2

Interest Rate

2.1 Introduction to Interest Rates

As we will see, there exists many different definitions of interest rates in the markets. A repo trader talks about the simple rate, an option trader of the continuous compounding rate and a bond trader of yield-to-maturity (YTM). We will briefly 0name some of the rates and give a short description. Some of these rates will be discussed in detail in later sections.

2.1.1 Benchmark Rate, Base Rate (UK), Prime Rate (US)

This is the lowest interest rate an investor is willing to take to make an investment in a risk-less security. These rates are given as a yield curve of instruments with different maturities. Usually this yield curve is built from government securities or Over-Night Index Swaps (OIS) and is used to compare against other (risky) interest rates.

2.1.2 Deposit Rate

A typical deposit contract is a standardized agreement of a loan between two banks. It is a credit for the party who placed it, and it may be taken back, transferred to another party, or used for a purchase. Deposits are usually banks main source of funding. The rate for such loan is called deposit rate.

J.R.M. Röman, *Analytical Finance: Volume II*,
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2.1.3 Discount Rate, Capitalization Rate

This is the rate used to discount a given cash flow in the future to a present value (*PV*). This rate reflects the time-value of money. This rate is not uniquely defined. For a certain deal it depends on how this deal is financed and your counterparty. If you have a collateral agreement with the counterparty you should discount with the collateral rate specified in the collateral agreement. A typical rate would be an OIS rate. Without a collateral agreement a typical rate would be the funding rate, like the inter-bank rate.

2.1.4 Simple Rate

The simple rate is the yield, expressed as a percentage per annum of an invested amount. If we receive all interest rates at the end of an investment period, we have the following relationship to the annual compounding rate

$$(1 + r_{annual})^t = (1 + r_{simple} \cdot t)$$

The relation to the discount function is then given by

$$p(t) = \frac{1}{1 + r_{simple}(t) \cdot t}$$

The difference between discount rate and simple rate is that the discount rate is applied to the nominal amount, while the simple rate is applied to the invested amount of a discount instrument. If we, for example, pay \$900 for a \$1,000 nominal amount maturing in 1 year (day-count Act/Act), the simple rate would be:

$$\left(\frac{1000}{900} - 1\right) \cdot 100 = 11.11\%$$

and the discount rate would be

$$\left(1 - \frac{900}{1000}\right) \cdot 100 = 10.00\%$$

2.1.5 Effective (Annual) Rate

The effective rate is the yield expressed as a percentage of the invested amount based on a year including the effect of compounding. If we receive interest, we have to ask us how often we get payments. If we let f be the period, for example, the number of annual payments we get

$$(1 + r_{annual})^t = \left(1 + \frac{r_f}{f}\right)^{f \cdot t}$$
$$(1 + r_{annual})^t = \left(1 + \frac{r_{quarterly}}{4}\right)^{4 \cdot t}$$

In continuous compounding this is expressed as

$$\left(1 + \frac{r_f}{f}\right)^{f \cdot t}, \quad f \to \infty \Rightarrow (1 + r_{annual})^t = e^{r_c \cdot t}$$

The annual rate is related to the discount function as

$$p(t) = \frac{1}{(1 + r_{annual}(t))^t}$$

The semi-annual rate is related to the discount function as

$$p(t) = \frac{1}{\left[1 + \frac{r_2(t)}{2}\right]^{2t}}$$

and the n-annual rate as

$$p(t) = \frac{1}{\left[1 + \frac{r_n(t)}{n}\right]^{nt}}$$

The continuous compounding rate is given as

$$p(t) = e^{-r_c(t) \cdot t}.$$

Each of these formulae can be inverted in the same way as for the annually compounded interest rate. The formulas also define the implicit relationship between the different interest rate types. Since there is a mathematical relationship between the concepts *discount function* and *yield curve*, both of these will be used in this text when we describe the information necessary to perform zero-coupon pricing.

When valuing options and other derivatives, like in Black-Scholes model, we use the continuous compounding. If r_c is the continuous compounded interest rate and r_m the same interest rate paid m times every year we have the relationship

$$r_c = m \cdot \ln\left(1 + \frac{r_m}{m}\right)$$
$$r_m = m \cdot \left\{\exp\left(\frac{r_c}{m}\right) - 1\right\}$$

In the interest rate markets it is very important how to discount cash flows, even for a short period as one day. So if you are given an overnight rate you have to know how this rate is quoted. Is it a simple rate, an effective annual rate or a continuous compounding rate? It is also important to know what day-count conversion that is being used. A one day discount rate can be expressed in many ways, such as

$$\frac{1}{(1+r)^{1/360}} \neq \frac{1}{(1+r)^{1/365}} \neq \frac{1}{1 + r \cdot \frac{1}{360}} \neq \frac{1}{1 + r \cdot \frac{1}{365}} \neq e^{-r/360} \neq e^{-r/365}$$

2.1.6 The Repo Rate

This is the interest rate for a repurchase agreement, that is, the rate you have to pay by selling a security and at the same time commit to buying it back after a short period. The period is usually one of

- O/N (Over-Night)
- T/N (Tomorrow-Next)
- C/W (Corporate-Week)
- S/N (Spot-Next)

The O/N rate is the rate for the period between now, sometime today until the closing time on the next business day. On a Friday, the O/N rate period will be 3 days (if the following Monday is a business day). The T/N will start on the next business day and end on the next following business day. All other rates usually begin two business days from today (if we use a spot lag of 2) and last for a given period time. We say that we are using two spot days.

A government repo rate is the rate at which the government buys their own bills, notes or bonds. Sometime this rate is used to calculate the carry cost for instruments with underlyings.

2.1.7 Interbank Rate

The interbank rate is the average rate at which XIBOR[1] rated banks can borrow from each other. We will discuss this in detail in a later section. XIBOR is a general name convention for the different interbank rates. LIBOR is the London Interbank Offered Rate, STIBOR is the Stockholm Interbank Offered Rate and EURIBOR is the Euro Interbank Offered Rate. Before the financial crises in 2008–2009 the interbank rate was considered as the risk-free interest rate. But at that time XIBOR rated banks such as the Lehman Brothers made default. After the crises more and more banks required collateral agreements for interbank loans. We will discuss this in a later section.

2.1.8 Coupon Rate

The coupon rate is given as the percentage of the nominal amount that is paid to the holder of a bond. These coupons are received with a certain frequency, usually one, two or four times per year. The coupons are paid by the issuer.

2.1.9 Zero Coupon Rate

The zero-coupon rate, or just zero rate, is the YTM on a zero-coupon bond, that is, a bond that pays no coupon. This rate can be bootstrapped from coupon bonds. The zero-coupon rates are often used for the discounting o future payments. Also risk managers use these rates to calculate the risk by making shifts of the curve.

2.1.10 Real Rate

The real rate is the interest rate adjusted for inflation. This rate can be found by bootstrapping Inflation linked bonds, sometimes referred to as Index linked bonds where the index is the Consumer Price Index, CPI.

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[1] XIBOR are used in general for Inter Bank Offer Rates in different currencies where X = L for London, ST for Stockholm EUR for EURO etc.

2.1.11 Nominal Rate

The interest rate including inflation. This means that the nominal rate is equal to the real rate plus inflation.

2.1.12 Yield – Yield to Maturity (YTM)

There are such a variety of fixed-income products, with different coupon structures, amortization, fixed and/or floating rates, that it is necessary to be able to consistently compare different products. One way to do this is through measures of how much each contract earns. There are several measures of this all coming under the same name, the yield.

2.1.13 Current Yield

The current yield have many other names such as interest yield, income yield, flat yield, market yield, mark to market yield or running yield: This yield is a financial term used in reference to bonds and other fixed-interest securities such as swaps. It is the ratio of the annual interest payment and the bond's current clean price of the bond.

The current yield therefor refers to the yield of the bond at the current moment. It does not reflect the total return over the life of the bond. In particular, it takes no account of reinvestment risk (the uncertainty about the rate at which future cashflows can be reinvested) or the fact that bonds usually mature at par value, which can be an important component of a bond's return.

For example, consider the 10-year bond that pays 2 cents every 6 months and $1 at maturity. This bond has a total income per year of 4 cents. Suppose that the quoted market price of this bond is 88 cents. The current yield is simply

$$0.04/0.88 = 4.5\%.$$

2.1.14 Par Rate and Par Yield

The par rate r_{par} is the (fixed) rate payments with the same value as a number of opposite floating rate payments so that their total values sum up to 0 as in Fig. 2.1. The typical instrument here is a plain vanilla interest rate swap.

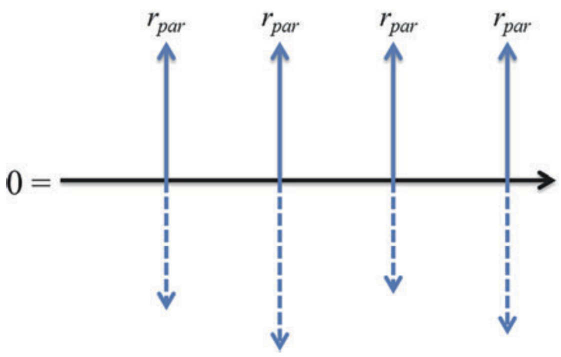

Fig. 2.1 The par rate r_{par} is the constant rate that equalizes the value of the floating leg (dotted arrows) to the fixed leg over the lifetime of the swap

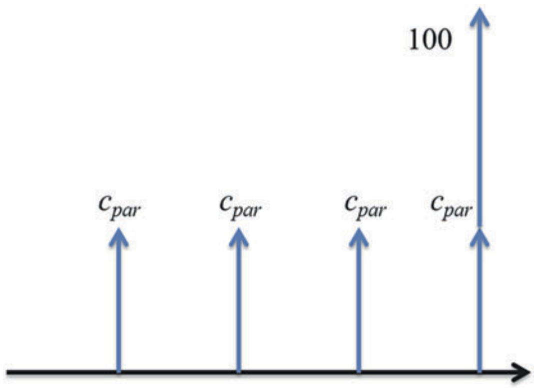

Fig. 2.2 The par yield is the yield that equals the coupon rate c_{par} so that the price

The par yield is the fixed coupons of an instrument so that the total (discounted) value, included the nominal value equalize the nominal value itself as in Fig. 2.2

The par rate for a swap is calculated as

$$\sum_i (p(0,t_i) \cdot r_{par}) = \sum_i (p(0,t_i) \cdot r_{forward}^{t_i - t_{i-1}}) \Rightarrow r_{par} = \frac{\sum_i (p(0,t_i) \cdot r_{forward}^{t_i - t_{i-1}})}{\sum_i p(0,t_i)}$$

Where $p(0, t_i)$ is the discount factor at time t_i, that is, the price of a zero-coupon bond with maturity at t_i. The rate $r_{forward}^{t_i - t_{i-1}}$ represent the floating rate, given by the forward rate (see next) between time t_i and (with maturity) t_{i-1}.

Similarly, the par rate of a bond is calculated as

$$100 = \sum_{i=1}^{n} \left(p(0,t_i) \cdot c_{par}\right) + (p(0,t_n) \cdot 100) \quad \Rightarrow \quad c_{par} = \frac{100 \cdot (1 - p(0,t_n))}{\sum_{i=1}^{n} p(0,t_i)}$$

2.1.15 Prime Rate

The prime rate or prime lending rate is a term applied in many countries to reference an interest rate used by banks. The term originally indicated the interest rate at which banks lent to favoured customers, that is, those with good credit, but this is no longer always the case.

2.1.16 Risk Free Rate

This is defined as the rate you can earn by taking a risk-less position. Many time, this rate is based on treasury bonds with the same time to maturity as the period used (see benchmark rate). If any rate really is risk-free can be discussed, and is discussed a lot in the literature. Some uses the swap rate, or the OIS rate as risk free and other says that also government zero-rate is not risk-free, since also a governments too can default.

2.1.17 Spot Rate

The spot rate or short rate is defined as the theoretical profit given by a zero-coupon bond. We use this rate when we calculate the amount we will get at time t_1 (in the future) if we invest X today (i.e. at time t_0)

$$X_{t_1} = \left(1 + r_{spot}\right)^{t_1} X_{t_0}$$

$$PV\left(X_{t_1}\right) = \frac{1}{\left(1 + r_{spot}\right)^{t_1}} X_{t_1}$$

where $PV(X_t)$ is the present value of X_t. The relation between the spot rate and the discount function is

$$p(t) = \frac{1}{\left(1 + r_{spot}(t)\right)^t}$$

The spot rate is calculated by bootstrapping, by fitting the yield curve. We also see that this rate is the same as the annual effective rate.

2.1.18 Forward Rate

From a yield curve describing the interest rates that apply between the current date and the set of future dates ordered by maturity, it is possible to calculate an implied forward rate curve, that is, the rate that "should" apply between two future dates. The formula for implied forward rates is based on an arbitrage argument, where the rate for a specific nominal amount between two future dates can be locked in by borrowing and lending at the current rates to the future dates.

A projection of the future interest rate, from one time to another, calculated from the spot rate (as shown earlier) or a yield curve is given by

$$(1+r_{t_1}^{spot})^{t_1}\cdot(1+r_{t_2-t_1}^{forward})^{t_2-t_1} = (1+r_{t_2}^{spot})^{t_2} \Rightarrow r_{t_2-t_1}^{forward} = \left(\frac{(1+r_{t_2}^{spot})^{t_2}}{(1+r_{t_1}^{spot})^{t_1}}\right)^{\frac{1}{t_2-t_1}} -1$$

An easy way to represent the forward rate is via the discount function. We then have

$$p(0,t_1)\cdot p(t_1,t_2) = p(0,t_2) \quad \Rightarrow \quad p(t_1,t_2) = \frac{p(0,t_2)}{p(0,t_1)} \equiv \frac{p(t_2)}{p(t_1)}$$

In terms of continuous compounding we then have

$$e^{-r(t_1)\cdot t_1}\cdot e^{-f(t_2,t_1)\cdot(t_2-t_1)} = e^{-r(t_2)\cdot t_2} \quad \Rightarrow \quad f(t_2,t_1)\cdot(t_2-t_1) = r(t_2)\cdot t_2 - r(t_1)\cdot t_1$$

or

$$f_{t_2,t_1} = \frac{r_2\cdot t_2 - r_1\cdot t_1}{t_2-t_1} = \frac{r_2\cdot t_2 - r_2\cdot t_1 + r_2\cdot t_1 - r_1\cdot t_1}{t_2-t_1} = r_2+(r_2-r_1)\frac{t_1}{t_2-t_1}$$

where $p(t, T)$ represent a pure discount bond or zero-coupon bonds at time t with maturity T. We have the boundary condition $p(T, T) = 1$, that is, the zero-coupon bond pays 1 cash unit (CU) at maturity.

2.1.19 Swap Rate

The fixed rate used to price a swap to zero value. A swap is a contract where the buyer and the seller exchange their cash flows, typical floating interest rate cash flows against fixed rate cash flows. Sometimes such a rate is used as the risk-free interest rate. We will discuss swaps in a later section.

2.1.20 Term Structure of Interest Rates

The term structure of interest rates is a set of market interest rates ordered by maturity, that is, the rates on for example, treasury bonds with different times to maturity. An instant term structure is shown in Fig. 2.3. This yield curve is used to discount cash flows to a present value.

2.1.21 Treasury Rate

Treasury rate is the rate you get if you lend money to a government in their own currency. Sometimes, this is used as the risk-free interest rate.

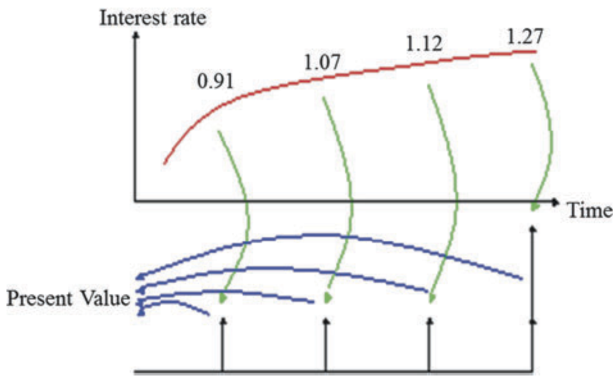

Fig. 2.3 Here we use a yield curve to discount a number of cash-flows PV

2.1.22 Accrued Interest

Accrued interest is calculated for the holder of a coupon bond every day as part of the market convention for the sharing of the annual or semi-annual coupon payment when the bond is bought and sold. For example, from a bond since the last coupon payment.

2.1.23 Dividend Rate

Dividend rate is the fixed or floating rate paid by a preferred stock in Great Britain.

2.1.24 Yield to Maturity (YTM)

The rate an investor will earn if he keeps an interest paying security, typically a bond, until maturity. This only holds true if the received cash flows during the lifetime of the security can be reinvested at the same interest rate. The YTM depends on the coupon rate, time to maturity and the market price of the instrument. Some instruments are quoted in yield to maturity since there is a one-to-one relationship between YTM and the price. This means that you can express the price in YTM.

2.1.25 Credit Rate

The credit rate is an interest rate depending on the ranking of a company. This is in many cases defined as a spread on some benchmark rate.

2.1.26 Hazard Rate

The Hazard rate is the rate based on the risk that the lender might default. If we model the probability that the counterparty will default and therefore cannot pay all of the obligations we use

$$r_{t_1}^{discount} = \frac{1}{\left(1 + r_{t_1}^{spot}\right)^{t_1}} \cdot [(1 - P(t_1)) + R \cdot P(t_1)]$$

where $P(t)$ is the probability that the counterparty will default between the time 0 (today) and the time t, and R the amount we will receive if default occur. R is given in per cent and is called recovery rate.

2.1.27 Rates and Discounting Summary

We summarize the most important discounting methods as follows:

Simple annualized rate: $p(t) = \frac{1}{1+r_{simple}(t)\cdot t}$

Annual compounding rate: $p(t) = \frac{1}{(1+r_{annual}(t))^t}$

Periodically compounding rate: $p(t) = \frac{1}{\left(1+\frac{r_f(t)}{f}\right)^{f\cdot t}}$

Continuous compounding rate: $p(t) = e^{-rt}$

To be able to compare two different yields or interest rates, they have to be in the same day-count basis and method. The "golden rule" for converting yields is: *The discount factor must be equal before and after the conversion.* This means, that we are able to solve the equation by setting the discount factors before and after the conversion equal, and then solve for the unknown yield. For example, to convert from a simple interest rate, Actual/360 basis to simple interest rate, Actual/365 basis, we can do it in the following way

$$1 + r_1 \cdot days/360 = 1 + r_2 \cdot days/365$$

After some simple calculations, we find

$$r_2 = r_1 \cdot 365/360$$

Therefore, for example simple interest rate 5% in actual/365 basis is equivalent to 5% · 365/360 = 5.069444% in actual/365 basis.

The following equations summarize the conversion formulas between periodically compounded and simple interest rates. For annually compounded rates, the f is 1 and the formulas take a simpler form. The formulas assume that year fractions for original and destination basis have been calculated.

From compounding yield basis to simple interest basis and vice versa

$$\left(1+\frac{r_f}{f}\right)^{t_1 f} = 1 + r_{simple} \cdot t_2$$

giving

$$r_f = f \cdot \left(1 + r_{simple} \cdot t_2\right)^{\frac{1}{t_1 \cdot f}} - 1 \qquad \Leftrightarrow \qquad r_{simple} = \frac{1}{t_2}\left(\left(1+\frac{r_f}{f}\right)^{t_1 \cdot f} - 1\right)$$

From compounding yield basis to another compounding yield basis we have

$$\left(1+\frac{r_{f_1}}{f_1}\right)^{t_1 \cdot f_1} = \left(1+\frac{r_{f_2}}{f_2}\right)^{t_2 \cdot f_2} \qquad \Leftrightarrow \qquad r_{f_1} = \left(\left(1+\frac{r_{f_2}}{f_2}\right)^{\frac{t_2 \cdot f_2}{t_1 \cdot f_1}} - 1\right) \cdot f_1$$

2.1.28 Black-Scholes Formula

In almost all literature in option theory, Black-Scholes formula (without dividends) for a call option is given by

$$C = S \cdot N(d_1) - e^{-rT} X \cdot N(d_2)$$

Here r is the risk free interest rate. We can rewrite the Black-Scholes equation as

$$C = e^{-r \cdot T}\left[S e^{r \cdot T} N(d_1) - X \cdot N(d_2)\right]$$

where we have moved the discount factor outside the bracket. The first term inside the bracket is recognized as the forward price of the underlying security, times the probability that the option will be at-the-money at maturity. In practical situations it is favourable to rewrite the equation as

$$C = e^{-r_{discount} \cdot T} \cdot \left[S \cdot e^{r_{repo} \cdot T} \cdot N(d_1) - X \cdot N(d_2)\right]$$

As we can see, we use two different interest rates. The discount rate $r_{discount}$ is used for discounting to a PV, and the repo rate r_{repo} as the risk-free rate in the valuation of the forward.