Swiss Solvency Test in Non-life Insurance

Luder Thomas
Federal Office of Private Insurance
Schwanengasse 2
3003 Bern
Switzerland
Tel. ++41 31 325 01 68
Fax. ++41 31 323 71 56
thomas.luder@bpv.admin.ch

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Abstract

The Swiss Solvency Test (SST) is a tool of the Swiss insurance regulator to measure the risk of insurance companies. The two major elements of SST are the available economic capital called risk bearing capital (RBC) and the target capital (tc). Their calculation is presented with a focus on non-life insurance.

Keywords: Swiss Solvency Test, Solvency II, Insurance Supervision, Non-Life Insurance

1 Introduction

The Swiss Solvency Test (SST) is a tool of the Swiss regulator (Federal Office of Private Insurance) to improve the identification of risks to which an insurance company is exposed (FOPI, 2004). It is set up as a stochastic risk model including scenarios for (i) market and ALM risk, (ii) insurance risk, and (iii) credit risk. The SST contains the definition of a risk bearing capital (RBC) and a target capital (tc), where the first is the available economic capital and the latter measures the risk. The time horizon is set to one year.

The SST defines a standard stochastic model together with a set of scenarios to actually determine the target capital. However, since the SST aims to be principle based, each insurer has got the possibility to deviate from this standard framwork. The degree of deviation can be anything between minor changes in some default parameter values on the one side and a complete replacement of the standard model by the company's internal risk model on the other side.

The standard model was developed in a close collaboration between the regulator and the Swiss insurance industry, which provided major and material contributions. After the work had begun in 2003, it lead to a first prototype of the SST in 2004. This prototype was tested in a field test in summer 2004 with some large Swiss insurers. The results and experiences were taken to identify errors and pitfalls and to determine additional parameters. In 2005 a next field test is performed with a broader range of insurers.

This article describes the standard model for non-life insurance. After setting up the risk bearing capital, we introduce the target capital. That part is then followed by a description of the stochastic modeling and an introduction into the scenario evaluation and aggregation.

2 Risk Bearing Capital

The risk bearing capital (RBC) is the available economic capital of the insurer which can be used as a reservoir to survive all the imponderabilities and randomness lying ahead on its one year trajectory in the business environment. RBC therefore is a value over which the insurer can dispose freely, i.e. it is capital not being legally bounded by any stakeholder of the company.

This leads to the following definition of the risk bearing capital at time t. It is defined as the difference between the value $A(t)$ of asset and the value $L(t)$ of the liabilities:

$$RBC(t) := A(t) - L(t) \tag{1}$$

This definition immediately leads to two questions: firstly, which types of liabilities are measured in $L(t)$, and secondly, how are the values of assets and liabilities measured?

The RBC shall contain all types of capital which can freely be used by the company in a situation of distress. Therefore, L contains the values of all types of liabilities which are legally bound or are promised to any of the stakeholders except the shareholders. Usually, the largest positions within L are the claims provisions and dept capital.

There are liability positions on which the insurer holds all options. Such positions have the same quality as equity capital, therefore, they are not part of L but of RBC. Examples are hybrid capital, convertible bonds or catastrophe reserve.

The second question can be answered easily: the purpose of the SST is to measure the economic risk, so the valuation has to be a economic valuation. This means that values of assets have to reflect current market values. While this is simple for the category of daily traded assets such as shares, there is a need of additional guideline how to provide a value for assets which are not traded or hardly traded. Examples of this second category is real estate, mortgages and loans. Such guideline is given in Geissühler et al. (2005). Claims reserve have to be valued by a discounted best estimate. First, the best estimates of the nominal cash flows of future claims payments has to be determined. Then, for the purpose of the market consistent valuation, the present value of these future payments is considered. Analogously, the value of dept capital is defined a the value of discounted cash flows related to the dept.

The best estimate L of a liability is less than the price of that liabily on a market. The reason is that the liability carries a (run off) risk for which the liability taker wants to be indemnified. Therefore, the market value of liabilities is considered to be the sum of the best estimate and the so called risk margin, which we will introduce below. However, we stress at this point that the risk margin is not part of L in equation (1).

3 Measuring the Risk: Target Capital

3.1 Definition of the Target Capital

For the purpose of the SST field test 2005 we identify today with t_0 = January 1, while t_1 equals December 31, 2005.

According to the time horizon $t_1 - t_0$, we are interested in how the insurer's position can change within one year from 01.01. to 31.12. The position is measured by the RBC as described above, so it is natural to consider the possible values of $RBC_{31.12}$, given the current value $rbc_{01.01}$. This is depicted in figure 1. While $rbc_{01.01}$ is a deterministic quantity and can be read off from the current economic balance sheet, the variable $RBC_{31.12}$ is a one dimensional stochastic quantity. Each possible outcome of $RBC_{31.12}$ falls within one

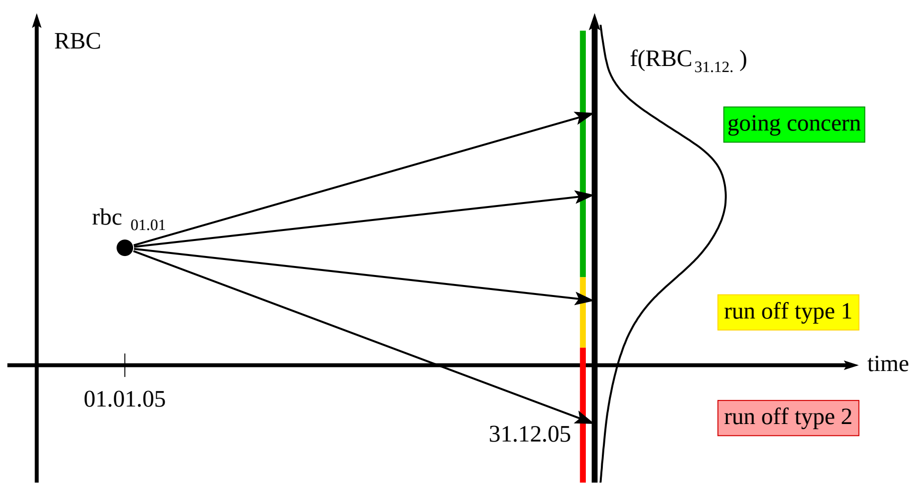

Figure 1: RBC at times $t_0 = 1.1.2005$ and $t_1 = 31.12.2005$. Starting from t_0 the arrows indicate four possible but arbitrary examples of outcomes at t_1. See text for explanation of the meaning of the three different levels.

of three regimes, which are shown in figure 1 in green, yellow and red.

The green regime is the set of all situations where the value of RBC is sufficiently high such that the insurer can continue its business ("going concern situation"). Obviously, in that situation, the policyholders with a claim will receive their money from the insurer.

The complement of the green regime is the red and the yellow zone. In a yellow and in a red situation the insurer is forced to stop taking new risks, i.e. it can not continue writing new business. Hence we are in the situation of a run-off, which means that the existing portfolio of old claims has to be developed to the end until the last payment is made. The difference between the yellow and the red regime is the happiness of the policyholder. In the yellow regime, the value of the assets is strictly larger than the expected value of the future payments. Therefore, the policyholder will get his money for his claim with a very high probability. This is called "run off type 1" in the figure.

The situation is different in the red regime ("run off type 2"). Obviously, if the value of the assets is below the expected value of the future payments, the insurer is not able to refund 100% of the claim amount of the policyholder. The policyholder suffers a financial loss.

However, the limit between the yellow and the red regime is not exactly at zero, but at a positive value. The reason is that there is still risk in the run-off. At time $t_1 = 31.12.2005$ we only have a best estimate of future payments. The true amount can be lower or higher with rougly 50% probability each. Therefore, if $RBC_{31.12.2005} = 0$, the policyholder will suffer a financial loss sometime in the future with 50% chance. Hence we are in the red regime.

In the framework of the SST, the limit between yellow and red is called the "risk margin" (rm). It is defined as the sum of present values of costs for future (regulatory) risk capitals. If the assets exceed the sum these future cost of capitals and the discounted best estimate of the liabilities, an investor can be found to provide risk capital for the run-off of the business. The assets are used to pay for the claims, and the excess is used to pay dividends to the investor for providing the risk capital (his investment). In this situation, it is the investor who takes the long term run-off risk. Therefore, the policyholder will not suffer a financial loss.

The aim of the regulator is to protect the policyholder. He requires that the probability that the insurer ends up in the red regime is low. In the SST this requirement is transformed into a requirement for $rbc_{1.1.2005}$, i.e. the current risk bearing capital. This requirement is called the "target capital" (tc) at t_0. One possible definition for tc was given by Gisler (2005):

$$ES_\alpha \left(RBC_{31.12.2005} | rbc_{1.1.2005} = tc_{1.1.2005} \right) = rm, \tag{2}$$

where the expected shortfall of a random variable X is defined as

$$ES_\alpha(X) := E[X | X \leq VaR_\alpha(X)]. \tag{3}$$

This implicit definition of target capital states: given that today the available capital is sufficient, the expected shortfall of the future risk bearing capital shall be equal to the risk margin. this means, if $\alpha = 1\%$, that available capital at year end is equal to or larger than the risk margin, even on the average of the worst 1% possbile outcomes of a year.

In order to avoid the implicity, the SST-definition of the target capital is

$$tc_{1.1.2005} = ES_\alpha \left(\frac{RBC_{31.12.2005}}{1 + r_1^{(0)}} - rbc_{1.1.2005} \right) + rm, \tag{4}$$

where $r_1^{(0)}$ is the one year risk free interest rate at t_0. Gisler (2005) showed that (4) is a good approximation of (2). The safety level $1-\alpha$ is typically about 99%. Whether the value of α has to be recalibrated will be decided at a later stage.

3.2 Change in RBC for a non-life insurer

3.2.1 Accident Year Principle

The SST model for non-life insurers is based on the accident year principle. This means that claims are grouped according to the date when the claim occurred. Other principles to group the claims are the underwriting year basis and the reporting year basis.

The accident year principle leads naturally to the distinction of (i) claims which have occurred in previous years ($PY = (-\infty, t_0)$), i.e. 2004, 2003, …, and (ii) claims which have not yet occurred but which will eventually occur in 2005. The latter are the current year (CY) claims.

Both PY and CY claims are related to a specific risk. The risk of PY claims is that their best estimate provision ($r_{PY}^{(0)}$) at the beginning of the year differs 2005 from the best estimate at the end of the year. The difference originates from the increase in information about the claims settlement process. The risk of CY claims consists of the uncertainties about the number of claims and about the amount of each single claim. The annual nominal incurred loss in 2005 is denoted with S_{CY}

3.2.2 Assumptions and Notation

The business model for a non-life insurer is based on the following assumptions.

Assets and Liabilities on January 1, 2005: On January 1. 2005, the insurer has got assets of value $a(0)$ and claims provisions (discounted best estimate) $l(0)$. The provisions include IBNyR (incurred but not yet reported) and provisions for future claims handling expenses/costs. These costs are composed of "allocated loss adjustment expenses" (ALAE) and "unallocated loss adjustment expenses" (ULAE). It is suggested to use the "New Yorker"-method to estimate the ULAE's value.

Costs and Earned Premiums: The SST assumes that the earned premium (p) for 2005 have already flown in a first part in 2004 and will flow in a second part on January 2. 2005. The first part is booked but not yet earned in 2004. Therefore it is called the "unearned premium". Because of that, there is an "unearned premium reserve" (upr) on the liability side of the balance sheet 31.12.2004. The remaining premium for 2005 is then $p - upr$. Immediately after the cash-in the value of the assets is $a(0) + (p - upr)$

Beside the claims adjustment expenses mentioned above there are the ordinary costs k to run the business. Similar to the premium, the SST assumption is that the cash flow related to the costs takes place on January 2. Overall, the value of the assets after premium inflow and cost outflow is

$$a(0) + (p - upr) - k. \tag{5}$$

This is the amount of assets which is available to invest at the beginning of 2005. The investment leads to a (stochastic) performance over the year. We denote this by R_I in relative terms, or by

$$R_I \cdot (a(0) + (p - upr) - k) \tag{6}$$

in absolute terms.

Payout Patterns: The non-life portfolio is divided into 12 lines of business (LoB.) In some of these LoB, the settlement of claims takes more than one year. This is especially true for all types of liability and accident annuity business. Therefore, we define for each LoB the payout pattern. Since there is a difference in the settlement of CY- and PY-claims, we also have to distinguish between CY

$$(\alpha_1^{(l)}, \alpha_2^{(l)}, \alpha_3^{(l)}, \ldots) \tag{7}$$

and PY payout patterns:

$$(\beta_1^{(l)}, \beta_2^{(l)}, \beta_3^{(l)}, \ldots) \tag{8}$$

for LoB $l = 1, \ldots, 12$. The patterns are normalised, i.e.

$$\sum_{i=1} \alpha_i^{(l)} = 1, \qquad \sum_{i=1} \beta_i^{(l)} = 1, \qquad \forall l \in 1, \ldots, 12. \tag{9}$$

If the superscript l is omitted in the following, we mean the patterns for the sum of all lines of business. It is assumed that the claims payment flow at the end of the year. For instance, the cash flows for CY claims are

$$\alpha_0 S_{CY}, \alpha_1 S_{CY}, \alpha_2 S_{CY}, \ldots \tag{10}$$

at the end of 2005, 2006, 2007, etc.

Yield Curves: To describe the yield curve of riskfree zero coupon bonds, we use the notation

$$\left(r_0^{(0)} \equiv 0, r_1^{(0)}, r_2^{(0)}, \ldots\right) \tag{11}$$

for the (observable) interest rates at time t_0 and

$$\left(R_0^{(1)} \equiv 0, R_1^{(1)}, R_2^{(1)}, \ldots\right) \tag{12}$$

for the unknown, hence stochastic, interest rates at time t_1. This immediately leads to the discount factors

$$v_i^{(0)} = \frac{1}{\left(1 + r_i^{(0)}\right)^i}, \qquad V_i^{(1)} = \frac{1}{\left(1 + R_i^{(1)}\right)^i}, \qquad i = 0, 1, \ldots \tag{13}$$

at time t_0 and t_1, respectively.

Discounted Incurred Loss and Discounted Best Estimate Reserves: If we combine payout patterns for CY and PY with the discount factors, we get the discounted incurred loss for the CY claims

$$d_{CY}^{(0)} \cdot S_{CY} = v_1^{(0)} \alpha_0 S_{CY} + v_2^{(0)} \alpha_1 S_{CY} + v_3^{(0)} \alpha_2 S_{CY} + \ldots = \left(\sum_{i=0} v_{i+1}^{(0)} \cdot \alpha_i \right) S_{CY} \qquad (14)$$

and the discounted best estimate for the PY claims provisions

$$d_{PY}^{(0)} \cdot r_{PY}^{(0)} = v_1^{(0)} \beta_0 r_{PY}^{(0)} + v_2^{(0)} \beta_1 r_{PY}^{(0)} + \ldots = \left(\sum_{i=0} v_{i+1}^{(0)} \cdot \beta_i \right) r_{PY}^{(0)}. \qquad (15)$$

$d_{CY}^{(0)}$ and $d_{PY}^{(0)}$ are discount factors at date t_0 for CY- and PY-claims. At the end of 2005, just before the payment for 2005 flows, we obtain for the discounted values

$$D_{CY}^{(1)} \cdot S_{CY} = V_0^{(0)} \alpha_0 S_{CY} + V_1^{(0)} \alpha_1 S_{CY} + V_2^{(0)} \alpha_2 S_{CY} + \ldots = \left(\sum_{i=0} V_i^{(0)} \cdot \alpha_i \right) S_{CY} \qquad (16)$$

and

$$D_{PY}^{(1)} \cdot C \cdot r_{PY}^{(0)} = V_0^{(0)} \beta_0 C r_{PY}^{(0)} + V_1^{(0)} \beta_1 C r_{PY}^{(0)} + \ldots = \left(\sum_{i=0} V_i^{(1)} \cdot \beta_i \right) C r_{PY}^{(0)}, \qquad (17)$$

$C \cdot r_{PY}^{(0)}$ is the best estimate at $t_1 =$ December 31 of the future cash flows for PY claims. Due to the increase in information during 2005, C will deviate from its expected value $E[C] = 1$. If an insurer thinks at t_1 that the provisions can be reduced then chooses a $C < 1$ and obtains a profit on the loss reserves. Note that in $D_{CY}^{(1)} \cdot S_{CY}$ and $D_{PY}^{(1)} \cdot C r_{PY}^{(0)}$, there is not only a insurance risk (S_{CY} and $C r_{PY}^{(0)}$) but also an interest rate risk ($D_{CY}^{(1)}$, $D_{PY}^{(1)}$).

3.2.3 Change in Risk Bearing Capital

Putting the assumptions above into the term $RBC_{31.12.05}/(1+r_1^{(0)}) - rbc_{1.1.05}$ and applying two approximation yields

$$\begin{aligned}
& \frac{RBC_{31.12.05}}{1+r_1^{(0)}} - rbc_{1.1.05} \approx \frac{E[R_I] - r_1^{(0)}}{1+r_1^{(0)}} \cdot (a(0) + (p - upr) - k) \\
+ \quad & \frac{R_I - E[R_I]}{1+r_1^{(0)}} \cdot (a(0) + (p - upr) - k) - \frac{D_{CY}^{(1)} - E[D_{CY}^{(1)}]}{1+r_1^{(0)}} \cdot E[S_{CY}] - \frac{D_{PY}^{(1)} - E[D_{PY}^{(1)}]}{1+r_1^{(0)}} \cdot r_{PY}^{(0)} \\
+ \quad & (p - k) - d_{CY}^{(0)} \cdot E[S_{CY}] \\
- \quad & d_{CY}^{(0)} \cdot (S_{CY} - E[S_{CY}]) - d_{PY}^{(0)} \cdot (C - 1) \cdot r_{PY}^{(0)} \qquad (18)
\end{aligned}$$

The first line on the right hand side is the expected investment performance above the one year risk free rate.

The second line refers to the financial and ALM risk. The first term is the difference between the stochastic investment performance R_I and its expectation. The second and the third term are the interest rate risk contribution from the insurance liabilities.

On the next line we have the expected technical result on a discounted basis. The last line then refers to the insurance risk on the new claims ($S_{CY} - E[S_{CY}]$) and on the change in claims provisions ($(C-1)r_{PY}^{(0)}$). Note that the expected values of the risk terms for market risks and insurance risks are zero by construction.

Above we mentioned two approximations. The first one refers to the fact that insurance liabilities are discounted. Therefore the risk in a liability consist of the interest rate risk in the discount factors ($D_{PY}^{(1)}$, $D_{CY}^{(1)}$) and the risk in the nominal value of the liability (S_{CY},$C \cdot r_{PY}^{(0)}$). The assumption made is that the products of two random variables like $D_{CY}^{(1)} \cdot S_{CY}$ can be replaced by their first order Taylor approximations

$$D_{CY}^{(1)} \cdot S_{CY} \approx E[D_{CY}^{(1)}]E[S_{CY}] + E[D_{CY}^{(1)}](S_{CY} - E[S_{CY}]) + (D_{CY}^{(1)} - E[D_{CY}^{(1)}])E[S_{CY}] \quad (19)$$

in case of the current year risk. Applying the same approximation for the reserving risk term yields

$$\begin{aligned} D_{PY}^{(1)} \cdot C &\approx E[D_{PY}^{(1)}]E[C] + E[D_{PY}^{(1)}](C - E[C]) + (D_{PY}^{(1)} - E[D_{PY}^{(1)}])E[C] \\ &= D_{PY}^{(1)} + E[D_{PY}^{(1)}](C-1), \end{aligned} \quad (20)$$

where we have used that provisions are best estimate provisions ($E[C] = 1$).

The second approximation applied in equation (18) is the assumption that

$$\frac{D_{PY/CY}^{(1)}}{1 + r_1^{(0)}} \approx d_{PY/CY}^{(0)}. \quad (21)$$

This means that discounting to time t_1 with the yield curve at t_1 and then discounting to time t_0 using the discount factor at t_0 can be replaced by directly discounting to time t_0 with the yield curve at t_0. Note that this assumption is only applied in the insurance risk terms.

The standard model of the SST provides the distribution of the change in the risk bearing capital. In what follows, we give a overview over the market and ALM risk model as well as the insurance risk model.

4 Stochastic Model for Market Risks

The market and ALM risk model aims to model the second line on the right hand side of equation (18). We observe firstly that on this line, the investment performance R_I and the discount factors are the only stochastic variables. Secondly, the value of that line is zero in expectation.

4.1 Modeling the Asset Term

We describe how to obtain the distribution of the first term

$$\frac{R_I - E[R_I]}{1 + r_1^{(0)}} \cdot (a(0) + (p - upr) - k)\,. \tag{22}$$

To simplify the notation, we abbreviate $(a(0) + (p - upr) - k)$ by the symbol a in this section. Thus, in essence, our aim is to model

$$(R_I - E[R_I]) \cdot a. \tag{23}$$

First we write the value at the end of the year as some function

$$R_I \cdot a = g(R_1, R_2, \ldots R_n). \tag{24}$$

of a set of n underlying stochastic market risk factors $R_1, \ldots R_n$. These risk factors are interest rates, equity indices, real estate indices, foreign exchange rates, etc. In total, the field test 2005 uses $n = 74$.

We linearise the dependence of $R_I \cdot a$ on the risk factors R_j by replacing the function g by its first order Taylor approximation

$$R_I \cdot a \approx g(E[R_1], \ldots E[R_n]) + \sum_{j=1}^{n} k_j^A \cdot (R_j - E[R_j]). \tag{25}$$

where

$$k_j^A = \left.\frac{\partial g}{\partial R_j}\right|_{R_j = E[R_j]} \qquad j = 1, \ldots, n \tag{26}$$

are the n partial derivatives in the point of expectation.

If the linearisation above is not too harmful, we can also approximate the expected value by the the function g evaluated at the point of the expected risk factors:

$$E[R_I] \cdot a \approx g(E[R_1], E[R_1], \ldots, E[R_n]). \tag{27}$$

Combining this with equation (25) yields for expression (23)

$$(R_I - E[R_I]) \cdot a \approx \sum_{j=1}^{n} k_j^A \cdot (R_j - E[R_j]) \tag{28}$$

Hence it is sufficient to determine the partial derivatives k_j^A and to model the distribution of the risk factors R_j. The SST model uses the normal approach

$$\begin{pmatrix} R_1 - E[R_1] \\ \vdots \\ R_n - E[R_n] \end{pmatrix} \sim N(0, \mathbf{\Sigma}), \tag{29}$$

where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the risk factor changes. It is determined and given annually by the regulator. The normal approach implies that $(R_I - E[R_I]) \cdot a$ is normally distributed with some variance $\sigma^2_{R_I}$ and expectation 0.

The individual part for the insurer is given by the partial derivatives. They can be obtained approximately using difference quotients

$$k_j^A \approx \frac{g(.., R_j + \Delta R_j, ..) - g(.., R_j, ..)}{\Delta R_j} = \frac{1}{\Delta R_j}\left[(R_I \cdot a)\big|_{E[R_j]+\Delta R_j} - (R_I \cdot a)\big|_{E[R_j]}\right]. \quad (30)$$

Hence, what remains to be done for the insurer is to determine for each risk factor R_j by how much the value of the assets change if the risk factor is shocked by ΔR_j.

Once the k_j^A have been determined, the variance of $(R_I - E[R_I]) \cdot a$ can be determined.

4.2 Modeling the Liability Term

The model for the dependence of liability

$$-(D_{CY}^{(1)} - E[D_{CY}^{(1)}]) \cdot E[S_{CY}] - (D_{PY}^{(1)} - E[D_{PY}^{(1)}]) \cdot r_{PY}^{(0)} \quad (31)$$

on the interest rates is identical to the method we considered in the former section 4.1 about the asset risks.

The underlying market risk factors are the same as discussed before, however, only the interest rates are important. Instead of determining the partial derivatives of the asset values, the partial derivatives k_j^L of the discounted liabilities with respect to the risk factors have to be evaluated. Thus, analogously to equation (28), we obtain a linear function in the risk factors

$$(D_{CY}^{(1)} - E[D_{CY}^{(1)}]) \cdot E[S_{CY}] + (D_{PY}^{(1)} - E[D_{PY}^{(1)}]) \cdot r_{PY}^{(0)} \approx \sum_{j=1}^{n} k_j^L \cdot (R_j - E[R_j]) \quad (32)$$

for the liabilities (31).

4.3 The Asset-Liability Model

The former two sections might suggest that assets and liabilities are modeled separately. However, in the economic view, it is unimportant how the assets alone or how the liabilities alone move. It is only sensible to consider their changes simultaneously.

Putting the representations (28) and (32) into equation (18) provides the model for the second line, the asset-liability risk:

$$\sum_{j=1}^{n}(k_j^A - k_j^L) \cdot (R_j - E[R_j]), \quad (33)$$

where the distribution of $(R_j - E[R_j])$ is given by equation (29).

This model is very similar to the well established risk metrics approach. Its simplicity is a advantage, but there are two major drawbacks. Firstly, it is too optimistic to model the market using normal distributions, and secondly, the model is linear in the risk factors. This implies that often it is impossible to take financial hedging into account.

5 Stochastic Model for Insurance Risks

The insurance risk model provides a distribution for the incurred loss (S_{CY}) on the claims occurring in the interval $CY = [t_0, t_1) = [1.1.05, 31.12.05)$ and the change in provisions $(C-1)r_{PY}^{(0)}$ for the claims which have occurred in $PY = (-\infty, t_0)$.

5.1 PY Risk

If a claim has occurred and there are still payments to be made to the policyholder, it is usual in non-life insurance to set back a provision for these payments. In most circumstances the future payments are not known at present. They may vary in number and amount. Others, such as annuities, are well known in advance, however, in total, there is uncertainty about future payments.

In principle there are three types of policies how to define the amount of provision. The first type is to be prudent in the sense that the insurer tries to estimate a value which is higher than the sum of future payments with high probability. The second type is located at the other end of the spectrum, setting the provisions very optimistic. This leads to a systematic under-reserving. The third type is the one favoured by SST: provisions are defined as the estimator for the expected value of the sum of future payments. Such a provision is called "best estimate provision". It is important to stress that the parameters of the probability distribution and in particular the expected value of future payments is unknown. Therefore we can only operate with estimators for the parameters.

Due to the increase of information and due to changes in external boundary conditions (e.g. changes in the legal environment) the value of the estimator for future payments changes over time. In general, there will be a difference between the provision $r_{PY}^{(0)}$ at t_0 and the sum of the payment at t_1 and the remaining provision. This sum is denoted by $Cr_{PY}^{(0)}$. Hence, $(1-C)r_{PY}^{(0)}$ is what is called the run-off profit.

Since the provision $r_{PY}^{(0)}$ at t_0 is an estimator for the expected future payments, it is a deterministic value. From the point of view at time t_0, the correction factor C is a random variable expressing the possible need at t_1 for changing the provisions. Two types of risk are covered in the SST run-off risk model. First there is a risk because of the fluctuation around the expected value. The second risk is the possible differences between the estimator and the expected value. While it is reasonable to assume that the expected value of the sum of future payments is a martingale, the estimator for the expected value is not.

The first type of risk is captured by estimating the variance of the distribution from historic run-off results based on best-estimate provisions. The question how to quantify the second type of risk in the framework of SST is not fully answered, yet. At the current stage, it is proposed to use an approach described by Mack (Mack, 1997).

5.2 CY Risk: Risk of Small Claims

The risk of claims occurring in 2005 is fundamentally different from the run-off risk of PY claims. Nature has already realised the latter, so as discussed in the previous section, the risk lies in the estimation about the proper provision.

The risk in the CY claims can best be shown by writing the annual incurred loss as the well known stochastic sum

$$S_{CY} = \sum_{j=1}^{N} Y_j \tag{34}$$

over individual loss events. S_{CY} is stochastic due to the randomness of the number of events and of the claims amount of the individual event.

In principle, one could consider to obtain the distribution of S_{CY} directly (i) with some normal approximation by applying the law of large numbers, or (ii) with the Panjer algorithm. However, in practice, both methods fail at this stage. In non-life insurance the Y_j are random over many orders of magnitude, their distribution tends to be heavy tailed. Therefore, S_{CY} is far from being normally distributed. On the other hand the numerics of the Panjer algorithm breaks down because the number of losses is huge (e.g. 10^6).

To overcome these obstacles in practice, S_{CY} is split up into a sum of small losses (SL) and a sum of large losses (LL):

$$S_{CY} = S_{CY}^{SL} + S_{CY}^{LL}. \tag{35}$$

The threshold between small claims and large claims was chosen to be 1 MCHF or 5 MCHF. Each insurer can select its preferred value.

The decomposition is such that firstly the number of large losses is small, thus the distribution of their sum can be treated using the Panjer algorithm. Secondly, the number of small losses is still large, but their variability is bounded. This allows to treat their sum using light-tailed distribution such as a Γ or a Log-normal. This assumption it is supported by observation and by limit theorems applicable under certain weak constraints. The next section 5.3 will outline the model for large claims. Here we focus on the modeling of the sum of small claims.

Again the sum of small claims can be written as

$$S_{CY}^{SL} = \sum_{j=1}^{N} Y_j^{SL} \tag{36}$$

In general, we cannot assume mutual independence between individual claim amounts Y_j^{SL}. The reason is that there is an underlying risk characteristics Θ. For instance, during a cold winter we expect systematically different claims in motor insurance than during an average winter. Additionally, also the distribution of the number of claims is different: in the cold winter the expected number is larger than on average. Taking the dependence on an underlying risk characteristics into account is one explanation why we often observe over-dispersion in the number of claims ($Var[N] > E[N]$).

The SST models the S_{CY}^{SL} using a Gamma distribution or a Log-normal distribution. Both of these are fixed by the first two moments. Using the relation

$$Var\left(S_{CY}^{SL}\right) = Var\left(E[S_{CY}^{SL}|\Theta]\right) + E\left[Var(S_{CY}^{SL}|\Theta)\right], \tag{37}$$

we observe that the risk in S_{CY}^{SL} consists of two contributions. They are called "parameter risk" and "stochastic risk". The first term refers to the variability of the expected value over different years or different risk characteristics. The second term is an average of the stochastic variance over a set of years. Both types of variability are shown in figure 2.

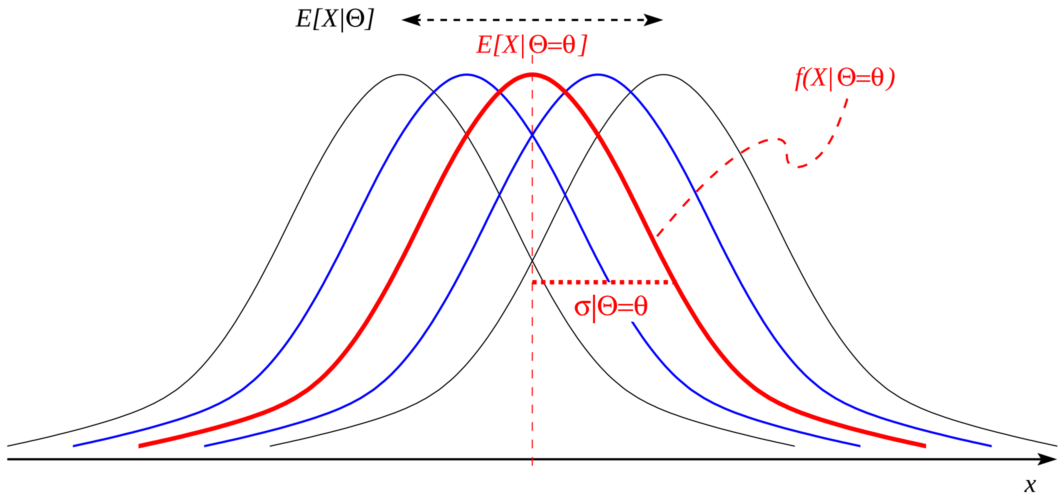

Figure 2: Uncertainty of X owing to stochastic variability around the expected value and due to uncertainty in the expected value. For a given risk characteristic $\Theta = \theta$, the density of X is given by the red curve (stochastic risk). However, the density itself is variable (parameter risk), shown here as the uncertainty of $E[X|\Theta]$ (black arrow).

Instead of the variance of a variable X, we consider the coefficient of variation which is defined as $V(X) = \sqrt{Var(X)}/E[X]$. Equation (37) becomes

$$V^2\left(S_{CY}^{SL}\right) = V_P^2\left(S_{CY}^{SL}\right) + V_S^2\left(S_{CY}^{SL}\right), \tag{38}$$

where V_P and V_S are the coefficients of variation due to parameter risk and stochastic risk, respectively. The important point about the parameter risk is that it cannot be reduced by volume diversification. This means that a small insurer is exposed to parameter risk in about the same percentage as a large insurer. Therefore it is reasonable to derive default values for V_P being valid for the whole Swiss market. This has been done for the lines of business considered in the SST.

To derive the stochastic risk contribution, we assume that given the risk characteristic Θ, the usual model assumption hold:

$$\begin{aligned} N|\Theta &\sim \text{Poisson}(\lambda(\Theta)), \\ Y_j^{SL}|\Theta &\sim F(\mu_Y(\Theta), \sigma_Y(\Theta)) \qquad iid., \end{aligned} \tag{39}$$

(for some distribution function F), and that $N|\Theta$ is independent from $Y_j^{SL}|\Theta$. Combining these assumptions with the well known formula

$$Var\left(S_{CY}^{SL}|\Theta\right) = Var(N|\Theta) \cdot (\mu_Y(\Theta))^2 + E[N|\Theta] \cdot (\sigma_Y(\Theta))^2 \tag{40}$$

for the variance of a stochastic sum, we get

$$V_S^2\left(S_{CY}^{SL}\right) = \frac{1}{E[N]} \cdot \left([V_S(Y_j^{SL})]^2 + 1\right). \tag{41}$$

This expression has a number of practical advantages. Firstly, it is reasonable to assume that if one LoB is considered, the coefficient of variations ($V_S(Y_j^{SL})$) of the individual small events is similar from insurer to insurer. Hence, a default value can be derived from large insurance portfolios and provided to insurers with a small portfolio. Secondly, it is sufficient then to estimate the expected number of claims to evaluate the right hand side.

The result of expression (41) for the stochastic risk together with the default values for the parameter risk can be put into formula (38) to obtain the total variance of S_{CY}^{SL}. This is done for each LoB separately. Using a correlation matrix between the LoB, we get the total variance of the annual incurred loss of the small claims.

5.3 CY Risk: Risk of Large Claims

We start the section with a specification of what is considered to be a "large loss" in the realm of the SST: a large loss can be (i) a large single claim or (ii) the sum of more than one claim which have been caused by the same event, if this sum is larger than the threshold for large claims (1 MCHF or 5 MCHF, respectively). Such a sum is called "event loss" or "cumul claim". Examples are a large liability claim in case (i) and the sum of thousands of claims owing to a flood in case (ii). Note that in the second case the single claims are mostly far below the large claim threshold.

5.3.1 Single large claims

The possibility of single large claims have been identified in the lines of business of motor liability, property without elementary damage, general liability, health, transport, and financial loss.

The annual incurred loss is modeled using a compound Poisson distribution with a Pareto distribution for the single claim amount. Default values for the Pareto parameters have been derived from the experience of large market participants. Cutting-off the Pareto distribution is possible, and W-XL or Cat-XL reinsurance cover can be easily taken into account.

5.3.2 Event losses

If an event loss occurs, it is often the whole insurance market that is concerned by the loss. The individual insurer participates in the market loss according to its own market share.

This observation has an important implication for the modeling within the SST. It means that the number of event losses is the same for all insurers having business in the corresponding LoB. Additionally, it is possible to simulate a market loss which is then broken down to each insurer by multiplying the realisations by the market shares.

Similar to the model for the single large claims, the annual market losses are modelled by a compound Poisson distribution with a Pareto distribution for the event loss amount. The individuality of the insurer comes in only by its market share.

Areas with this type of behaviour are elementary damage, accident and motor hull due to hail storms. Beside the motor hull damages a hail storm can lead to severe property damages. Most of the latter are taken by state-run building insurers, which are not under supervision. Therefore, property hail losses are of such low importance for the private insurers that, in the SST, they can be treated in the motor hail model.

Modeling elementary damages is very similar to the hail loss model. However, there exists a loss limitation at 500 MCHF per event. It is the policy holder who carries the risk of a event loss exceeding that limit. All elementary damages are then pooled in the elementary damage pool, which is protected by an annual stop loss treaty by an external reinsurer. The individual insurer has to carry its pool share of the loss net of the stop loss. The event loss limit as well as the stop loss treaty are taken into account by the SST model.

6 The Use of Scenarios

Beside the stochastic models presented in section 4 and 5, the SST bases on the evaluation of a small set of scenarios. The purpose of the scenarios is to consider risks which are not contained in the stochastic model and to overcome the optimism in some parts of the model. For instance, modeling market and ALM-risk using a normal distribution does probably not reflect sufficiently the large events on the market.

The scenarios are selected in order to be roughly 100 year events for the risks that are not covered in the stochastic model. Examples for scenarios are the breakout of a pandemic, the collapse of a water barrage, market shocks, etc.

In what follows we describe how the results from the scenario evaluation can be combined with the distribution resulting from the stochastic modeling. The aim is to obtain an overall distribution function which includes the risks represented by the scenarios.

We denote the scenarios with S_j, $j = 1, \ldots, n$, and S_0 is defined as the set of states of the world where no scenario occurs. The scenarios are selected such that their annual probability is very small ($p_j := P[S_j] \ll 1, j = 1, ..., n$). In reality, the probability of two scenarios occurring in the same year is larger than zero, however, we neglect it for reasons of simplicity ($P[S_j \cap S_j] = 0, i \neq j$). If this is assumed, the scenarios together with S_0 form a partition of the probability space, and

$$p_0 := P[S_0] = 1 - \sum_{j=1}^{n} P[S_j]. \qquad (42)$$

The first step in the scenario evaluation by the insurer is to determine the change c_j in RBC caused by scenario S_j for $j = 1, ..., n$. Then we assume that in a year when the scenario event takes place, the distribution of the change in RBC is given by

$$F(x|S_j) = F(x - c_j|S_0), \tag{43}$$

where $F(x|S_0)$ is the result out of the stochastic model

$$F(x|S_0) := P\left[\frac{RBC_{31.12.05}}{1 + r_1^{(0)}} - RBC_{1.1.05} < x \middle| S_0\right], \tag{44}$$

the distribution function of the change in RBC given that no scenario takes place.

The assumption (43) is based on the idea, that the occurrence of the scenario event does not change the probability of the remaining randomness working in the world. Obviously this is an approximation which does not hold in all situations. For instance one might think that large insurance losses have an impact on the capital markets. This is true on the short term, but it is argued that already on a time horizon of one year the dependency is strongly damped.

Given the distributions (43) we can easily obtain the unconditional distribution by mixing the scenarios

$$F(x) := P\left[\frac{RBC_{31.12.05}}{1 + r_1^{(0)}} - RBC_{1.1.05} < x\right] = \sum_{j=0}^{n} p_j \cdot F(x|S_j). \tag{45}$$

From this distribution the expected shortfall at the level α is determined.

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