Chapter 20

Converibles

20 Convertible Bonds

A convertible bond is a security issued by a company that may be converted from debt to equity (and vice-versa) at various prices and stages in the life cycle of the contract (for example, the time to maturity). There are many types of convertible bonds with various conversion properties and complex structures. Common examples of convertible bonds are **Convertible Preferred Stock bonds**, **Zero-Coupon Convertibles**, **Mandatory Convertibles**, to name but a few. The traditional (simplest) convertible bond is one that is a fixed coupon paying bond when the stock price S is below some predetermined conversion price K (i.e. $S < K$) , and may be converted to a predetermined number of stocks (the conversion ratio) when above (i.e. $S > K$). It may immediately become apparent that the traditional convertible bond is effectively a bond with an imbedded call warrant (option) on the stock of the issuing company with strike price K.

Since a convertible bond is a liability of the issuer, if the company goes into liquidation, the convertible bondholder has priority over most other parties except pure bondholders.

Returning to our traditional convertible bond example, for the moment we ignore credit rating issues, the convertible price B_{con} is close to its fixed income value (bond) B of an equivalent pure bond from the same issuer when deep out-of-the-money. Hence,

$$B_{con} \sim B \quad \text{for} \quad S << K.$$

When the stock price is very high, and exceeds the conversion price K, the convertible bond becomes stock-like when deep in-the-money, hence,

$$B_{con} \sim S \quad \text{for} \quad S >> K.$$

Moreover, given that the convertible is effectively a bond plus a warrant on the stock, the price of a convertible must be comparable to the sum of the two individual components. From arbitrage considerations for options, we know that the price of a call warrant W must satisfy

$$W > \max[S - K, 0]$$

prior to expiry. Ignoring issues related to credit risk, when the warrant element is out-of-the-money the convertible is worth the fixed income value B. Therefore, the convertible has to satisfy the relation

$$B_{con} > \max[S - K, 0] + B$$

given the hybrid nature of the convertible bond. Here the price of the convertible bond has been approximated as

$$B_{con} = B + W$$

In Figure 20-1 we show the price track of a traditional convertible bond. The convertible price (solid red line) is always above the stock price track as already discussed. The red dotted line demonstrates the price track of a bond plus warrant in the absence of credit risk.

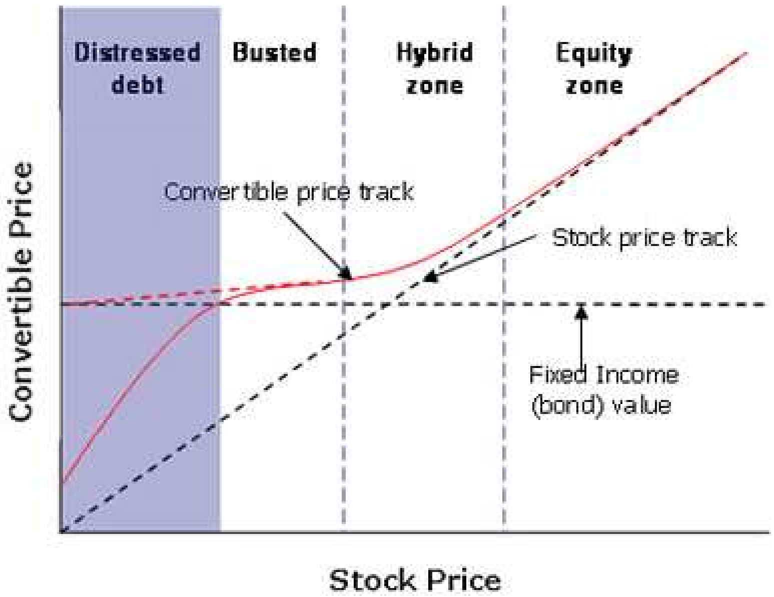

Figure 20-1 The price track of a Convertible Bond

The true price track of the convertible is found to fall below the fixed income value ($B_{con} < B$) when the stock price falls to low levels due to the widening of credit risk spread when the company's stock price falls. As the stock price falls to low levels there is an increasing correlation between the price of the convertible and the stock, however, as the price rises to very high values the correlation becomes insignificant (as the credit rating improves). A fall below the fixed income value is also seen in the pure bonds due to poorer credit rating.

The following terms are used to describe various sections of the price track of the convertible:

1. Distressed debt - In this region the convertible is on close to a default event. If a default event occurs, a sum proportional to the recovery rate R is paid out to the holder of a convertible. The value of the convertible is highly sensitive to the credit risk spread (a parameter often referred to as omicron) in this region.
2. Busted Convertible - A term often used to describe a convertible that is out-of-the money but above the distressed zone.
3. Hybrid zone - The convertible shows behaviour between the stock and a pure bond.
4. Equity zone - The convertible price is more equity-like than debt. Credit risk factors become insignificant since the company's credit rating is high due to the high stock value.

The hybrid nature of convertibles is often exploited by arbitrageurs in the rather popular practice of Convertible Arbitrage. Convertible arbitrage is especially successful at times of high volatility in stock price, producing high returns with relatively low risk. Delta, gamma, and other more sophisticated hedging strategies are used to Capture the low risk profits (though not completely risk-free). Instances can occur when the equity options imbedded are priced different to those of the equivalent pure options that may exist in the market for various reasons. These situations, when a relative price difference is observed, are exploited by arbitrageurs in a long-short trade.

20.1 A model for convertibles

The stock is modelled as a lognormal Brownian process

$$dS = \mu S dt + \sigma S dZ_1$$

and the interest rate as

$$dr = u(r,t)dt + w(r,t)dZ_2$$

where Z_1 and Z_2 are two independent Wiener processes with a correlation ρ. The drift $u(r, t)$ and volatility $w(r, t)$ is dependent on the interest rate model. We make the choice

$$dr = (a_1 - b_1 r)dt + w dZ_2$$

where all parameters are time dependent. The value of the convertible V depends on the stock price, the interest rate and of time, $V(S, r, t)$ and given by the following PDE

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + \rho\sigma S w \frac{\partial^2 V}{\partial S \partial r} + \frac{w^2}{2}\frac{\partial^2 V}{\partial r^2} + rS\frac{\partial V}{\partial S} + (u - \lambda w)\frac{\partial V}{\partial r} - rV = 0$$

This is found by hedging the two processes against each other and with the introduction of the market price of risk. The value of the convertible must be: $V(S, r, t) \geq nS$ where n is the number of stocks on exercise. We get

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + \rho\sigma S w \frac{\partial^2 V}{\partial S \partial r} + \frac{w^2}{2}\frac{\partial^2 V}{\partial r^2} + rS\frac{\partial V}{\partial S} + (a_1 - b_1 r)\frac{\partial V}{\partial r} - rV = 0$$

This equation can be solved with a finite difference method.