convert from annuity payments to lump sums. From the FAS 87 perspective, this is a subsidy, since the lump sums are typically larger than if they were based on corporate bond rates.[9]

One actuarial practice is to project lump-sum benefits using an assumed lump-sum conversion rate and then discount the projected lump-sum cash flow by the use of a preretirement discount rate.[10] The preretirement discount rate in this case would be comparable to the discount rate for a plan without lump sums. The practitioner would combine that rate with an assumed lump-sum conversion rate that has a spread below the preretirement discount rate consistent with historical spreads between long corporate yields and long Treasury yields, for example, 1%.[11] In most cases it turns out that this practice provides a reasonable value for lump sums. This paper will provide the detailed analysis supporting this intuitive approach.

The mathematics of calculating lump sums with interest subsidies can be complex, at least if a theoretical approach is used. Fortunately some simple approximations provide results for many common situations.

The Appendix develops a methodology for the exact valuation of interest-sensitive lump sums by showing, first, that, in addition to valuing nonsubsidized lump sums directly from the annuity payment cash flow, lump sums can also be valued using the lump-sum conversion rates implied by today's yield curve.[12] The values derived in either approach are identical. This is Basic Principle 3a.

Basic Principle 3a: The present value of future lump sums can be calculated by projecting lump-sum amounts with implied lump-sum conversion rates and then discounting the lump-sum amounts by the spot rates in the current corporate yield curve.

Given that principle, we can calculate the effect of an interest subsidy simply by using a different set of implied lump-sum conversion rates—the rates implied by today's Treasury yield curve. This is Basic Principle 3b and provides the basis for exact calculation of subsidized lump-sum benefits.

Basic Principle 3b: The present value of future lump sums with adjustments for interest subsidies can be calculated by projecting annuity benefits, converting to lump sums by using implied future 30-year Treasury bond yields, and then discounting the lump-sum amounts by the spot rates in the current corporate yield curve.

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[9] The relationship for a specific situation depends on the age at retirement and other demographic factors. In some individual cases, the lump-sum amount based on 30-year Treasury yields could be less than the lump-sum amount based on corporate spot rates.

[10] This paper uses the term "lump-sum conversion rate" to refer to the interest rate used to calculate lump-sum values. The term "preretirement discount rate" refers to the period from the valuation date to each participant's retirement. Since our hypothetical plan consists only of active participants, there is no ambiguity. In a plan with both active and inactive participants, there would likely be two rates, an assumed lump-sum conversion rate and a discount rate for calculating the present value of all projected annuity and lump-sum benefits.

[11] Another approach is to assume that the future lump-sum conversion rates are equal to today's Treasury yields. On December 31, 2003, that rate was 5.2%.

[12] The implied lump-sum conversion rates at any future date are calculated by "backing out" the spot rates prior to the specified date.

Table 1 summarizes the results of the methodologies as applied to variations of the hypothetical plan. The bottom row indicates the result of the incorrect methodology, resulting in a present value that is significantly higher than the correct value. Note that the detailed approach for valuing interest-sensitive lump sums results in a preretirement discount rate of 6.0%, whereas the approximate approach results in a preretirement discount rate of 6.3%. The difference in preretirement rates could affect the calculation of pension expense.

Table 1
Summary of Results for Hypothetical Plan

Plan Description	Lump-Sum Conversion Rate Methodology	Preretirement Rate	Lump-Sum Conversion Rate	Present Value
Annuity payments		6.3%	—	$163,000
Lump-sum payments:				
No interest subsidy	Implied future corporate spot rates	6.0	6.8% average[13]	163,000
Interest subsidy, based on 30-year Treasury yield	Approximation (lump sum= preretirement rate -1%)	6.3	5.3	175,000
	Exact (implied future Treasury yields)	6.0	5.9 average	174,000
	Future lump-sum rate = today's lump-sum rate (incorrect duration)	5.9	5.3	182,000

[13] The year-by-year implied future lump-sum conversion rates are used; however, the same answer would be obtained by using a lump-sum conversion rate of 6.8%.

Use of Approximate Methodology for Valuing Lump Sums

The present value of interest-sensitive lump sums without an interest subsidy can be readily calculated by using the underlying annuity payments. The use of implied lump-sum conversion rates to calculate future lump-sum amounts will also provide the correct answer but requires unnecessary complexity.

As shown in the Appendix, the present value of interest-sensitive lump sums with an interest subsidy can be calculated directly by projecting lump-sum amounts based on implied Treasury yields.

The use of implied lump-sum conversion rates could be incompatible with existing actuarial valuation systems. How can we simplify the calculations?

Three possible approaches come to mind; one uses projected lump sums, and the other two use projected annuity payments.

1. Calculate implied future Treasury bond yields and estimate the average lump-sum conversion rate. Then project the lump-sum cash flow with the average lump-sum conversion rate and use corporate spot rates to value the cash flow. For example, our hypothetical plan could be valued with an average lump-sum conversion rate of 5.9%; the present value of the cash flow would be the same as if year-by-year implied lump-sum conversion rates had been used. Inspection of the chart of implied 30-year Treasury yields in the Appendix indicates that the implied yields vary between 5.6% and 6.1%; given the distribution of benefit payments, 5.9% would be a reasonable value for the weighted average lump-sum conversion rate. The resulting preretirement discount rate for the sample plan would be 6%.
2. Project the underlying annuity payments and use the corporate spot-rate curve to calculate the present value of the cash flow. For this method, the preretirement (and postretirement) discount rate for the hypothetical plan would be 6.3%. Then apply a load factor to the annuity cash flow to represent the cost of the interest subsidy. For example, applying a load factor of approximately 7% to the projected annuity payments for new retirees would result in the correct present value for the hypothetical plan.
3. Project the underlying annuity payments and use the corporate spot-rate curve to calculate the present value of the cash flow. As in the prior method, the preretirement discount rate for the hypothetical plan would be 6.3%.Then use an interest spread between the preretirement discount rate and the lump-sum conversion rate to approximate the cost of the interest subsidy. For our hypothetical plan, a 1% spread would be appropriate, setting the assumed lump-sum conversion rate at 5.3%.

Valuation of Lump Sums for Funding

To complete the list of valuation methodologies for lump-sum benefits, the last discussion is with respect to funding liabilities. Funding liabilities may or may not be regulated by federal or state law. In either case they are valued differently than FAS 87 obligations.

Actuaries typically use assumed investment return and expected cash flow to value funding liabilities. Funding valuations typically do not reference spot rates and yield curves. Unlike accounting obligations, funding liabilities use the "expected present value" concept and not the "settlement value" concept. Therefore, to be consistent with the principles of funding valuation, future lump-sum values should be based on assumed future lump-sum conversion rates and need not be consistent with today's yield curve. However, the assumed future conversion rates should be consistent with the inflation assumptions used for the valuation.

Conclusion

Plans with non-interest-sensitive lump-sum benefits should use the projected lump-sum benefits to determine the appropriate discount rate and obligation value.

For plans with interest-sensitive lump sums, this paper shows how to calculate present values in the context of FAS 87 valuations. The methodology uses the current spot-rate curve to value benefits consistent with standard financial practice. Practitioners can use the principles stated in this paper to validate actuarial judgment as applied to specific plans.

The following principles are stated:

Basic Principle 1: The present value of a future benefit, which may be either a defined annuity or an equivalent lump sum, is equal to the present value of the annuity, as long as

- No interest rate subsidies are used; that is, the lump-sum values are determined from a future yield curve that is consistent with the yield curve that is used to set the discount rate, and
- Benefit subsidies (e.g., early retirement subsidies), if any, are equally applied to annuities and lump sums.

Basic Principle 2: The duration of the lump-sum benefit described above is equal to the duration of the benefit payable as an annuity.

Basic Principle 3a: The present value of future lump sums can be calculated by projecting lump-sum amounts with implied lump-sum conversion rates and then discounting the lump-sum amounts by the spot rates in the current corporate yield curve.

Basic Principle 3b: The present value of future lump sums with interest subsidies can be calculated by projecting annuity benefits, converting to lump sums by using implied 30-year Treasury bond yields, and then discounting the lump-sum amounts by the spot rates in the current corporate yield curve.

Finally, the paper shows three methodologies to approximate the theoretically correct value of interest-sensitive lump sums.

Appendix: Valuing Interest-Sensitive Lump Sums

This Appendix contains a detailed analysis of the underlying financial principles for valuing defined benefit plans with interest-sensitive lump sums. The second part of the Appendix discusses the valuation of lump sums with interest subsidies. The term "subsidy" refers to the use of lump-sum conversion rates based on Treasury bond yields and preretirement discount rates based on corporate bond yields.

Hypothetical Plan Example for Annuity Payments

Here is a stylized example of the valuation methodology for a hypothetical plan. We will create a plan where all current participants are between the ages of 25 and 65 and everyone retires exactly at 65. At the start only active participants are in the plan. The actuarial assumptions forecast that exactly $1,000 worth of annual benefits is expected to go into force each year for the closed group for the next 41 years. In this first example, all benefits are paid as annuities.

Figure 2 shows the pattern of expected cash flow for this plan for the next 100 years, compared to the spot rates in the current yield curve.[14] The annuity cash-flow stream reaches a peak of about $18,000 in year 41 and then tails off because of the absence of new retirees. The total amount of benefits payable over 100 years is $740,000.

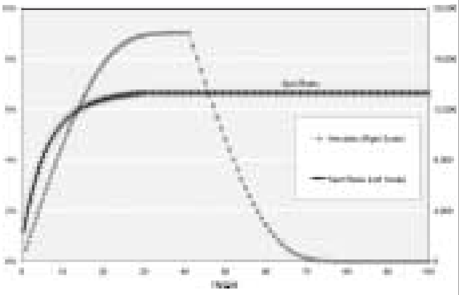


Figure 2—Spot Rates and Projected Benefits

To value this benefit stream, we will use the spot-rate curve for high-quality, noncallable corporate bonds discussed in the body of the paper. Applying the spot rates to the cash flow results in a present value of $163,000. The level equivalent discount rate (i.e., the internal rate of return) is 6.3% in this example. (If this rate seems slightly high, it is due to the assumption of a group of all active employees, which has a longer duration than typical plans.) The modified Macaulay duration of the cash flow comes in at 19 years,[15] as compared to about 14 years for a 30-year bond.

Hypothetical Plan with Interest-Sensitive Lump-Sum Payments

As in many traditional defined benefit plans, participants in the hypothetical plan may elect lump-sum benefits, based on 30-year Treasury bond yields at the retirement date. Since the Treasury rates provide an interest subsidy, we would expect the present value of the lump sums to be somewhat greater than $163,000, the discounted value of the annuity payments.

A common approach to selecting valuation assumptions for this plan would be the following steps:

1. Select a preretirement discount rate as if the plan did not pay lump sums, which in this example would be 6.3%

[14] Technically the cash flow should be based on projected ABO or PBO benefits. This paper assumes that the expected cash flow is appropriate for measuring the particular obligation.

[15] Salary-related benefits would typically have a longer duration. The hypothetical example is based on level benefits.

2. Assume a 1% spread between the preretirement discount rate and the lump-sum conversion rate
3. Thereby use 5.3% for the lump-sum conversion rate.

Using that combination of discount rate and lump-sum conversion rate, the present value of the lump-sum benefits would be $175,000. As expected, the result is greater than $163,000; the detailed analysis shown below indicates that the judgmental result often can be very close to the theoretically correct value.

An incorrect method of validating this common approach would be to discount the projected lump-sum benefits by the corporate spot-rate curve and solve for the internal rate of return. That methodology would result in a present value of $182,000 and a preretirement discount rate of 5.9%. The lower discount rate could be judged naively to be due to the perceived shorter duration of the lump-sum payments (12 years).[16] As indicated above, the true present value is closer to $175,000 than $182,000.

The reason that the common approach works is twofold:

- The appropriate discount rate for the plan, if there were no interest subsidies, would be 6.3%
- The cost impact of a lump-sum discount rate of 5.3% instead of 6.3% is very close to the true cost of the interest subsidy.

The following sections will discuss the determination of the cost of the interest subsidy.

Interest-Sensitive Lump-Sum Payments: Using Implied Lump-Sum Conversion Rates

Returning to the example for participant X in the body of the paper, we used the 15-year spot rate to calculate the present value of the annuity benefit and lump-sum benefit; the resulting value was $4,114. We also can calculate the present value by projecting the lump-sum benefit with a lump-sum conversion rate and then applying a preretirement discount rate to the lump-sum benefit. If the preretirement rate and lump-sum conversion rate are consistent with the spot-rate curve, then the result will be the same present value of $4,114.[17] For example, if the preretirement discount rate were the 10-year spot rate (5.5% in this example), then a lump-sum conversion rate of 7.4% would result in the correct present value. The 7.4% rate is the implied lump-sum conversion rate for 10 years in the future. The 7.4% rate is not a forecast of increasing interest rates from today's five-year rate of 4.0%—it is simply the implied rate, also known as a forward rate.

Implied conversion rates are not needed to value "plain vanilla" benefits, that is, benefits without interest subsidies. However, the implied conversion rates provide the basis for valuing benefits with interest subsidies.

Given the starting yield curve, an equivalent spot-rate curve can be calculated easily by standard formulas. Then, for any future horizon, a full implied spot-rate curve can be calculated by "backing out" the spot rate for that horizon.[18] In formula terms,

[16] This duration measure does not take account of the interest sensitivity of the lump-sum payments.

[17] In fact, there are an infinite number of combinations of preretirement rates and lump-sum conversion rates that will yield the correct present value. However, not all combinations make financial sense.

[18] Technically a risk-free yield curve should be used to calculate implied future spot rates. Since corporate bonds are subject to default, they are not considered risk free. However, this paper follows the FAS 87 principle that the corporate yield curve is the appropriate yield curve for discounting liabilities.

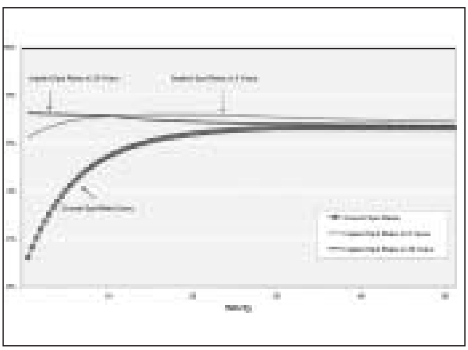
- (1 + implied spot rate for maturity *t* at time *T*)^t= $\dfrac{(1+S(T+t))^{T+t}}{(1+S(T))^T}$, where S($x$) = the current spot rate for maturity of x

- In our numerical example: $1.074^5 = \dfrac{1.061^{15}}{1.055^{10}}$ (after rounding)

- Forward rate = 7.4%

Figure 3 uses the above formula to calculate the full implied spot-rate curves for horizons of five and 20 years in the future, based on the spot-rate curve at December 31, 2003. In other words, the implied spot-rate curve at the 20-year horizon is determined by backing out the 20-year spot rate from the current spot-rate curve. Curves 30 or more years in the future are perfectly flat, because of the assumption of level spot rates beyond 30-year maturities. Once the implied spot rates are determined for any future date, the practitioner can use them to calculate implied future yields and implied future lump-sum conversion rates.

Figure 3—Implied Future Spot Rate Curves
Based on December 31, 2003 Corporate Spot Rate Curves



Valuing Lump Sums for the Hypothetical Plan with Implied Lump-Sum Conversion Rates

Next, we return to the example of the hypothetical plan with $1,000 annual benefits and apply the concept of the implied lump-sum conversion rates. For the moment we continue to assume that the lump sums are based on life annuity factors calculated with corporate yields (i.e., without interest subsidy). The assumed mortality is the same as used to project the runoff of annuity payments.

Applying the current spot-rate curve to the projected lump-sum amounts results in a present value of $163,000 and an internal rate of return of 6.0%. Therefore, the preretirement discount rate would be 6.00%. Note that, as expected, the present value is identical to the present value calculated for annuity payments, while the preretirement discount rate has dropped from 6.3% to 6.0%.[19]

This example demonstrates that the correct present values can be calculated from either a discounted annuity cash flow or a discounted lump-sum cash flow, as long as the projected lump-sum amounts are based on the implied lump-sum conversion rates.[20]

Valuing Lump Sums with Interest Subsidies

So far the assumption for the hypothetical plan has been that the lump-sum amount is based on the same yield curve that is used for discounting the cash flow: the corporate yield curve. In many plans the lump-sum amount includes an interest subsidy, usually by determining the conversion rate by reference to 30-year Treasury bond yields.

[19] The result is equivalent to a preretirement discount rate of 6.00% and a level lump-sum discount rate of 6.8%, which is the weighted average future lump-sum conversion rate. As stated above, this is not a forecast of higher future interest rates, but simply the rates consistent with today's term structure.

[20] Some actuaries believe that both approaches are counterintuitive. However, the simple example of participant X shows the validity of the analysis.

This section of the Appendix shows how to value lump sums with an interest subsidy. Where there is no interest subsidy, the actuary simply can use the annuity payments (or implied lump-sum conversion rates, if desired) for determining the lump-sum cash flow.

Where the lump-sum amount is determined by reference to Treasury yields, the present value of the subsidy depends on the relationship between future corporate spot rates and future Treasury spot rates. The spreads between corporate and Treasury yields may vary widely over time, so there is significant uncertainty involved in the valuation of the subsidy.[21]

For the example of the $10,000 benefit payable five years after retirement and where the lump-sum value at retirement is based on the 30-year Treasury yield, consider the following logic: From our Basic Principle 1, we know that, if the lump-sum amount were based on corporate spot rates, then the present value would be

$$\frac{PV = 10{,}000}{(1+ \text{Corporate Spot Rate (year 15)})^{15}}$$

However, the lump sum is actually based on the future 30-year Treasury yield. Our simplifying assumption of parallel yield changes allows us to "splice" the two yield curves by projecting lump sums at the implied future Treasury yield and then discounting by the preretirement corporate spot rate:

$$PV = \frac{\text{Lump sum discounted by implied 30-year Treasury yield}}{(1+ \text{Corporate Spot Rate (year 10)})^{10}}$$

For this calculation the implied 30-year Treasury yield 10 years in the future is derived from the full Treasury spot-rate curve 10 years in the future. That spot-rate curve, in turn, is based on the current Treasury yield curve.

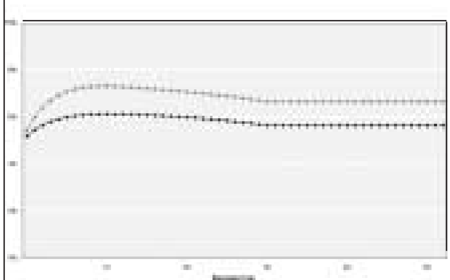
The cost of the subsidy is the ratio of the lump sum based on implied 30-year Treasury yield to the lump sum based on implied corporate spot rates minus one. Therefore, the following equation holds:

PV of subsidized benefit = PV of unsubsidized benefit $\times$ (1 + cost of subsidy).

As in the case of corporate spot rates, the implied Treasury spot-rate curves and yields can be calculated for any future date. Those curves then can be used to calculate implied lump-sum conversion rates. The present value of the lump sum is calculated by projecting the lump-sum amount with the implied 30-year Treasury yield and then discounting the lump sum at corporate spot rates to the valuation date.

Figure 4 shows the relationship of implied lump-sum conversion rates for corporate spot rates and implied 30-year Treasury bond yields, based on the yield curves at December 31, 2003.[22] Note that the lump-sum conversion rate based on corporate spot rates depends on the form of the annuity, age and sex of the annuitant, and mortality table; the implied future Treasury-bond yield is independent of demographic factors.

Figure 4—Implied Future Lump-Sum Conversion Rates for Life Annuity and 30-Year Treasury Yields Based on December 31, 2003 Yield Curves



Note: The lower line on this chart is the implied future 30-year Treasury yield, based on the December 31, 2003 Treasury yield curve. The ultimate rates converge to the respective ultimate spot rates. The ultimate spread is approximately 1 %.

[21] In the FAS 87 accounting context, we are assuming that corporate bonds are relatively risk free. Since we know that Treasury bonds are actually risk-free, it makes sense to make a simplifying assumption that the two yield curves move in parallel in the future. Specifically we need to assume that the future short rates move in proportion to the forward rates in each yield curve. With this simplifying assumption, the present values can be calculated by means of a "spliced spot-rate curve."

[22] The example lump-sum conversion rates are based on retirement age 65 and a commonly used mortality table.

Applying corporate discount factors to the projected lump sums, the present value is $174,000, and the preretirement discount rate is 6.0%. The subsidized benefits in this example increase the present value by about 7%. Assuming a preretirement discount rate of 6.0%, a level lump-sum conversion rate of 5.9% would be required to obtain the same present value.[23]

This Appendix demonstrates that implied lump-sum conversion rates provide the theoretically correct present value of interest-sensitive lump sums. The body of the paper suggests how simplified approaches can approximate the theoretically correct values.

[23] That is, a preretirement interest rate of 6% and a lump-sum conversion rate of 5.9% results in a present value of $174,000.

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