1
Financial Instruments

In the previous book, we studied derivatives in the equity markets and in this book, we will study the available instruments in the interest rate markets. First, we will shortly group the various instruments.

In order to group the wide variety of instruments that exist adequately, it is necessary to break the interest rate asset classes into two subdivisions: *long-term* and *short-term* debts. In addition, it is necessary to divide the derivatives into two groups: *standard derivatives* and *over-the-counter* (OTC) *derivatives.*

- Standard derivatives are traded on exchanges. In such trades, a clearing house act as a counterparty to both buyers and sellers. These trades have a daily settlement[1] to protect the clearing house for losses, if one of the counterparties cannot fulfil its obligations. The clearing house guarantees the delivery of payments or underlying securities to its counterparties.
- OTC derivatives are typically traded over telephone or via a broker firm. They are known as OTC instruments because each trade is an individual contract between the two counterparties making the trade. These contracts are *privately negotiated* which means that they are not negotiable, for example, if I lend you some money, I cannot trade that loan contract to someone else without your prior consent.

[1] Some exchanges use monthly settlement, for example, Nasdaq-OMX in Stockholm.

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Table 1.1 Instrument types and asset classes

Instrument-type asset class	Cash	Standard derivatives	OTC derivatives
Interest Rate (Long Term)	Bond, note Floating rate note	Bond futures Options Bond futures	Swaps, swaptions, caps & floors, IRG, Cross Currency swaps, Exotics
Interest Rate (Short Term)	Deposit/Loan, Bill, CD (Certificate of Deposit), CP (Commercial Paper)	Interest rate futures	Forward Rate Agreement FX-swap, Euro Dollar futures
Equity	Stock (Index)	Equity Options Equity futures	Equity Options Exotics
Foreign Exchange			

- The International Swaps and Derivatives Association (ISDA) provides standard contracts to facilitate the trading of OTC derivatives.
- Many clearinghouses also clear OTC instruments. In this case they are said to use central clearing. By using central clearing the counterparty risk can be minimized. Also the Capital requirements for buyers and sellers will be minimized by using central clearing

Further subdivisions of the categories give rise to the matrix as shown in Table 1.1.

1.1.1 Money

Money, in wholesale banking, exists only as an electronic entity in the banking systems. The reason is that paper money does not earn interest and is therefore not money in a financial view. Therefore, we consider paper money as an interest free loan to the government. An analogy is the old type of share certificates that was physical delivered between the counterparties who have made a deal. Nowadays, share certificates are no longer used, instead all ownership is registered electronically.

Also, dollars only exist in the US banking system, pound sterling only in the British banking system and Euro in European banks.

Every bank that accepts US dollar has a Nostro account in its correspondent bank in the US. Similar accounts exist in all currencies in banks in all countries. If for example Sanwa in London transfers 1 USD to Barclays in London, Sanwa instructs its correspondent bank in US to transfer the 1 USD to Barclays. The money therefore never leaves the US.

It is important to notice that payments can only be made or received when the banking system is up. Therefore, we have to consider when the banking holidays for all countries exist, because then, no money transactions can be made in that specific country.

1.1.2 Valuation of Interest Rate Instruments

We will start to study interest rate instruments and how to value them. The following instruments are examples of cash-flow instruments:

- Bonds, bills and notes
- Floating Rate Notes (FRN)
- Swaps, Currency swaps and FX swaps
- Swaptions
- Caps, floors, collars and Interest Rate Guarantees
- Forward Rate Agreements (FRA)
- Convertibles
- Deposits and Certificates of Deposits (CD)
- Repos and reverses
- Credit Default Swaps/Indices (CDS, CDI, CDX etc.).

Many of these instruments are treated only as cash-flow sequences. Some of them are treated as derivatives. That is, no assumption is made on the pattern of how the cash flows looks like in the valuation process. In this way, the description of how to value a single cash flow can be generalized for all cash-flow instruments.

The advantage of such method is its generality. It can be applied to any kind of cash-flow pattern, whether it is amortized, has non-consecutive interest rate periods or broken dates.

There are a number of different cash-flow types as well:

- Fixed amount
- Fixed rates
- Floating rates
- Caplets
- Floorlets
- Total return
- Credit default
- Return
- Redemption amount
- Call fixed rates
- Call float rates
- Zero-coupon fixed rates.

The different cash-flow types are described in terms of various parameters as shown in Table 1.2.

1.1.2.1 Parameters

Common parameters for all cash-flow types are the *Pay Date* – the calendar date when the cash flow is paid – and the *Currency* of the cash flow. All cash flows are discounted using a *zero-coupon curve* from the payout date to the valuation date.
The simplest cash-flow type is a single fixed payment, *Fixed Amount*. All other cash flows are related to interest rates payments in some way. They have the common attributes:

- day count – the day-count convention used for a certain period
- start day – the date on which the interest rate period starts
- end day – the date on which the interest rate period ends

The simplest interest payment is the fixed coupon rate, using the attribute, *Fixed Rate* – the fixed interest rate that applies for a specific period.

Table 1.2 Parameters for different cash flows

Parameter	Zero-coupon fixed	Return	Redemption amount	Fixed Amount	Fixed Rate	Float Rate	Caplet/ floorlet	Total Return	Credit Default	Call fixed rate	Call float rate
Pay Date	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Currency	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Fix Amount				Y							
Start Day	Y	Y			Y	Y	Y	Y	Y	Y	Y
End Day	Y	Y			Y	Y	Y	Y	Y	Y	Y
Day count	Y				Y	Y	Y			Y	Y
Fix Rate					Y	Y	Y			Y	Y
Float Rate						Y	Y				Y
Reset Day	Y	Y				Y	Y	Y			Y
Spread						Y	Y				Y
Strike							Y				

The different pay types are:

- Spot — An instant pay order (to pay in 2 days, the spot days)
- Future — "Mark to Market", daily
- Forward — Pay on expiration date
- IMM — On IMM days (International Monetary Market days that is the third Wednesday in March, June, September and December.
- Forward/Periodically — Make payments on certain days, for example, the 3rd Friday on each month

There are two delivery or exercise types for derivatives:

- Physical delivery — Typically a stock (equity) option.
- Cash settlement — Typically, an option with an index as underlying.

There are three types of option exercise:

- European — Exercise only at expiration date
- American — Exercise any time
- Bermudan — Exercise in pre-defined periods or days

There are two types of option underlying:

- The underlying asset itself
- A future or a forward (on the underlying asset)

We can arrange the types as in Table 1.3.

Table 1.3 Pay types, deliveries and underlying for different instruments

Instrument	Pay type	Delivery Eur./Am.	Underlying
Stocks	Spot		
Bonds	Spot		
Index forwards	Forward	Cash	
Index futures	Future	Cash	
Bond futures	Future	Physical	
Commodity	Future	Future	Physical
Stock options	Spot	Physical/American	Stock
Index options 1	Spot	Cash/European	Index
Index options 2	Future	Physical/American	Index futures
Bond options	Future	Physical/American	Bond futures
OTC derivatives	Spot/forward	European	...

1.1.2.2 Future Value and Present Value

When we value different financial instruments, we use different expressions for their rates of return. If we calculate the rate of return of an equity to find the payoff, we often use a simple period rate r over the holding period. This rate is the percentage return on annual basis of the invested amount P. To calculate the present value of this amount we use

$$F = P \cdot (1 + r)$$

where F is the value at the end of the period. It is also common to annualize the rate using some convention for counting the length of the holding period, that is, the number of days, as a fraction of a year. In the money market, we usually use the following measure for the yield

$$F = P \cdot \left(1 + r \cdot \frac{d}{360}\right)$$

where d is the number of days to maturity. Since no compounding was used above the rate is referred as the simple rate. If we use annual compounding with the same number of days, we can express this as a fraction of a 360-day year. We then use the compounded annual rate r_c

$$F = P \cdot (1 + r_c)^{\frac{d}{360}}$$

For money market instruments, such as treasury bills and CD, which have fixed dates of expiry, the quoting convention relating market prices to rates typically does not use compounding. Their values upon expiry equals their nominal amounts so we can solve for their current price

$$P = \frac{N}{1 + r \cdot \frac{d}{360}}$$

where N is the nominal amount, paid on expiry and r the simple annualized rate on a yearly basis. The simple rate r can then be expressed as

$$r = \frac{N - P}{P} \frac{360}{d}$$

1.1.3 Zero Coupon Pricing

The concept behind zero-coupon pricing is the evaluation of all individual cash flows as if they were zero-coupon bonds. The evaluation is made using a yield curve or, alternatively, a discount function, which accurately describes current market conditions.

The pricing of liquid, standardized instruments are quite simple – the current market price is used. The zero-coupon pricing methodology becomes important when pricing OTC instruments, for which no market prices are available. It is also needed for pricing standardized instruments, which do not have reliable market prices. In this case, zero-coupon pricing will be used to price these instruments consistently alongside the liquid instruments. This is a kind of relative pricing where user preferences only need to be taken account of to a small extent. Many risk management techniques also require the use of a yield curve to aggregate correctly the risk over several different instrument types.

1.1.3.1 The Discount Function

The discount function, $p(t_0, t)$, describes the present value at time t_0 of a unit cash flow at time t. This is a fundamental function that can be given, for each time in the future, as individual components, the discount factors. These factors are non-random and should be equal for all banks due to arbitrage conditions.[2]

In most cases, t_0 is the current time (equal zero) and is therefore dropped for notational convenience. The remaining variable $t(t - t_0)$ then refers to the time between $t_0(= 0)$ and t. The discount function is, as we will see, used as the base for all other interest rates. For any future date t this function also represent the value of a zero-coupon bond (also called a pure discount bond) at time $t_0(= 0)$ with maturity t. At maturity, a zero-coupon bond pays one cash unit (in USD, GBP, EUR SEK etc.). So therefore $p(t, t) = 1$. A discount function with rate $r = 2.0\%$ is shown in Fig. 1.1.

[2] Since the financial crisis in 2008, this is not really true, since some currencies are more risky than others. Therefore, we have to add, a so-called cross currency basis spread to the discount function. This basis spread is set against the most liquid currency in a trade. Only USD will have a zero basis spread. We will discuss that later. But now we think about the discount function as generic.

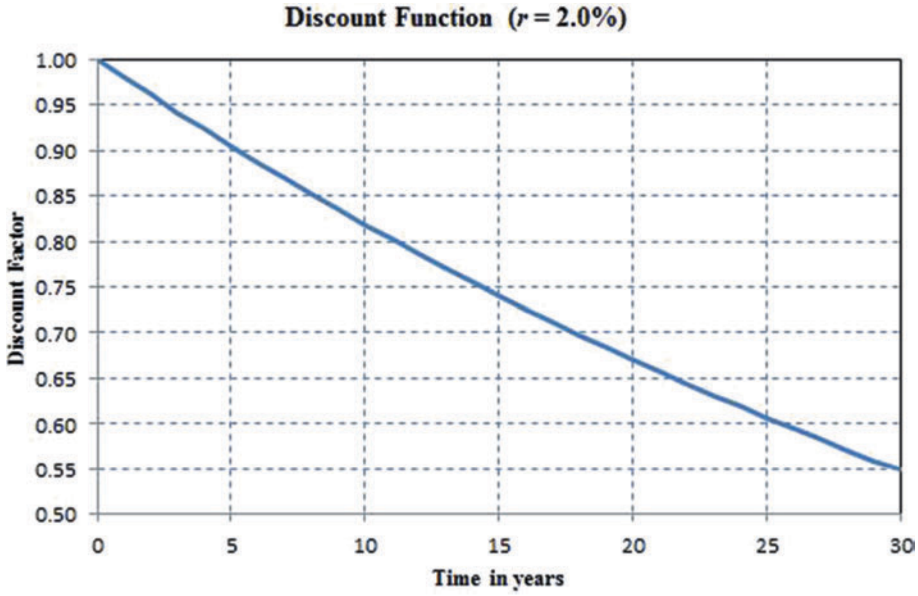

Fig. 1.1

At $t = 0$, the discount function always has the value 1 ($p(0, 0) = 1$). One unit of cash today must have the value of one unit by definition. The discount function is monotonically decreasing, which corresponds to the assumption that interest rates are always positive. It never reaches zero since all cash flows, no matter how far in the future they are paid, should always be worth something.

The discount function has a mathematical relationship to the spot yield curve, although the "yield curve" is not a well-defined concept. The relationship between the discount function and the annually compounded yields of matching maturity, using a day-count convention that reflects the actual time between time t_0 and t measured in years, can be written as

$$p(t) \equiv p(0, t) = \frac{1}{(1 + r_1(t))^t}$$

This formula can be inverted to give

$$r_1(t) = P(t)^{-1/t} - 1.$$

Other used yields have a mathematical relationship to the discount function.

1.1.4 Day-Count Conventions

When using the discount function to express yield or interest rates, it is very important to known and consider the day-count convention used for each instrument and each market. The day-count convention is a user-defined, instrument-specific parameter and will typically have a substantial impact on the valuation of particular instruments.

1.1.4.1 Date Arithmetic

Dates are usually integers starting from 1900-01-01. Following the Excel convention, May 8, 2006 is day 38845, and May 9, 2006 as day 38846, etc. We therefore use an *Add-function* between different dates

$$t_{th} = \text{Add}\left(t, n, unit, EOMFlag, \ldots\right)$$

which adds n units (days, months, years or business days) to date t, where n can be positive, zero or negative.

We have the following End-Of-Months rules (*EOMFlag*)

1. If we add months or years and t_{th} ends up beyond the end of a particular month, we replace this date with the last day of month. Example:

   May 31 + 1 month = June 30
   December 31 + 2 month = February 28 (or 29 for a leap year)

2. If date t is the last day of a month, then

   If *EOMFlag* is *true*: adding months or years always gives the last day of the month:

   February 29 + 1 month = March 31
   April 30 – 1 month = March 31

   If *EOMFlag* is *false*: adding months or years always gives the same day of the month, provided that it exists:

   February 29 + 1 month = March 29
   April 30 – 1 month = March 31
   May 31 – 1 month = April 30

3. The *EOMFlag* is irrelevant if t is not the last day in a month.

We also must have a general add functionality

$$t_{act} = \text{Add}\left(t, n, unit, EOMFlag, BDR, Hol1, Hol2, Hol3 \ldots\right)$$

where *BDR* is the Business-Day-Rule. We first compute

$$t_{th} = \text{Add}\left(t, n, unit, EOMFlag, \ldots\right)$$

If t_{th} is not a business day, we apply the *BDR* rule to resolve the date. t_{th} is a bad day if it is a bad day in any holiday *Holx*.

We have the five business day rules:

1. *none*: return t_{th} (banks can go into default also on non-banking days!)
2. *following* (*succeeding*): t_{act} is the first valid business day on or after t_{th}.
3. *proceeding*: t_{act} is the first valid business day on or before t_{th}.
4. *modified following*: t_{act} is the first valid business day on or after t_{th} if it is the same calendar month as t_{th}. Otherwise t_{act} is the first valid business day before t_{th}
5. *modified proceeding*: t_{act} is the first valid business day on or before t_{th} if it is the same calendar month as t_{th}. Otherwise t_{act} is the first valid business day after t_{th}

The *modified following* is the standard rule for payments. Typically, dates are generated backwards from the theoretical end date. Otherwise, it is difficult to do a rewind of a trade with a number of cash flows with another customer.

First, we get the theoretical end date. For an M month leg, starting at t_0 we have

$$t_n^{th} = \text{Add}\left(t_0,\ M,\ month,\ no,\ none,\ ccy1,\ ccy2 \ldots\right)$$

For a leg with m months per period, we have

$$t_j^{th} = \text{Add}\left(t_n, -m\left(n-j\right),\ month,\ no,\ none,\ ccy1,\ ccy2 \ldots\right)$$
$$t_j^{act} = \text{Add}\left(t_n, -m\left(n-j\right), month,\ no,\ modfol,\ ccy1,\ ccy2 \ldots\right)$$

If an odd period is needed, the default is ***short first*** period, other possibilities are *long first, short last* and *long last*. The last two requires that we generate the dates from t_0, which we do not want. The holiday parameters *ccy1* and *ccy2* have to be used for the different currencies.

When dealing with interest rate payments, accrued from t_{j-1} to t_j and paid at t_j, we use the following rule:

1. If the swap leg is *adjusted* (which is the default situation), interest accrued from t_{j-1}^{act} to t_j^{act}.
2. If the swap leg is *unadjusted*, interest accrues from t_{j-1}^{th} to t_j^{th}.
3. Interest payments are $\alpha_j rN$ paid at t_j^{act} for $j = 1, 2, \ldots, n$ where N is the notional, r the interest rate and α_j the day-count fraction.

$$\alpha_j = DayCountFrac(t_{j-1}, t_j, basis)$$

Day-count basis are rules assigning official fractions of a year to any two dates. Some alternative day-count conventions are (there exist about 80 more day-count bases than those listed below):

- 30/360 corporate bonds, Eurobonds etc.
- 30E/360 money market Switzerland
- Act/360 US T-bills US, Euro and Switzerland money, etc.
- Act/365 US Treasury bonds/notes, UK gilts, German bunds etc.
- NL/365 Actual/365 with no leap year
- Act/Act New Euro bonds, LIFFE UK bond/bund futures etc.

The meaning of the abbreviations used in the naming of the above conventions is as follows:

- **Act**: Actual number of calendar days.
- **NL**: Actual number of calendar days, with no leap year.
  - Exception: If the year is a leap year then February is considered to have 28 days (instead of 29).
- **30**: Each month is considered to have 30 days.
  - Exception 1: If the later date is the last day of February, that month is considered to have its actual number of days.
  - Exception 2: When the later date of the period is the 31st and the **first day is not** the 30th or the 31st, the month that includes the later date is considered to have its actual number of days.
- **30E**: Each month has 30 days.
  - Exception: If the later date is the last day of the month of February, that month is considered to have its actual number of days.

Credit cards always use Act/360, which gives them five extra days of interest per year.

Interest rates are typically expressed for annual periods. The time period measured in years between two dates, t, is described as the fraction of the number of days between two dates, t_d, and the number of days in a year, t_y

$$t = \frac{t_d}{t_y}$$

t_d and t_y are determined according to the specified day-count convention.

Example 1.1
What is the time period between 11 January and 31 March?

30/360: Number of days in January = 19 + 30 in February + 31 in March = 80: $t = 80/360$

30E/360: Number of days in January = 19 + 30 in February + 30 in March = 79: $t = 79/360$

NL/365: Number of days in January = 20 + 28 in February + 31 in March = 79: $t = 79/365$

Example 1.2
If we let $t = (d_1, m_1, y_1)$ (date, month and year) and $T = (d_2, m_2, y_2)$, then the 30/360 convention can be calculated as

$$\frac{\min(d_2, 30) + (30 - d_1)^+}{360} + \frac{(m_2 - m_1 - 1)^+}{12} + y_2 - y_1$$

where $(x)^+ = \max(x, 0)$. The time between t = January 4, 2005 and T = July 4 2007 is then

$$\frac{4 + (30 - 4)^+}{360} + \frac{(7 - 1 - 1)^+}{12} + 2007 - 2005 = 2.5.$$

1.1.4.2 International Monetary Market (IMM) Days

Many Fixed Income instruments have start days and maturities on International Monetary Market (IMM) days. IMM days are the third Wednesdays in Mars, June, September and December.

1.1.5 Quote Types

When pricing interest rate instruments, a number of different quote types are used. Quotes are the market prices traders do observe on screen from their trading system or from other price sources. We will now define some of them.

1.1.5.1 Per Cent of Nominal Amount

Quote is taken as a per cent of the nominal amount (also called the face value). This is used for bonds and can be given with or without accrued interests.

1.1.5.2 Clean Price

Quote is taken as a per cent of the nominal amount without the accrued interest. This is the normal quotes of bonds and other similar instruments.[3]

1.1.5.3 Price

Quote is taken as a per cent of the nominal amount included the accrued interest. This is also called the dirty price. Therefore the (dirty) price equals the (clean) price plus the accrued interest rate since the last coupon payment for a bond.

1.1.5.4 Coupon Rate

Quote given as the coupon rate. This can be used when comparing different bonds with similar maturities. With known coupon rate, the price can be calculated by discounting of the cash flows, included the nominal payout at redemption.

[3] Swedish bonds are quoted in yield (to maturity).

1.1.5.5 Yield/Yield-to-Maturity

Quote is given as a flat yield used to discount all future cash flows. This is how Swedish bills and bonds are quoted. Yield-to-maturity has a one-to-one relationship with the dirty price. It's based on that all the coupons can be reinvested at the same (flat) yield.

1.1.5.6 Volatility

This quote type is available for Options/Warrants, swaptions and caps/floors. The quote of an instrument with quote type "Volatility" is interpreted in terms of the implied volatility structure used for the instrument. Normally this is Black (lognormal) or Normal volatility. The reason for different volatilities like Black volatility and Normal volatility is that whatever the model used to value an instrument the price must be the same.