

# Modeling assets and liabilities of a Finnish pension company: a VEqC approach

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## Abstract

This paper develops a stochastic model for assets and liabilities of a Finnish pension company. The assets and liabilities are expressed in terms of seven economic factors from Finland and EU-area. The development of these factors is modeled with a Vector Equilibrium Correction model, that incorporates statistical information with expert views in the form of user specified growth rates and long term equilibria. The forecast performance of the resulting model is tested and the model is used in long-term solvency and asset liability simulations.

Key words: time series analysis, VAR, Vector equilibrium correction model, asset liability management.

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# 1 Introduction

Pension insurance companies manage enormous investment funds. Their main goal is to invest policyholders' pension insurance premiums safely and profitably in order to meet their liabilities in the future. The duration of the liabilities is usually very long (over 20 years), which calls for realistic models for long-term scenarios of investment returns and liability flows. Such models form the basis for pension companies' asset and liability management (ALM). Typically, these models have a macroeconomic flavor in that they try to describe the development of larger investment classes like interest rates and broad equity indices along with wage indices and inflation.

To a large extent, the existing models for pension companies have been based on the well known Wilkie's stochastic investment model; see e.g. Wilkie (1986, 1995), Yakoubov et al. (1999) and Ranne (1998). The drawback of these models is their cascade structure, which allows the modeling of one-way causalities only. Vector autoregression (VAR) and their generalizations, vector equilibrium correction (VEqC) models, do not have this limitation; see for example Sims (1980), Engle and Granger (1987), Johansen (1995) and Clements and Hendry (1999). VAR-models for asset liability management have been used for example in Dert (1998), Wright (1998) and Harris (1999), but to our knowledge, only Boender et al. (1998) have proposed using a VEqC model for these purposes.

This paper develops a model for assets and liabilities of a Finnish pension company. The stochastic variables in the model are expressed in terms of seven economic factors whose development is modeled with a linear time series model in VEqC form. The model incorporates statistical information with expert views and it has been successfully implemented in a case where the available data is scarce and subject to changing economic conditions. The expert views are given in the form of drift parameters (for such quantities as interest rates, equity and property prices), and certain long-run equilibrium relations (e.g. average levels of interest rate spread and dividend yield). This is especially important in situations where the available data displays characteristics that are believed to change in the future (e.g. declining interest rates in EU-area during the 90's). Indeed, according to Hendry and Doornik (1997), deterministic factors like drift parameters and equilibrium values matter most for predictive failure.

We will consider assets in the level of larger investment classes, namely, cash, bonds, equities, property and loans to policyholders. We treat the cash-flow and the change-in-value components of total asset returns separately. This is essential in the presence of significant transaction costs. Also, this enables the modeling of such terms as dividend yield (dividend-price ratio) that has been shown to have predictive power in describing future changes of dividends; see Campbell and Shiller (1988). On the liability side, we model the cash-flows of a pension company and some technical quantities that have an important role in the Finnish statutory earnings-related pension scheme. In general, these terms depend on the development of salaries of the pensioners as well as population dynamics. These two factors are assumed independent, so that their development can be modeled separately. In this paper, we concentrate on modeling salaries and their co-movement with asset returns.

All the stochastic parameters in our model will be expressed in terms of the following seven factors: short-term interest rate, bond yield, stock price index, dividend yield, property price index, rental yield and wage index. These (or more precisely, simple transformations of them) will, in turn, be modeled by a structured VEqC model that allows for simple economic interpretations. The data for estimation is taken from Finland and the EU-area that are of greatest interest to Finnish pension companies.

The rest of this paper is organized as follows. In the next section, we describe the asset classes and liabilities and show how their development can be described with the above seven factors. In

Section 3, we describe the data and study its stationarity properties. We start the development of our time series model in Section 4, where we build a VAR model for differences. The purpose there is to show how to specify the average drift in the model and to demonstrate the importance of correct specifications through simulations. In Section 5, we augment the VAR model with equilibrium correction terms thus obtaining our complete model in VEqC form. Section 6 compares the forecast performance of the developed VEqC-model with three rival models in out-of-estimation-sample forecast experiment. In Section 7, we examine by simulation the statistical properties of the long-term asset returns produced by the model and calculate the company's long-term cash-flows and reserves. In Section 8 the developed model is used in an ALM framework. We compare different dynamic portfolio allocation strategies and evaluate the company's long-term solvency and bankruptcy risks. Concluding remarks are presented in Section 9.

## 2 Assets and liabilities in terms of economic time series

Finnish pension companies update their investment and bonus strategies on a yearly basis. These decisions are linked with the company's liabilities which extend far into the future. On one hand, this link comes from the company's goal of meeting the liabilities, and on the other, from the legislation that aims at regulating the solvency risks of the company. This section describes the main investment classes, the liabilities, and other quantities that are of interest in the strategic financial planning of a Finnish pension company. Our aim is to express all these quantities in terms of seven economic factors that will then be modeled with a time series model.

### 2.1 Assets

A pension company's assets can be divided into five main investment classes: cash, long-term bonds, stocks, property and loans. Our goal is to model the return per wealth invested in each asset class. The total returns on the assets are split between cash income and change in value components, which, in general, require separate treatment due to transaction costs etc.

**Cash:** Pension companies keep a proportion of their assets in cash (short-term deposits) to ensure a reasonable level of liquid financial resources. Because of the short term nature of these investments, the change in value can be ignored and the whole return can be modeled as cash income. The return on cash investments can be well approximated by the 3-month Euribor.

**Bonds:** Currently, about one half of the Finnish pension companies' wealth is invested in long-term bonds. The primary source of income on bond investments are the coupon payments, which is cash income. Usually, newly issued bonds sell at par, which implies that coupon payments equal the current yield. We approximate the cash flow component of the whole bond portfolio by the bond yield which is denoted by  $br_t$ .

According to (Campbell et al., 1997, page 408) the price of a bond can be approximated by

$$\ln B_t \approx D[k + (1 - \rho)c - \ln(1 + br_t)], \quad (1)$$

where  $D$  and  $c$  are the duration and coupon payments, respectively, of the portfolio, and  $k$  and  $\rho$  are constants. When the bond is selling at par, we have  $\rho = 1/(1 + br_t) \approx 1$ ; see (Campbell et al., 1997, page 407). If we assume in addition that the bond portfolio is updated so that its duration is fairly constant (which is not far from reality in Finnish pension companies), we get

$$\ln B_t - \ln B_{t-1} \approx -D \ln \frac{1 + br_t}{1 + br_{t-1}}.$$

Based on this, we approximate the value change of the bond portfolio by

$$\frac{B_t}{B_{t-1}} \approx \left( \frac{1 + br_{t-1}}{1 + br_t} \right)^D$$

The duration will be set equal to the duration of the bond portfolio of a company being modeled. We will use the yield on German government benchmark bond, whose duration is close to  $D$ . In our calculations we will use  $D \approx 5$  years.

**Stocks:** The riskiest but historically the most profitable long-term investment class is stocks. In stocks, the majority of the total return comes from the change in value and the dividend payments constitute the cash income component. We will model the change in value component with a stock price index and the cash income with the corresponding dividend yield.

Finnish pension companies invest in stocks mainly in Finland and the rest of the European Union (EU) area. The development of the value of the company's stock portfolio is modeled with a "fixed mix" stock price index  $S$  which gives the value of a portfolio that is sequentially rebalanced to have a fraction  $\theta^F$  of stock investments in Finland and  $\theta^E$  in the EU area. The quarterly change in the fixed mix index is

$$\frac{S_t}{S_{t-1}} = \theta^F \frac{S_t^F}{S_{t-1}^F} + \theta^E \frac{S_t^E}{S_{t-1}^E}$$

where  $S^F$  and  $S^E$  are stock indices in Finland and EU, respectively. For  $S^E$ , we use the Datastream Europe market index, and for  $S^F$  the Helsinki Stock Exchange (HEX) portfolio price index.

The annual dividend yield corresponding to the fixed mix stock portfolio is calculated as a weighted average of dividends from Finland and EU as

$$Y_t^S = \theta^F Y_t^F + \theta^E Y_t^E.$$

The Finnish dividend yield  $Y^F$  is based on the HEX portfolio price index and the European yield  $Y^E$  is based on the Datastream Europe market index.

In this study, we assume that  $\theta^F = \theta^E = \frac{1}{2}$ , which has the interpretation that a company's stock portfolio is split evenly between Finland and the rest of the EU.

**Property:** As an investment class, property resembles stocks in many ways. The return on property investments consists of potentially large price fluctuations and fairly stable cash income. Therefore, the return components on property investments are modeled similarly to stocks: the change in value component is modeled with a property price index and the cash income is modeled with a rental yield. The main difference from stocks is that the cash income component forms the majority of the total gross return on property investments.

Finnish pension companies invest in property mainly domestically. Although the companies mostly invest in commercial property, we will use the Finnish residential property price index to model the price fluctuations of all property investments, This is because the commercial property market is rather illiquid and the property prices are hard to estimate accurately.

The cash income component will be modeled as the difference between the residential rental yield  $Y_t^P$  and the maintenance costs. The rental yield gives the rent paid per wealth invested and the maintenance costs are assumed to be a constant 3% of the property value. The property price index  $P_t$  (€ / $m^2$ ) and the corresponding rental index  $R_t$  (€ / $m^2$ ) are available from Statistics Finland. The rental yield is given by

$$Y_t^P = \frac{R_t}{P_t}. \tag{2}$$

**Loans:** The Finnish pension companies invest part of their funds by giving loans to policyholders. In the past, these formed a great majority of all investments, but currently they account for about 10 %. There are two kinds of loans, premium loans and investment loans. The premium loans are an arrangement where a customer can borrow back part of the paid premium according to fixed rules. For the investment loans, the terms are agreed freely between the company and the borrower. In the model, the two kinds of loans are combined to form one investment class. The change in value component for loans is zero. The cash income component will be approximated by a moving average of bond yield. This is based on the fact that the interest on newly given loans is usually set equal to current bond yield.

## 2.2 Liabilities

The Finnish pension insurance companies are responsible for managing the statutory earnings-related pension scheme. Because the system is statutory, many of its characteristics are common to all the companies. For example, the amount of the pension is determined by fixed defined-benefit rules independent on the company where the person is insured. Also the contribution rates and the technical reserves are calculated according to common formulas confirmed by the Ministry of Social Affairs and Health. The technical interest rate for the reserves is also common to all the companies and its value for each year is confirmed by the Ministry.

The pension companies are, however, able to choose their own investment policy. Depending on the investment returns, the company can give bonuses to its customers. These bonuses are paid as reductions of the contributions, and they are the most important element in the competition between the companies. The planning of the investment strategy is therefore essential for the success of the company. For this, the company must take into account the nature of its liabilities as well as its solvency position.

**Reserves:** The Finnish statutory earnings-related pension scheme is partly funded. Only part of the total amounts of old age, disability and unemployment pensions are funded, and the part time and survivors' pensions are not funded at all. As a whole, about 25% of the total pension expenditure is paid by the funded part. The rest is financed as a pay-as-you-go system.

In the model, the average amounts of the funded pensions are calculated by age and sex. The technical reserves before increases at the end of the year (see below) are

$$L = \sum_{x,s} \mathcal{V}(s,x) \mathcal{E}(s,x) a(s,x),$$

where summation is by sex  $s$  and age  $x$ ,  $\mathcal{V}(s,x)$  is the number of pensioners or active workers,  $\mathcal{E}(s,x)$  is the yearly average of the funded pension and  $a(s,x)$  is the actuarial present value (APV) function depending on mortality and a discount rate of 3%. The definition of the APV function is different for the old age, disability and unemployment pensions and it is given in the actuarial basis confirmed by the Ministry of Social Affairs and Health.

Because the funded pension is based on the salaries of the insured persons, the reserves in the model are dependent on the development of the wage index. The reserves are also dependent on the technical interest rate in a way explained below. Otherwise, the reserves in the model are deterministic.

Besides the old age, disability and unemployment pension reserves, the model contains some other special reserves:

- an equalization reserve for buffering the yearly surplus/deficit of the insurance business
- a clearing reserve for the pay-as-you-go part of the pension expenditure
- a bonus reserve for the bonuses paid to customers as reductions of pension contributions.

**Technical interest rate:** Besides the discount rate of 3% used for calculating the actuarial present value functions, there is a higher rate, the technical interest rate whose value varies yearly. The amount of the reserves is increased at the end of each year corresponding to the difference between the technical interest rate and the 3% discount rate. This means that the technical interest rate determines the actual total interest rate for the reserves, and the 3% discount rate is its minimum value.

The technical interest rate ( $r^{tech}$ ) is calculated by a formula dependent on the average solvency level of all the pension companies and funds. In the model, this formula cannot be used because the solvency level is one of the company's decision variables. Besides, the model contains only the company's own solvency and not its level in the whole TEL pension scheme. For these reasons, an approximation is used based on the investment variables, since these are correlated with the general solvency level of the system. The formula used in the model is

$$r_t^{tech} = \max\{3\%, \gamma_0 + \gamma_1 br_t + \gamma_2(\ln S_t - \overline{\ln S_t}) + \gamma_3(\ln P_t - \overline{\ln P_t})\},$$

where  $br_t$  denotes the bond yield,  $\ln S_t$  the logarithm of the stock price index and  $\ln P_t$  the logarithm of the property price index at time  $t$ . The  $\overline{\ln S_t}$  and  $\overline{\ln P_t}$  are the expected values of the variables  $\ln S_t$  and  $\ln P_t$  for year  $t$  calculated using average growth rates, defined in section 4. The formula was found using a separate, more detailed simulation model where the actual formula for the technical interest rate could be calculated. The approximation formula was fitted to the results of this model. The estimated parameters  $\gamma_i$ ,  $i = 0, \dots, 3$  are all positive suggesting, that the technical interest rate follows the long term bond yield and increases when the stock and property prices are above their expected values. The technical interest rate plays a crucial part in the model because, to a great extent, it determines the correlations between the investment variables and the reserves.

**Cash flows:** Besides the investment yields, money flows in the company as paid contributions and out as pension expenditure. The pension expenditure is calculated depending on the number of pensioners and the average funded pensions. The contributions depend on the total salaries of the insured persons. The contribution rates, which vary by age and sex, are confirmed by the Ministry of Social Affairs and Health. The combined cash flow depends on the development of the wage index. The Finnish wage index from Statistics Finland is used to represent the average wage development.

**Solvency capital:** The solvency capital is the amount by which the total assets of the company exceed the sum of its reserves. It functions as a buffer against the variation of the investment results. Because the reserves of a pension insurance company must always be fully covered by its assets, a non-positive solvency capital means a bankruptcy. Therefore, the development of the solvency capital is an important factor in policy evaluations.

The legislation prescribes various minimum and target levels for the solvency capital of a pension company. The basic quantity is the solvency border, which depends on the structure of the company's investment portfolio. The lower border of the target zone is twice the amount of the solvency border. The position of the solvency capital relative to these levels is an indicator of the solvency risk of the company.

## 3 Time series data

### 3.1 Historical data

As described above, the assets and liabilities of a Finnish pension company can be approximately expressed in terms of the following seven economic factors

1. Three month Euribor  $sr$ ;
2. Five year German government bond  $br$ ;
3. Fixed mix stock index  $S$ ;
4. Fixed mix dividend yield  $Y^S$ ;
5. Property price index  $P$ ;
6. Rental yield  $Y^P$ ;
7. Wage index  $W$ .

Our data set will consist of quarterly observations of these factors between 1991/1 - 2001/4. We have chosen such a short period because of the capital movement liberalization in the EU area during 1990, which resulted in significant changes in economic conditions. The data prior to 1991 corresponds to a more regulated economy.

Three month Euribor has been quoted only since the beginning of 1999. We extend this series backwards by using the German 3 month interest rate which is available from Datastream. Rental index  $R_t$ , used in (2) is reported only once a year. Therefore linear interpolation for  $\ln R_t$  is used to fill in the gaps in the time series. For the wage index we use a seasonally adjusted series in the model. We thus obtain an approximation of the full set of quarterly data for all the seven factors between 1991/1 - 2001/4. We take this as a description of the statistical parameters in our asset and liability model. The data is displayed in Figure 1.

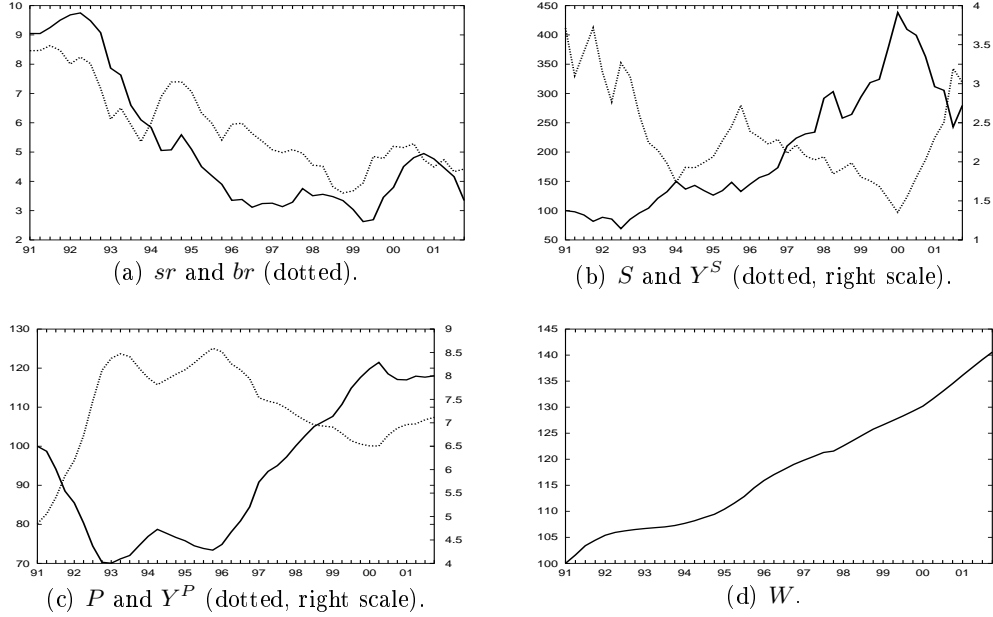


Figure 1: Time series.

### 3.2 Data transformations

The variations in dividend yield are roughly inversely proportional to the variations in the stock price index; see Figure 1. This is due to the fact that the dividend yield is, by definition, the dividend obtained per wealth invested in stocks where the latter follows the stock price index. Such multiplicative effects are not well modeled by the linear time series models that we are about to build. We will thus transform the dividend yield into the *dividend index*

$$D_t = S_t Y_t^S.$$

Similarly, instead of modeling the rental yield directly, we model the *rental index*

$$R_t = P_t Y_t^P.$$

We perform one more transformation, which is to take natural logarithms of all the seven time series, short-term interest rate, bond rate, stock price-, stock dividend-, property price-, property rental- and wage indices. This guarantees that the model never predicts negative indices or interest rates. The logarithmic time series are displayed in Figure 2.

### 3.3 Unit root tests

Before building an econometric model for the time series, we have to study their stationarity properties. We perform five unit root tests on  $x$  and its first difference. The tests are the augmented Dickey-Fuller test (ADF) (Dickey and Fuller, 1981),  $P_T$  and DF-GLS tests by Elliot et al. (1996) and  $Q_T$  and DF-GLSu tests, suggested by Elliot (1999). In the ADF test the lag length has been selected according to Schwarz information criterion with a maximum of five lags. The selected lag length is subsequently applied in the other reported tests. The  $P_T$  and DF-GLS tests are known to have improved power and better small sample properties compared to the ADF test (Elliot et al.,



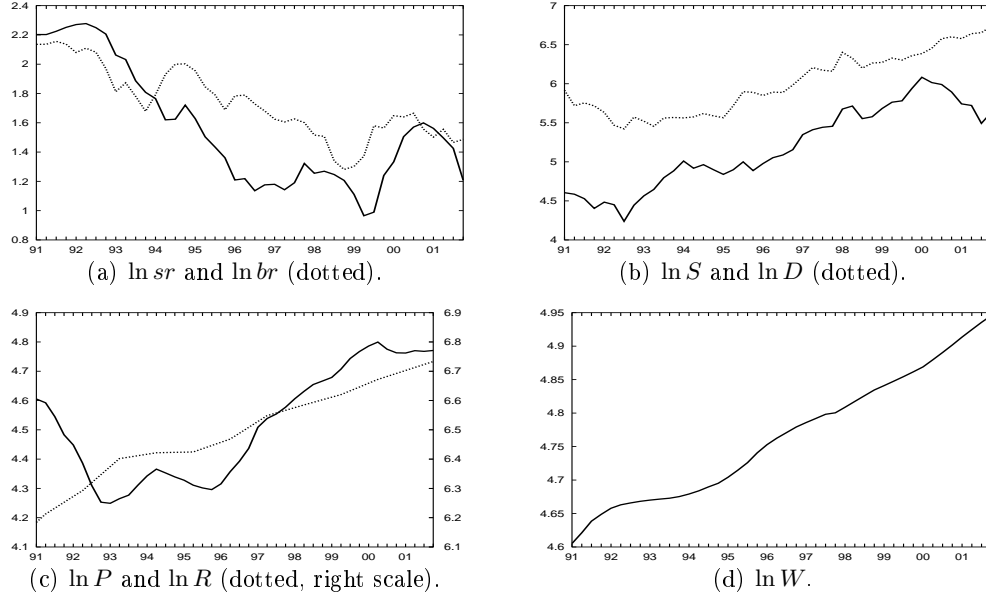


Figure 2: Transformed time series.

1996). The  $Q_T$  and DF-GLSu tests differ from the  $P_T$  and DF-GLS tests in the way the initial observation is treated in the derivation of the test statistics. In the  $Q_T$  and DF-GLSu tests the initial observation is drawn from its unconditional distribution whereas in the  $P_T$  and DF-GLS tests it is set to zero; see Elliot (1999). The results of the unit root tests were robust against different lag length selection methods, such as Akaike information criterion and General to simple, where the strategy is to select the highest significant lagged difference length e.g. in the ADF regression, less than or equal to some initial value. The value of  $z(t)$  in Table 1 indicates the deterministic terms included in the unit root regressions. When  $z(t) = 1$  a constant is included and with  $z(t) = (1, t)$  a constant and a trend are included.

The results of the unit root test are displayed in Table 1. They clearly indicate that  $\ln sr$ ,  $\ln br$ ,  $\ln S$  and  $\ln D$  need to be differenced once in order to achieve stationarity. This confirms the findings of Hall et al. (1992), Sherris et al. (1999), Kanioura (2001) and Montoro (2001) regarding the interest rates. With  $\ln P$  and  $\ln R$  the evidence is not so clear. The non-stationarity of these series cannot be rejected at 5% significance level, except according to  $Q_T$  statistic the  $\ln R$  is found to be trend stationary. The analysis of  $\Delta \ln P$  and  $\Delta \ln R$  cannot reliably reject the non-stationarity of the first differences either, which is not surprising considering the data used. The assumption, that the first difference of the logarithmic price index is stationary seems reasonable on economic grounds. Also, using quarterly data from 1970/1 to 1997/4, Barot and Takala (1998) concluded that  $\Delta \ln P$  is stationary. The problem with property data are the long cycles that follow closely the general economic conditions in Finland. During the deep recession of the 1990's the nominal property prices fell almost 40% by 1993. As a consequence of strong economic boom the property prices started to recover a few years later. These large long term fluctuations have caused the observed problems in unit root testing. We follow the findings in Barot and Takala (1998) and treat  $\ln P$  as a difference stationary process. Accordingly,  $\ln R$  is treated similarly. All the unit root tests suggest that the logarithmic wage index is trend stationary. However, since all the other time series are treated as difference stationary we adopt the same strategy for  $\ln W$ . This assumption is supported by  $Q_T$ , DF-GLSu and ADF tests, Table 1. Moreover, Clements and Hendry (2001) argues that difference stationary models are considerably more adaptive forecasting tools compared to trend stationary

models, when deterministic shifts occur during the forecast period.

Table 1: Unit root test statistics. \*\*\*, \*\*, \* indicate the rejection of the unit root null at 1%, 5%, 10% significance level.

Time series	Number of lags	Detrending, $z(t) =$	PT	DF-GLS	$Q_T$	DF-GLSu	ADF
$\ln sr$	1	(1)	11.07	-0.34	10.92	-1.35	-1.43
$\ln br$	1	(1)	10.48	-0.62	9.55	-1.67	-1.68
$\ln S$	0	(1,t)	9	-1.69	5	-1.75	-1.62
$\ln D$	0	(1,t)	20.85	-1.38	8.82	-2.32	-3
$\ln P$	1	(1,t)	6.74*	-2.08	3.51	-2.66	-3.22*
$\ln R$	1	(1,t)	5.88*	-2.32	2.60**	-2.7	-3.30*
$\ln W$	1	(1,t)	2.47***	-3.81***	1.31***	-3.88***	-3.5*
$\Delta \ln sr$	0	(1)	1.25***	-3.88***	2.06***	-3.82***	-3.97***
$\Delta \ln br$	0	(1)	0.69***	-5.18***	1.27***	-5.39***	-5.37***
$\Delta \ln S$	0	(1)	0.70***	-4.94***	1.34***	-5.85***	-6.22***
$\Delta \ln D$	1	(1)	0.44***	-6.04***	0.86***	-6.05***	-5.67***
$\Delta \ln P$	0	(1)	7.1	-1.2	5.48*	-2.78*	-2.44
$\Delta \ln R$	0	(1)	6.07	-1.25	5.33*	-2.5*	-2.42
$\Delta \ln W$	3	(1)	4.95	-1.37	1.69***	-2.92**	-5.2***

## 4 A VAR-model with specified drift

Denote the vector of logarithmic variables by

$$x_t = \begin{bmatrix} \ln sr_t \\ \ln br_t \\ \ln S_t \\ \ln D_t \\ \ln P_t \\ \ln R_t \\ \ln W_t \end{bmatrix}.$$

Based on the above observations, we assume that  $\Delta x_t$  is stationary. Our first attempt consists of building a VAR model of the form

$$\Delta_d x_t = \sum_{i=1}^k A_i \Delta_d x_{t-i} + \epsilon_t, \quad \epsilon_t \sim N(0, \Sigma), \quad (3)$$

where  $A_i \in \mathbb{R}^{7 \times 7}$ ,  $\Sigma \in \mathbb{R}^{7 \times 7}$  and  $\Delta_d$  denotes the shifted difference operator

$$\Delta_d x_t := \Delta x_t - d$$

with  $d \in \mathbb{R}^7$ ; see also (Clements and Hendry, 1998, page 160). This format is convenient in that the parameter vector  $d$  determines the average drift in simulations. Indeed, if  $\Delta_d x_t$  is stationary, (3) gives

$$E[\Delta_d x_t] = \left( \sum_{i=1}^k A_i \right) E[\Delta_d x_t],$$

so if (3) is free of unit roots,  $E[\Delta_d x_t] = 0$ , or

$$E[\Delta x_t] = d. \quad (4)$$

The above format is particularly natural for modeling indices.

**Example 1** If  $x_t$  is the scalar process  $\ln S_t$ , and  $A_i = 0$ , (3) becomes

$$\Delta \ln S_t = d + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma),$$

which is a discrete-time version of the geometric Brownian motion model of stock price; see for example Hull (2000).

Moreover, according to Hendry and Clements (2001) modeling differences instead of levels gives protection against structural breaks in data generating process. VAR-models for differences of logarithms of economic time series have been built for example by Eitrheim et al. (1999) for the Central Bank of Norway.

Looking only at our (far from ideal) data, might suggest that there is a strong negative drift in the interest rates. However, we believe that  $E[\Delta sr_t] = E[\Delta br_t] = 0$  in the long run, so we choose  $d_{sr} = d_{br} = 0$  rather than estimating these parameters from the data. On the other hand, the dividend yield satisfies

$$\Delta \ln Y_t^S = \Delta \ln D_t - \Delta \ln S_t = \Delta_d \ln D_t - \Delta_d \ln S_t + d_D - d_S,$$

so if  $\Delta_d \ln D_t$  and  $\Delta_d \ln S_t$  follow (3), and if (3) is stationary, we have

$$E[\Delta \ln Y_t^S] = d_D - d_S.$$

Since there is no reason to believe that the dividend yield would have a consistent drift, one way or the other, we require  $d_D = d_S$ . Similar reasoning for the rental yield suggests  $d_R = d_P$ . It seems thus reasonable to assume that  $d$  has the form

$$d = \begin{bmatrix} 0 \\ 0 \\ d_S \\ d_S \\ d_P \\ d_P \\ d_W \end{bmatrix}. \quad (5)$$

Simply estimating  $d$  from the data would not result in a vector of the form (5). This is a clear case where, “expert” information seems more reliable than statistical information.

The choice of the values of the remaining drift parameters  $d_S$ ,  $d_P$  and  $d_W$  is not quite as clear. Bewley (2000) and Landon-Lane (2000) have presented methods to restrict  $d$  when estimating the parameters of (3). In our case, one could make the restriction that  $d$  has to be of the form (5) and then the remaining parameters could be estimated from the data. In practice, however, pension companies’ managers often have their own estimates for the average drifts for the future development of various time series. Such estimates are rarely based on statistical data alone. Accordingly, we will take  $d_S$ ,  $d_P$  and  $d_W$  as *user-specified parameters*. This not only provides a convenient way of incorporating expert views into the model, but it also simplifies the estimation process considerably; see below.

Our experiments below will be based on the choice

$$d = \begin{bmatrix} 0 \\ 0 \\ 0.0114 \\ 0.0114 \\ 0.007 \\ 0.007 \\ 0.009 \end{bmatrix} .$$

The value of  $d_S$  corresponds to 4.6% average of yearly log-return<sup>1</sup>. It was argued in Barot and Takala (1998), that in the long run, there is no excess return in residential property prices over inflation rate. However, we believe that property investments will gain some real return over inflation and set the yearly growth rate of  $\ln P$  to 2.8%, which is 0.8% over the inflation target of the European Central Bank. Finally, the yearly growth rate of  $\ln W$  is set to 3.6%, which is 1.6% over the inflation target.

#### 4.1 Estimation

Having specified the drift parameter  $d$ , it remains to choose the lag length  $k$  and the matrices  $A_i$  and  $\Omega$  in model (3). We use PcFiml 9.0 for computing test statistics and parameter estimates; see Doornik and Hendry (1997). We start by selecting the appropriate lag length  $k$  in our VAR-model. Table 2 presents the lag length reduction test results starting from  $k = 4$ . Since, the sequential F-tests cannot reject the reductions to  $k = 1$  and since the Schwarz (SC) and Hannan-Quinn (HQ) information criteria in Table 3 have minimum values at  $k = 1$ , we select  $k = 1$ .

Table 2: Lag length reduction tests

System reduction		Test statistic	p-value
from $k =$	to $k =$		
2	1	$F(49, 126) = 1.342$	0.0982
3	2	$F(49, 90) = 0.8912$	0.6663
4	3	$F(49, 55) = 1.0998$	0.3645

Table 3: Information criteria values

$k =$	np	log-likelihood	SC	HQ
1	49	1167.7489	-48.865	-50.115
2	98	1214.5735	-46.779	-49.28
3	147	1258.4598	-44.56	-48.31
4	196	1334.5532	-43.805	-48.805

Note: np = number of estimated parameters

We then estimate  $A_1$  and  $\Sigma$  with the method of maximum likelihood (ML), starting with an unrestricted model and carry out an iterative procedure, where one insignificant parameter per iteration is removed and ML estimates for the remaining parameters are recomputed until all insignificant coefficients at 5% significance level have been removed. For a comparison and discussion

<sup>1</sup>In our model, the usual return  $S_t/S_{t-1}$  is log-normally distributed with mean  $\exp(d_S + \frac{1}{2}\sigma_S^2)$ , where  $\sigma_S$  is the stock price volatility (Hull, 2000). Our choice of  $d_S$  gives roughly 7% average yearly return when  $\sigma_S = 20\%$ .

of different model selection criteria; see Brüggemann and Lütkepohl (2000) and Lütkepohl (1991). The resulting estimates of  $A_1$ , their standard errors  $SE(A_1)$ , residual correlation matrix  $C$  and residual standard deviations  $\sigma$  are

$$\begin{array}{l}
 A_1 = 10^{-1} \left[ \begin{array}{ccccccc}
 3.665 & 4.887 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 7.392 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -2.832 & 0 & 9.122 & 0 & 0 \\
 0 & 0 & 0.571 & 0 & 6.999 & 0 & 0 \\
 0 & -0.162 & 0 & 0 & 0 & 8.538 & 0 \\
 0 & 0 & 0 & 0 & 0 & -0.825 & 8.424
 \end{array} \right] \\
 SE(A_1) = 10^{-1} \left[ \begin{array}{ccccccc}
 1.269 & 1.428 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 3.234 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0.842 & 0 & 2.811 & 0 & 0 \\
 0 & 0 & 0.273 & 0 & 0.868 & 0 & 0 \\
 0 & 0.080 & 0 & 0 & 0 & 0.619 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.246 & 0.521
 \end{array} \right] \\
 C = \left[ \begin{array}{ccccccc}
 1 & & & & & & \\
 0.0761 & 1 & & & & & \\
 -0.0157 & -0.0201 & 1 & & & & \\
 -0.1976 & -0.3084 & 0.4780 & 1 & & & \\
 -0.0341 & -0.0683 & 0.3192 & -0.0275 & 1 & & \\
 0.1519 & 0.1084 & -0.1433 & -0.0111 & 0.0300 & 1 & \\
 -0.1108 & -0.2356 & 0.0680 & 0.4057 & -0.0222 & -0.2113 & 1
 \end{array} \right] \\
 \sigma = 10^{-2} [ \begin{array}{ccccccc}
 7.5639 & 7.6128 & 11.1010 & 7.7848 & 1.9880 & 0.4381 & 0.1869
 \end{array} ]
 \end{array}$$

The likelihood ratio test of over-identifying restrictions  $\chi^2(38) = 37.26[0.503]$ , clearly accepts the made reductions. Table 4 reports the equation residual test results, where the numbers are the p-values of the test statistics. The reported tests are the  $F$ -test for 4th-order residual autocorrelation,  $\chi^2$  normality test,  $F$ -test for autocorrelated squared residuals and  $F$ -test for residual heteroscedasticity respectively, see Doornik and Hendry (1997). The results reveal some autocorrelation problems in  $\Delta \ln sr$ ,  $\Delta \ln S$  and  $\Delta \ln D$ , which is quite typical for financial time series data. The autocorrelation problems were persistent even in models with longer lag lengths, suggesting that VARMA models, (see e.g. Lütkepohl and Poskitt (1996), Lütkepohl and Claessen (1997)) could be more appropriate for the given data set. The residual normality assumption is rejected for  $\Delta \ln R$  due to one negative outlier. The residual distribution of  $\Delta \ln W$  has fatter tails compared to normal distribution, which would imply larger variance for the wage index forecasts than observed with normally distributed errors. This in turn would mean that the reserves would exhibit little larger fluctuations. However, we believe that this will not considerably affect the efficient asset allocation decisions obtained for the ALM problem in Section 8.

Table 4: Equation residual diagnostics

Equation	AR 1-4 F	Norm $\chi^2$	ARCH 1-4 F	HET F
$\Delta \ln sr$	0.0069	0.5134	0.7492	0.0984
$\Delta \ln br$	0.2533	0.6668	0.7999	0.9590
$\Delta \ln S$	0.0150	0.2902	0.9625	0.7573
$\Delta \ln D$	0.0393	0.2335	0.8165	0.9430
$\Delta \ln P$	0.0630	0.3898	0.1250	0.1043
$\Delta \ln R$	0.0727	0.0000	0.5932	0.5895
$\Delta \ln W$	0.1273	0.0042	0.7088	0.6960

## 4.2 Simulation experiment

To test the long term behavior of the model we performed 250 twenty-year simulations with the estimated model (DVAR<sub>mod</sub>) started from

$$x_0 = \ln \begin{bmatrix} 3.35 \\ 4.42 \\ 279.6 \\ 843.7 \\ 118.0 \\ 839.8 \\ 140.6 \end{bmatrix}, \quad x_{-1} = \ln \begin{bmatrix} 4.16 \\ 4.33 \\ 242.9 \\ 776.0 \\ 117.7 \\ 831.3 \\ 139.1 \end{bmatrix},$$

which was the situation in the beginning of 2002. For comparison we performed equivalent simulations with a model (DVAR<sub>sys</sub>) where all the regressors are retained and the drift parameters are estimated without restrictions. The outcomes of the simulations for the DVAR<sub>mod</sub> and DVAR<sub>sys</sub> models are displayed in Figures 3 and 4 respectively. The outcomes of the DVAR<sub>sys</sub> simulations highlights the importance of correctly specifying the form of the drift vector, see Figure 4. The short term interest rate is rapidly declining throughout the simulation period due to estimated negative drift. Also, the log interest rate spread, log dividend and rental yields are trending due to differences in the estimated drifts of the underlying time series. The average drifts of the DVAR<sub>mod</sub> were what we desired, but in some respects the model behaves strangely: in many scenarios the logarithmic short rate, log interest rate spread, log dividend and rental yields have deviated unrealistically from their usual values; see Figure 3.

Despite the fact that the drift parameters for interest rates were set to zero in DVAR<sub>mod</sub>, the short term interest rate is generally declining throughout the simulation period. This shows how strongly the future distributions depend on the initial values of the variables in a VAR model. This phenomenon could be avoided by changing the initial values, so that the simulation starts from some neutral conditions, as suggested by Lee and Wilkie (2000), but then the relevant market information essential to the present investment decisions is lost.

The problems even after specifying the drift vector result from the fact that the above models only look at the differences  $\Delta x_t$  and completely ignores the actual values of  $x_t$ , which is what we are really interested in. As demonstrated by the simulations, the average values for  $x_t$  are largely determined by the initial values  $x_0$  and  $x_{-1}$ , which may be poor estimates of the future. This leads us to consider VEqC-models, which avoid this kind of shortcomings.

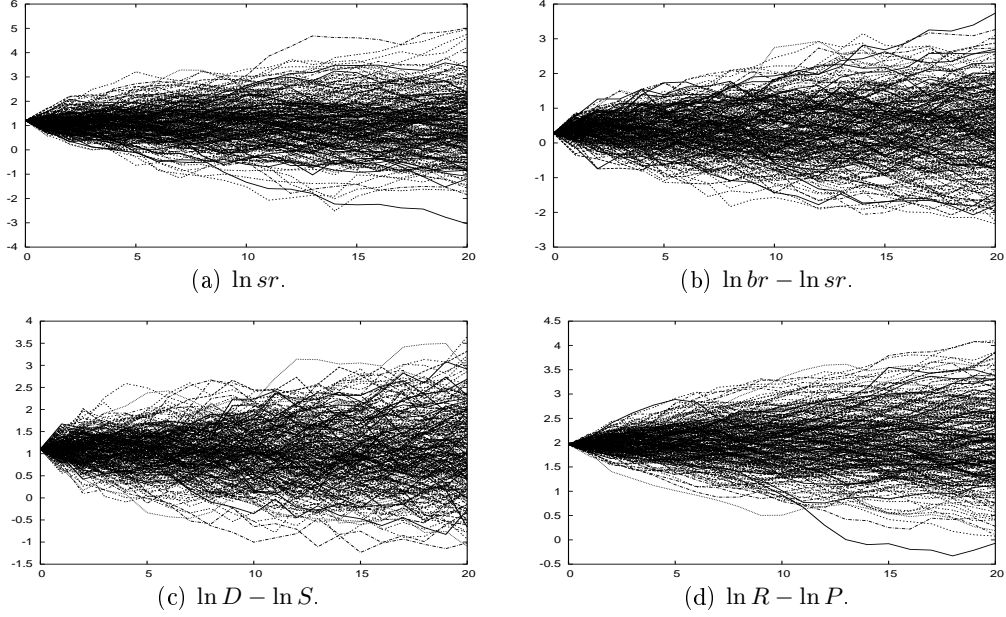


Figure 3:  $DVAR_{mod}$  simulations.

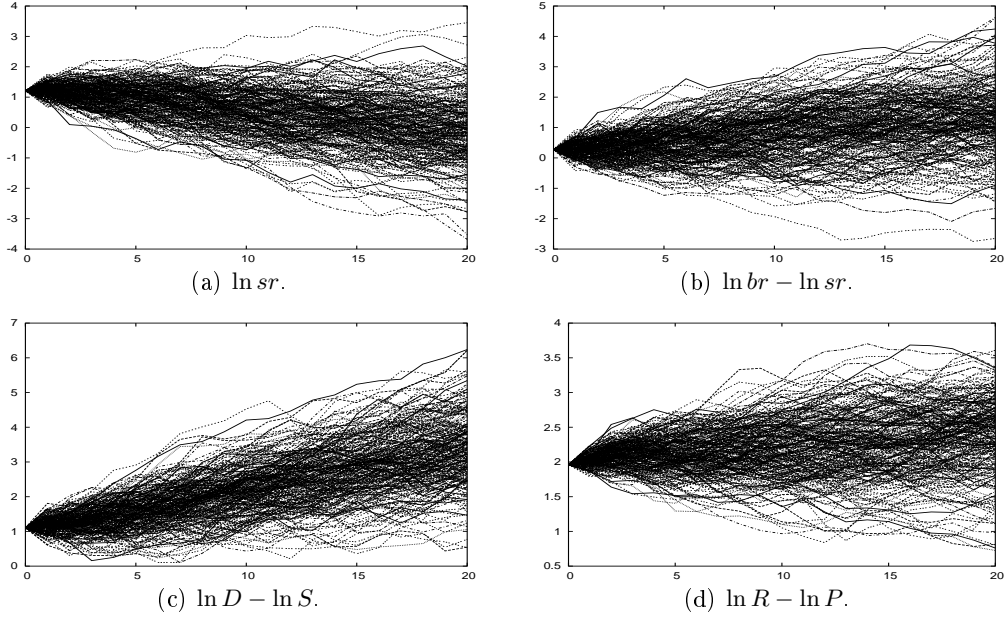


Figure 4:  $DVAR_{sys}$  simulations.

## 5 A VEqC-model with specified drift and cointegration relations

A *vector equilibrium correction model* is obtained from the VAR-model for differences by adding an “equilibrium correction term” to the right-hand side of the equations. In the case of our drift-specified VAR-model we get the model

$$\Delta_d x_t = \sum_{i=1}^k A_i \Delta_d x_{t-i} + \alpha(\beta' x_{t-1} - \mu) + \epsilon_t, \quad \epsilon_t \sim N(0, \Sigma), \quad (6)$$

where  $\beta \in \mathbb{R}^{7 \times l}$ ,  $\mu \in \mathbb{R}^l$  and  $\alpha \in \mathbb{R}^{7 \times l}$ . The additional term takes into account the long-term behavior of  $x_t$  around statistical equilibria described by the linear equations  $\beta'x = \mu$ . It is assumed that, in the long run,

$$E[\beta'x_t] = \mu, \quad (7)$$

and that if  $x_t$  deviates from the equilibria (due to shocks in economic conditions) it will tend to move back towards them. The matrix  $\alpha$  determines the speed of adjustment towards the equilibria. In this sense, VEqC-models incorporate long-run equilibrium relationships (often derived from economic theory) with short-run dynamic characteristics deduced from historical data. VEqC-models for logarithms of economic time series have been built for example by Eitrheim et al. (1999) for the Central Bank of Norway and by Anderson et al. (2000) for the Federal Reserve Bank of St.Louis. The results of Eitrheim et al. (1999) indicate that the inclusion of equilibrium-correction feedbacks may improve the forecast accuracy of VAR-models for differences, especially in the long-run.

The equilibrium correction term is particularly convenient when modeling interest rates.

**Example 2** *If  $x_t$  is the scalar process  $\ln r_t$ ,  $d = 0$  and  $A_i = 0$ , the model becomes*

$$\Delta \ln r_t = \alpha(\ln r_{t-1} - \mu) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma).$$

*With  $\alpha < 0$ , this is a discrete-time version of the mean-reverting interest rate model of Black and Karasinski (1991). With  $\alpha = -1$ , we obtain the memoryless model*

$$\ln r_t = \mu + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma).$$

Besides mean reversion effects for interest rates, the equilibrium correction term is useful also in controlling long-term averages of the interest rate spread and the yields, which behaved unrealistically in the simple VAR-model of the previous section. We follow the two-step Engle-Granger methodology (Engle and Granger (1987)) by first specifying the equilibrium relations, and then estimating  $A$ ,  $\alpha$  and  $\Sigma$  from the data. Note that by (4) and (7), the matrix  $\beta$  has to satisfy the consistency condition

$$\beta'd = \beta'E[\Delta x_t] = E[\beta'\Delta x_t] = E[\Delta(\beta'x_t)] = 0. \quad (8)$$

Based on our experiences with the VAR-model, we propose the following four equilibrium relations.

1.  $\ln sr_t = \mu_{mr}$ . Similarly to mean reverting interest rate models, this suggests that, in the long run, the short rate drifts towards certain equilibrium level. Although, the unit root tests in Section 4 indicated  $\ln sr$  to be non-stationary (due to the changing economic conditions during the 1990's), many studies have concluded that interest rates are mean reverting and stationary in the long run; see for example Wu and Zhang (1996) and Fama and Bliss (1987).
2.  $\ln br_t - \ln sr_t = \mu_{sp}$ . This relation means that the “geometric interest rate spread”  $\frac{br_t}{sr_t}$  has a long term equilibrium value. Various studies have concluded that the difference  $br - sr$  of the long and short term interest rates is stationary; see for example Campbell and Shiller (1987), Bradley and Lumpkin (1992) and Hall et al. (1992). Campbell et al. (1997) found also the logarithmic transformation  $\ln(1 + br) - \ln(1 + sr)$  of the interest rate spread to be stationary. To our knowledge, only (Kanioura, 2001, page 5) has studied the geometric spread. She found it to be stationary in the United States.
3.  $\ln D_t - \ln S_t = \mu_{dy}$ . Writing this as  $\ln Y_t^S = \mu_{dy}$ , we see that it corresponds to the existence of an equilibrium value for the dividend yield. This is supported by the findings of Campbell and Shiller (1988), Campbell et al. (1997) and Wilkie (1986).



4.  $\ln R_t - \ln P_t = \mu_{ry}$ . Similarly to dividend yield, this can be written as  $\ln Y_t^P = \mu_{ry}$  which corresponds to a stationary rental yield.

These choices correspond to

$$\beta = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (9)$$

This satisfies (8) with any  $d$  of the form (5). In fact, any  $d$  satisfying (8) for this choice of  $\beta$  has to be of the form (5). Similarly to the drift vector  $d$ , we take the equilibrium values  $\mu$  in the cointegration relations as *user-specified parameters*.

The historical values of  $\beta x_t$  are displayed in Figure 5. In our time frame 1991/1 – 2001/4, these series do not pass the stationarity tests on conventional significance levels. However, in the light of the above references and our own intuition, we believe that these series will be stationary in the long run.

In our experiments, we use the  $\beta$  in (9) and

$$\mu = \begin{bmatrix} \ln 3.7 \\ \ln 1.2 \\ \ln 2.5 \\ \ln 7.0 \end{bmatrix}.$$

This corresponds to long term equilibrium values of 3.7%, 1.2%, 2.5% and 7% for short rate, geometric interest rate spread, dividend yield and rental yield, respectively.

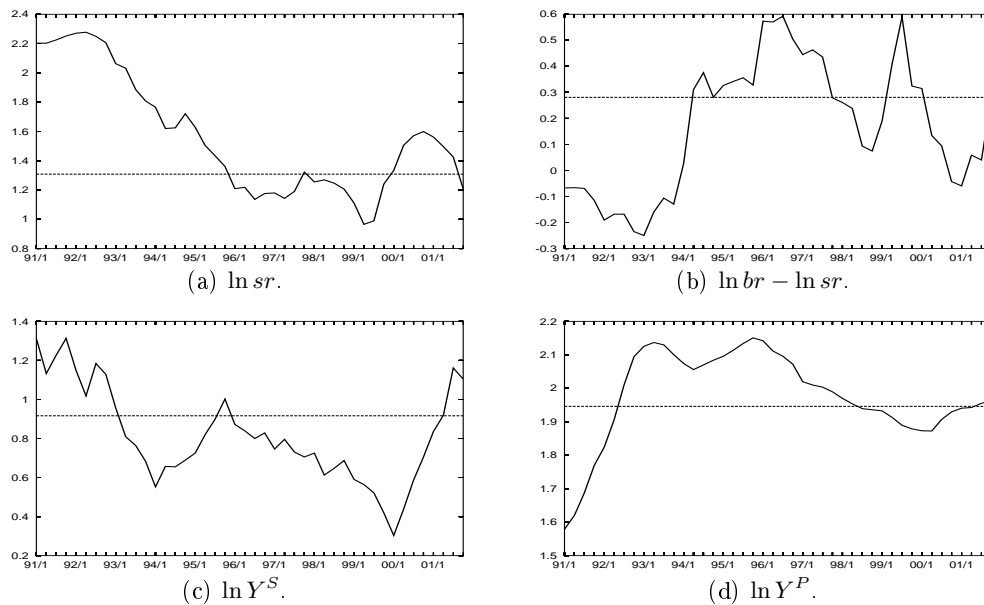


Figure 5: Historical values of the cointegration vectors. The horizontal lines mark the expected equilibrium levels  $\mu$ .

## 5.1 Estimation

Having specified the drift parameter  $d$  and the cointegration relations, we find the maximum likelihood estimates of the remaining parameters  $A$ ,  $\alpha$  and  $\Sigma$ , using the iterative model reduction procedure like in section 4.1. This results in the following values.

$$A_1 = 10^{-1} \begin{bmatrix} 3.672 & 3.467 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.855 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -59.11 \\ 0 & 0 & -2.425 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.629 & 0 & 3.617 & 0 & 0 \\ 0 & -0.209 & 0 & 0 & -0.663 & 8.533 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.638 & 8.712 \end{bmatrix}$$

$$SE(A_1) = 10^{-1} \begin{bmatrix} 1.222 & 1.466 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.469 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 26.50 \\ 0 & 0 & 0.836 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.231 & 0 & 1.065 & 0 & 0 \\ 0 & 0.082 & 0 & 0 & 0.222 & 0.682 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.246 & 0.589 \end{bmatrix}$$

$$\alpha = 10^{-1} \begin{bmatrix} 0 & 0.964 & 0 & 0 \\ -1.061 & -1.499 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1.449 & 0 \\ -0.238 & 0 & 0 & 0.637 \\ 0 & 0.080 & 0 & 0 \\ 0 & 0 & -0.024 & 0 \end{bmatrix}$$

$$SE(\alpha) = 10^{-1} \begin{bmatrix} 0 & 0.415 & 0 & 0 \\ 0.414 & 0.781 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.352 & 0 \\ 0.067 & 0 & 0 & 0.160 \\ 0 & 0.032 & 0 & 0 \\ 0 & 0 & 0.011 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & & & & & & \\ 0.1308 & 1 & & & & & \\ -0.0606 & 0.1332 & 1 & & & & \\ -0.2764 & -0.1978 & 0.5302 & 1 & & & \\ 0.0092 & 0.0184 & 0.354 & -0.0796 & 1 & & \\ 0.0911 & 0.1526 & -0.1329 & -0.0675 & -0.0102 & 1 & \\ -0.174 & -0.1929 & 0.0641 & 0.3586 & 0.0113 & -0.2396 & 1 \end{bmatrix}$$

$$\sigma = 10^{-2} \begin{bmatrix} 7.3288 & 7.4645 & 10.804 & 7.8889 & 1.6589 & 0.4085 & 0.1826 \end{bmatrix}$$

Some remarks on  $\alpha$ :

- The first cointegration vector has a significant negative coefficient in bond rate equation. This can be interpreted as a reaction to interest rate expectations: when the short rate is above its long term average  $\mu_{sr}$ , it is expected to decline in the long run, which causes a drop in the bond rate. Similarly, short rate being below its average, pushes the bond rate

up. The first cointegration vector appears also in the property price equation with a negative sign. This implies that low interest rates increases property prices and vice versa. This may result from the fact that low interest rates decrease the loan servicing costs, which encourages people/companies to invest in properties.

- The geometric interest rate spread enters the short rate equation with a positive sign (which is in line with the expectations hypothesis; see e.g. Campbell et al. (1997)), and the bond rate equation with a negative sign (which in turn contradicts the expectations hypothesis). If the spread is above its average value  $\mu_{sp}$ , these terms push the interest rates closer to each other, and if it is below  $\mu_{sp}$ , they push the short rate down and the bond rate up. This is in line with the findings of Campbell (1995) and (Campbell et al., 1997, Section 10.2.2).
- The third cointegration vector, the log dividend yield, appears in the dividend index equation with a negative sign, so large values of the dividend yield cause a decrease in the dividend index, and vice versa. This effect is similar to findings in Campbell and Shiller (1988). It causes the dividend index to follow the movements in the stock price index, keeping the dividend yield in a reasonable range.
- The log rental yield enters the property price equation with a positive sign, with the interpretation that large values of the rental yield anticipates an increase in property prices, and vice versa.

Table 5 reports the equation residual test results for the VEqC-model. Again, the numbers denote the p-values of the different test statistics. The results are similar to those obtained with the VAR-model in Section 4. The tests reveal some autocorrelation problems in  $\Delta \ln sr$ ,  $\Delta \ln S$  and  $\Delta \ln D$  and again the normality assumption is rejected in the residuals of  $\Delta \ln R$  and  $\Delta \ln W$  due to few outliers.

Table 5: VEqC-model equation residual diagnostics

Equation	AR 1-4 F	Norm $\chi^2$	ARCH 4 F	HET F
$\Delta \ln sr$	0.0029	0.3471	0.9232	0.8173
$\Delta \ln br$	0.1698	0.8191	0.9030	0.9862
$\Delta \ln S$	0.0070	0.2516	0.8918	0.8439
$\Delta \ln D$	0.0037	0.3504	0.6661	0.8877
$\Delta \ln P$	0.0551	0.2290	0.6195	0.8498
$\Delta \ln R$	0.1725	0.0004	0.5072	0.9563
$\Delta \ln W$	0.0530	0.0028	0.5326	0.9123

## 5.2 Simulation experiment

We computed 250 twenty-year simulations with the above VEqC-model (VEqC<sub>mod</sub>) started from the same initial values as in Section 4.2. The equilibrium correction terms effectively control the interest rates and the yields that were problematic in the DVAR models; see Figure 6. The mean reversion apparent in Figure 6 is caused by the inclusion of the equilibrium correction terms, which considerably reduce the variance of the interest rates as well as the dividend and rental yields.

## 6 Forecast tests

We test and compare the performance of our VEqC<sub>mod</sub> model with three rival models in an out-of-estimation-sample forecast experiment. The forecast test period covers seven new quarterly obser-

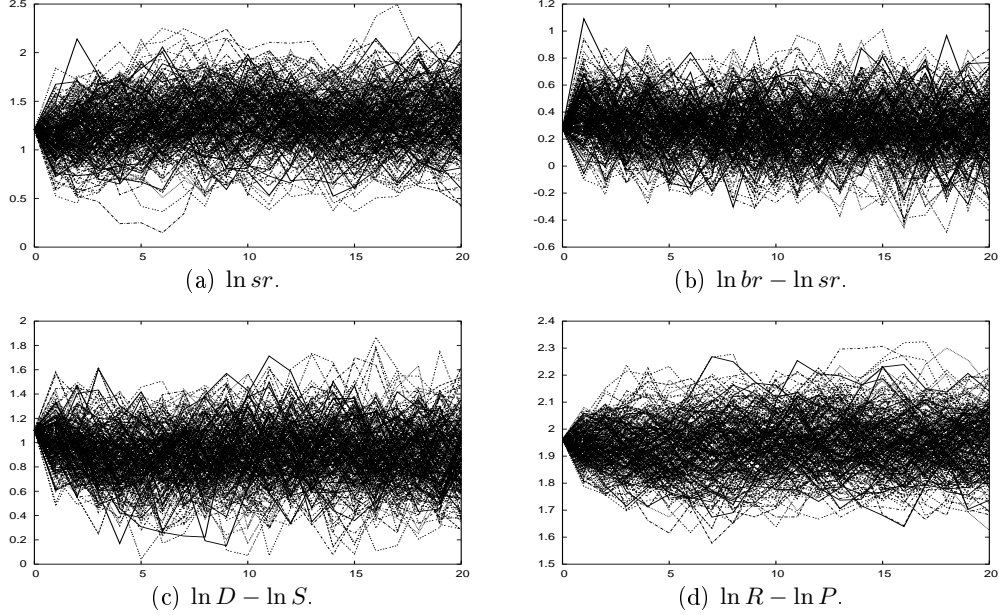


Figure 6:  $\text{VEqC}_{mod}$  simulations.

vations from 2002/1 to 2003/3. Although the short test period does not allow us to draw significant conclusions concerning the forecast performance of different models, the test gives us an indication how the developed  $\text{VEqC}_{mod}$  model would have performed in volatile financial markets during 2002–2003. For comparison, we report the results of the forecast tests for  $\text{DVAR}_{mod}$ ,  $\text{DVAR}_{sys}$  and  $\text{VEqC}_{sys}$  models.  $\text{VEqC}_{sys}$  denotes an unrestricted vector equilibrium correction system where the drifts and the equilibrium values for the equilibrium correction relations of Section 5 are estimated from historical data without restrictions.

We compare the four models' forecast accuracy by performing tests for structural stability (see Lütkepohl (1991)) during the forecast period  $T + 1, \dots, T + h$ , where  $T$  denotes the forecast origin and  $h = 7$  is the length of the forecast horizon. We calculate a test statistic of the form

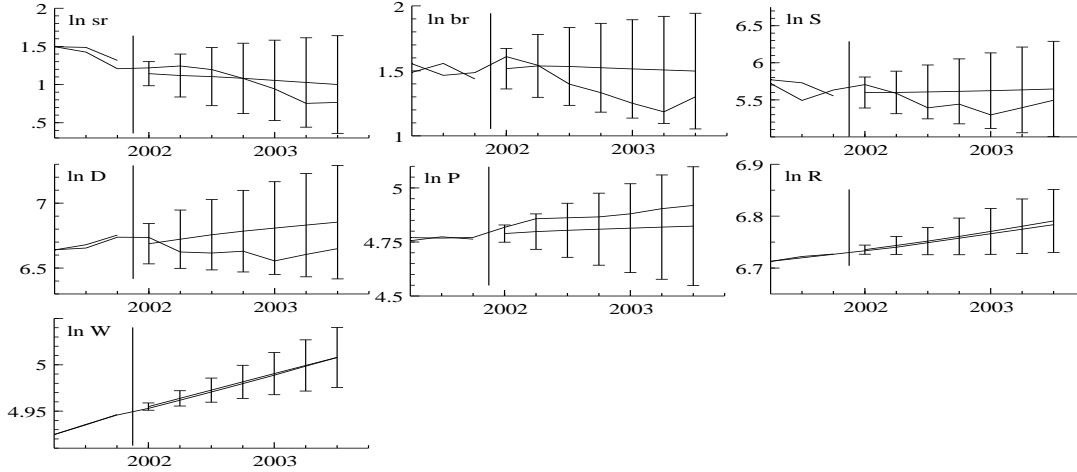
$$\lambda_h = \sum_{i=1}^h u'_{T+i} \Sigma^{-1} u_{T+i} \sim \chi^2(Nh), \quad (10)$$

where  $u_{T+i}$  can be interpreted as the 1-step ahead forecast errors as we move through the forecast period,  $\Sigma$  is the residual covariance matrix as in (3) and  $N = 7$  is the dimension of the model. The null hypothesis for the test is that the process generating  $(\Delta x_{T+1}, \dots, \Delta x_{T+h})$  is the same as that which generated  $(\Delta x_1, \dots, \Delta x_T)$  and the hypothesis is rejected if the forecasts differ too much from the actually observed values, see e.g. Lütkepohl (1991). The values of the approximate  $\chi^2$  and an  $F$  variant test statistics, where the unknown quantities in (10) are replaced by estimated values (see e.g. Lütkepohl (1991) or Clements and Hendry (1998)) together with the p-values of the tests are reported in Table 6.

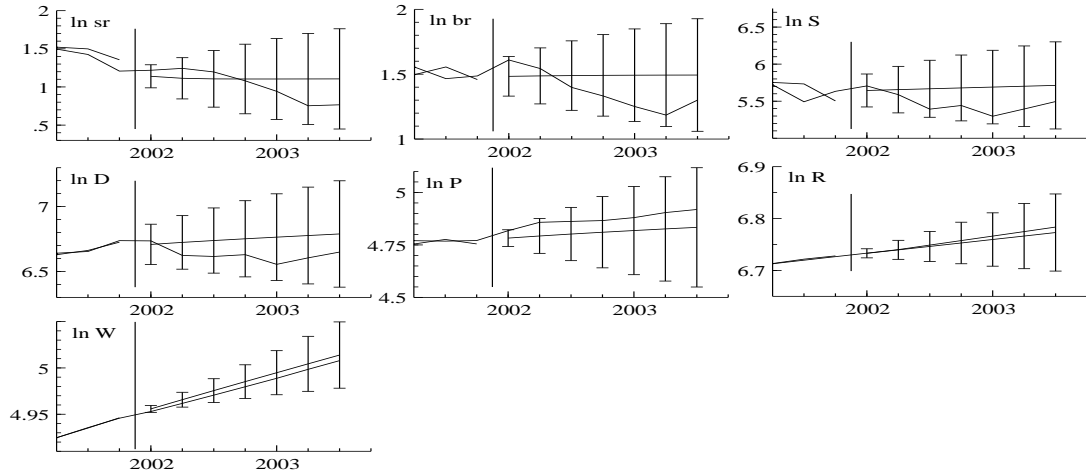
Table 6: Forecast test statistics

Model	$\chi^2$	p-value	$F$	p-value
DVAR <sub>sys</sub>	46.11	0.59	0.94	0.58
DVAR <sub>mod</sub>	54.94	0.26	1.12	0.35
VEqC <sub>sys</sub>	94.39	0.00	1.93	0.03
VEqC <sub>mod</sub>	47.47	0.54	0.97	0.55

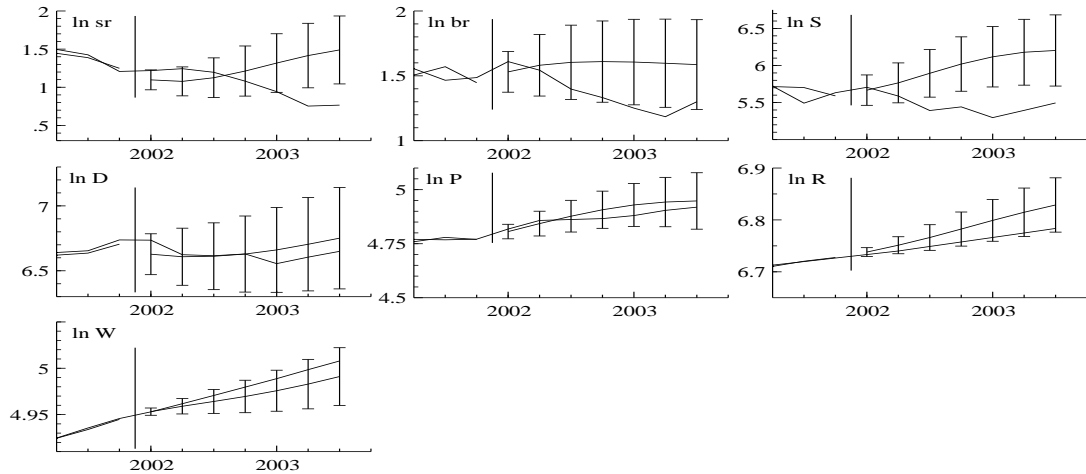
The null hypothesis of structural stability is rejected only for VEqC<sub>sys</sub> at the 1% and 5% significance levels according to  $\chi^2$  and  $F$  statistics, respectively, and VEqC<sub>mod</sub> and DVAR<sub>sys</sub> seems to produce the smallest forecast errors during the test period. Figure 7 displays 1- to 7-step ahead forecasts with their approximate 95% confidence intervals and the actually observed values of  $x_t$  during the forecast period for all the four models. The main reason for the forecast failure of VEqC<sub>sys</sub> is apparent from Figure 7(c), where the actually observed value for  $\ln sr$  and  $\ln S$  are clearly outside their forecast confidence intervals. For the other three models there are no striking differences in the forecast performance, although the confidence intervals for the VAR-models (Figures 7(a) – 7(b)) are wider than for the VEqC-models (Figures 7(c) – 7(d)). These findings together with the results of the long term simulation experiments of Sections 4.2 and 5.2 give support to our approach of specifying the drifts and equilibrium values for the equilibrium relations. It is also worth noting that the VEqC-models contain more economic insight than the pure DVAR models and especially the parameters of the VEqC<sub>mod</sub> model are easy to interpret. In the next Section we will use the developed VEqC<sub>mod</sub>-model in long term return and liability simulation.



(a) DVAR<sub>sys</sub>.



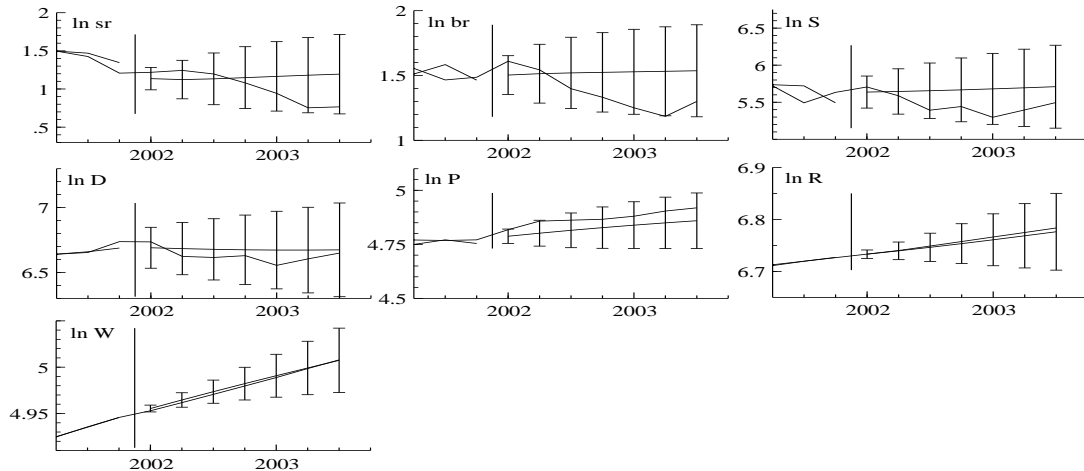
(b)  $DVAR_{mod}$ .



(c)  $VEqC_{sys}$ .

## 7 Long-term return and liability simulations

We will first study the behavior of total returns of the considered asset classes. This will be done by performing 1000 twenty-year simulations with the above model started from the initial values given in Section 4.2, and computing the corresponding yearly total returns for each asset class. The total return of an asset is defined as the sum of the change-in-value and the cash income components.



(d) VEqC<sub>mod</sub>.

Figure 7: 1- to 7-step ahead forecasts with 95% confidence intervals and observed values of  $x_t$  for the four models.

Using the approximations of Section 2.1, we get the following expressions for the total returns.

$$\begin{aligned}
 \text{Cash:} & \quad \sqrt{sr_t sr_{t-1}}, \\
 \text{Bonds:} & \quad \left( \frac{1 + br_{t-1}}{1 + br_t} \right)^D + \frac{1}{2}(br_{t-1} + br_t), \\
 \text{Stocks:} & \quad \frac{S_t}{S_{t-1}} + \frac{1}{2} \left( \frac{D_{t-1}}{S_{t-1}} + \frac{D_t}{S_t} \right), \\
 \text{Property:} & \quad \frac{P_t}{P_{t-1}} + \frac{1}{2} \left( \frac{R_{t-1}}{P_{t-1}} + \frac{R_t}{P_t} \right) - 0.03, \\
 \text{Loans:} & \quad \frac{1}{2}(br_{t-1} + br_t).
 \end{aligned}$$

Figure 8 displays the development of the means and standard deviations of the yearly total returns for cash, bonds, stocks, property and loans, based on 1000 twenty-year simulations. During the first few years, the average returns go through large changes, after which they converge to their equilibrium values. The variations in the average returns towards the end of the simulation horizon are simply effects of the finite sample size.

The initial conditions for the simulation affect the returns and correlation structures considerably in the first few years as the model starts from a disequilibrium. The correlations between the total returns of the asset classes at the end of years 1 and 20 are shown in Figure 9. Even the signs of some correlation coefficients change between years 1 and 20. Once the model converges back to an equilibrium the yearly correlation structure becomes stable.

The reserves and cash-flows can be computed based on the values of the time series according to the rules outlined in Section 2.2. The results of 1000 20-year simulations are displayed in Figure 10. The reserves grow consistently over time but the variation in the projected cash flows at the horizon are substantial. The decreasing trend in cash flows after a few years results from the retirement of the large age groups.

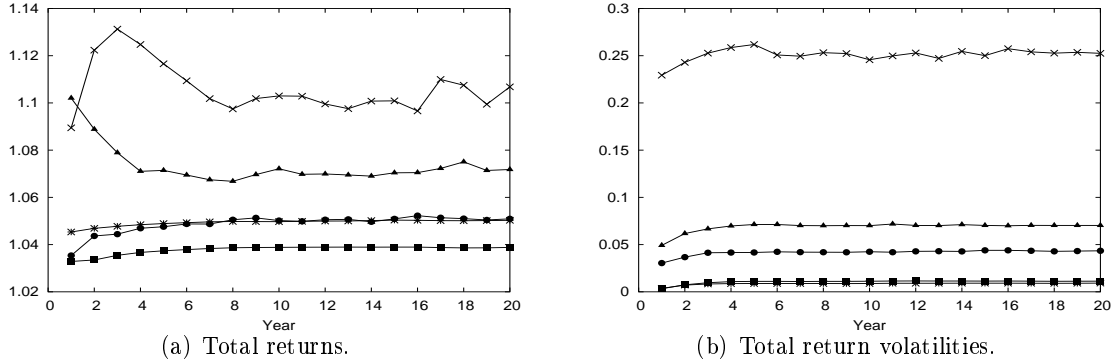


Figure 8: Simulated total returns and their volatilities for Cash (■), Bonds (●), Stocks (×), Property (▲) and Loans (\*) with VEqC(mod)

	Cash	Bonds	Stocks	Property	Loans
Cash	1				
Bonds	-0.530	1			
Stocks	0.014	-0.153	1		
Property	-0.123	-0.007	0.553	1	
Loans	0.530	-0.999	0.153	0.007	1

(a) Year 1.

	Cash	Bonds	Stocks	Property	Loans
Cash	1				
Bonds	0.240	1			
Stocks	-0.060	-0.133	1		
Property	-0.340	-0.113	0.515	1	
Loans	0.763	0.229	0.036	-0.218	1

(b) Year 20.

Figure 9: Simulated total return correlations.

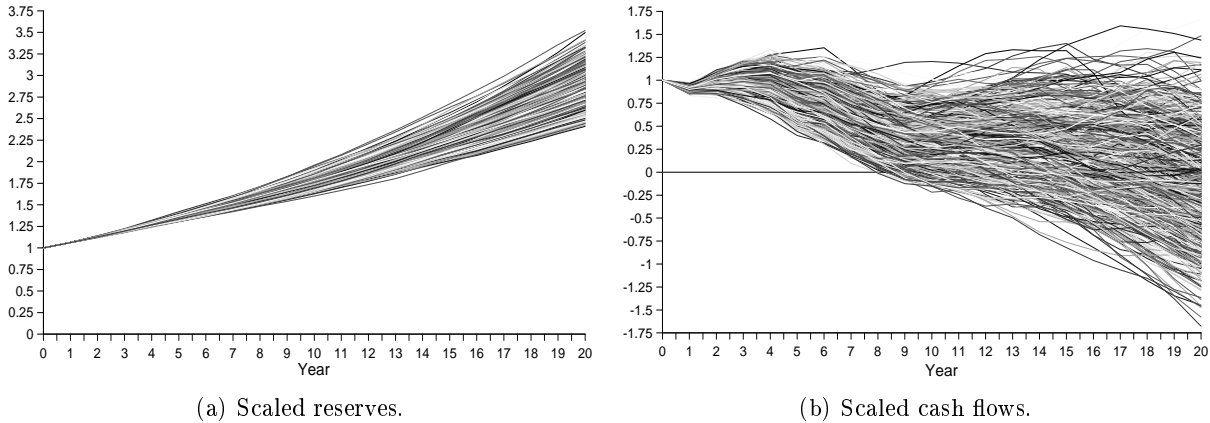


Figure 10: Simulated Reserves and Cash flows.

## 8 Evaluation of dynamic portfolio allocation strategies

Given a stochastic model for the asset returns and the liabilities, we would like to compute the distribution of a company's solvency in the future. This is a nontrivial task since the future solvency



depends on the values of the reserves and cash flows as well as the investment strategies that the company employs now and in all the possible states of the world in the future. Moreover, the Finnish legislation imposes complicated regulations that the companies' must take into consideration in their strategic asset allocation, see Hilli et al. (2003). We evaluate the company's long term solvency by considering two widely studied decision rules for dynamic portfolio allocation, namely *fixed-mix* and *portfolio insurance* strategies; see e.g. Perold and Sharpe (1988), Cesari and Cremonini (2003).

In fixed-mix strategy the portfolio is always rebalanced to a given (fixed) asset distribution (mix). So a fixed-mix strategy is given by a vector of numbers giving the fixed percentages that the asset allocations should satisfy now and in the future. In the present setting this is a vector of five numbers giving the portfolio weights for cash, bonds, stocks, property and loans.

The proportion of loans in the investment portfolio each year is kept fixed at 0.145% of the reserves, which corresponds to 11.5% weight in the initial asset portfolio. This means that the proportion of loans in the portfolio decreases if the total value of the investments increases faster than the value of the reserves and vice versa. We will examine by simulation the performance of different asset mixes obtained by different combinations of the following weights applied to the remaining portfolio.

$$\begin{aligned} \text{Cash: } w_C &\in \{0, 0.01, \dots, 0.03\}, ; \\ \text{Stocks: } w_S &\in \{0, 0.025, \dots, 0.5\}, ; \\ \text{Property: } w_P &\in \{0.1, 0.15, \dots, 0.4\}, ; \\ \text{Bonds: } w_B &= 1 - w_C - w_S - w_P. \end{aligned}$$

The upper bounds for stocks and property are statutory restrictions and the bond investments will be chosen so that the total weights in the remaining portfolio sum up to 100%.

The used portfolio insurance (PI) strategy is based on the constant proportion portfolio insurance framework of Perold and Sharpe (1988) and Black and Jones (1988). The portfolio weights for cash and property are varied according to the same rules as in the fixed-mix case. The rest of the wealth is allocated between the more liquid assets, bonds and stocks. The proportion of stocks in the portfolio at time  $t$  is given by,

$$w_{S,t} = \begin{cases} \min \left\{ (1 - w_C - w_P) \min \left\{ \rho \left( \frac{W_t - L_t}{W_t} \right), 1 \right\}, 0.5 \right\} & \text{if } W_t - L_t \geq 0, \\ 0 & \text{if } W_t - L_t < 0, \end{cases}$$

where  $\rho$  is a risk tolerance parameter indicating how the proportion invested in stocks increases with the company's solvency ratio,  $(W_t - L_t)/W_t$ , where  $W_t$  and  $L_t$  denote the values of the company's assets and reserves in the beginning of year  $t$ , respectively. The percentage invested in stocks is a constant multiple of the company's solvency ratio, which was close to 22% initially, with higher values of  $\rho$  resulting in higher stock market allocations and again at most 50% of the total wealth can be invested in stocks. When the company's wealth  $W_t$  is less than the value of its reserves  $L_t$  (floor) the stock market allocation is set to zero and the remaining wealth is invested in bonds. PI strategy seems appropriate for a pension company because it allocates more wealth to risky assets, stocks when the company's solvency ratio improves and reduces the stock market exposure as the company approaches insolvency.

For each fixed-mix portfolio combination and for PI strategies with varying risk tolerances,  $\rho \in \{1, 1.5, \dots, 20\}$ , we perform 1000 simulations with a 20-year time horizon. Figure 11 displays the average solvency capital/reserves ratio in 20 years versus the insolvency probability for each fixed-mix portfolio and PI strategy based on the sample of 1000 scenarios. Insolvency means that

the solvency capital has become negative at least once during the 20-years. The best PI strategies clearly dominate the best performing fixed-mix strategies at all reasonable risk levels.

The lower boundaries of the clouds of points can be interpreted as the *efficient frontiers* of the fixed-mix and PI strategies, which are displayed in Figure 12. For insolvency probabilities between 0–5% the efficient PI strategies improve the average solvency capital-reserves ratios from 15 to 35% compared to fixed-mix portfolios, and the performance gap between the two methods decreases as the bankruptcy risk increases.

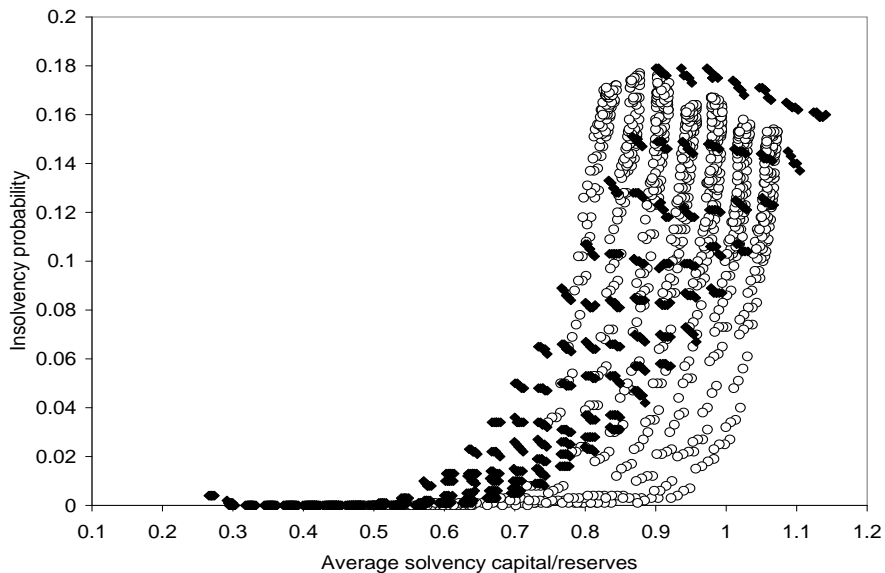


Figure 11: Average solvency capital/reserves at the horizon against insolvency probability for different fixed-mix (◆) and PI (○) strategies.

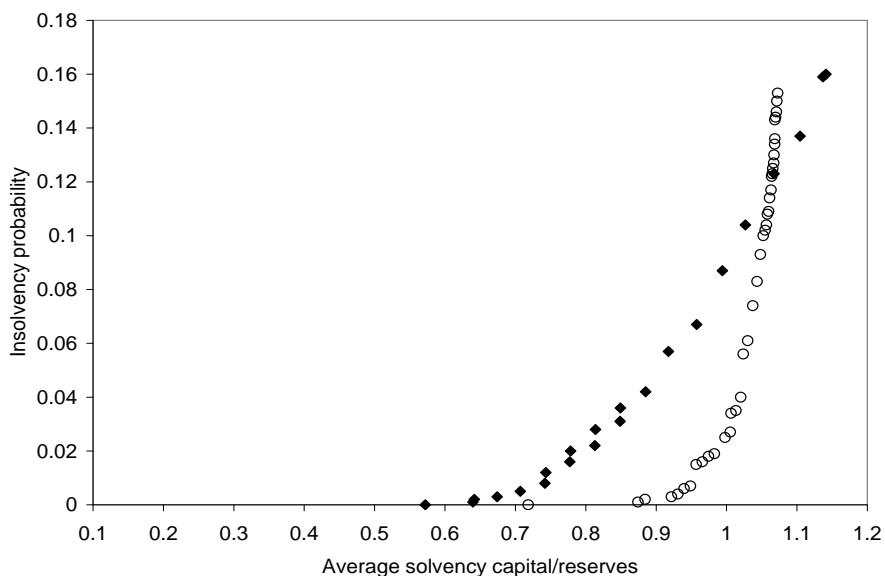


Figure 12: Efficient frontier of fixed-mix (◆) and PI (○) strategies.

The compositions of efficient fixed-mix portfolios and initial portfolio weights for efficient PI strategies are displayed in Figure 13 for varying insolvency probabilities. All the efficient fixed-mix

and initial PI portfolios have at least 35% of the wealth invested in property and in some portfolios a small fraction of money invested in cash. Due to PI strategies' ability to react dynamically to changing solvency situations, the initial stock market allocation in PI portfolios can be kept much higher compared to fixed-mix portfolios with similar insolvency probabilities.

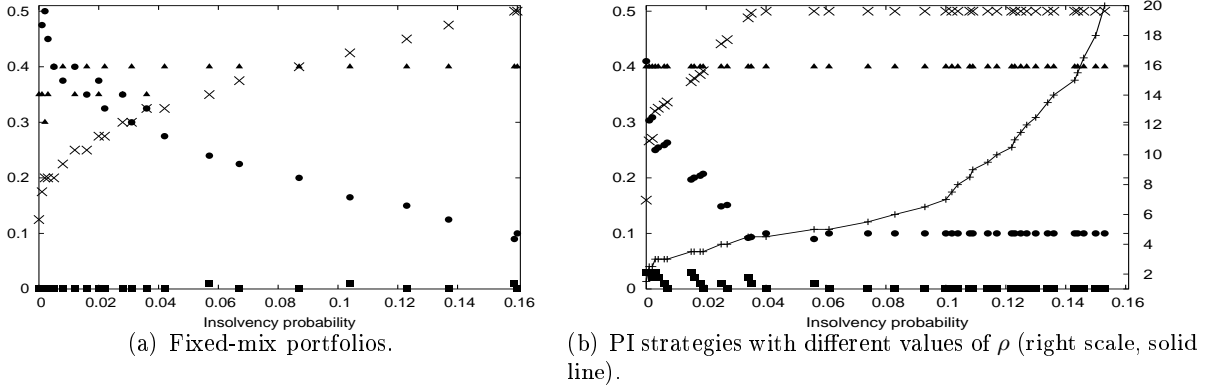


Figure 13: Efficient portfolios' weights for Cash (■), Bonds (●), Stocks (×) and Property (▲).

## 9 Conclusions

This paper proposed a stochastic model for future development of the main stochastic factors that are of interest in asset liability management of a Finnish pension company. Some of the most critical parameters in the model, namely the drift rates and certain long-term equilibrium values, are taken as user-specified instead of relying completely on statistical information. This is essential when the available data displays drifts or other characteristics that are believed to change in the future. The cointegration relations allow the modeling of causalities derived from economic theories and/or statistical studies.

The presented model should, of course, not be taken as the only possible model of reality. We would like to emphasize more the general model building procedure that combines statistical information with user-specified characteristics. In the proposed approach many variations are possible. For example, the model for bond investments returns is only a crude approximation of reality and it could probably be made more accurate by more careful analysis. Also, in modeling property investments, one could try to replace the residential property price index by something that better describes the value of the property investments of a Finnish pension company. Finally, instead of modeling nominal values of the time series, one could incorporate inflation to the model and model the real values of the time series<sup>2</sup>. An alternative possibility for modeling the relation between inflation and interest rates would be to use the fact that inflation is strongly related to logarithmic changes in the wage index which is already in our model. However, our model does not show any direct relation between the interest rates and the wage index. This is probably due to the fact that the three month Euribor series was extended backwards by using data from Germany while the wage index is that of Finland.

The decision rules for dynamic asset allocation considered in Section 8 give a fairly good approximation of the company's expected future solvency and bankruptcy risks, but are not of course the best way of designing investment strategies for a pension company. In Hilli et al. (2003) we describe

<sup>2</sup>The authors are grateful to professor David F. Hendry for suggesting this.

an optimization model that better takes into account the freedom to update the portfolio in the future as well as all the relevant constraints that the Finnish legislation imposes. In this approach, known as stochastic programming, one tries to find the best initial portfolio given the objectives of the company, various portfolio (and other) constraints and the stochastic model for the uncertain factors.

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