

Examination in MMA707, Analytical Finance I

Wednesday 3 of November 2010, 14:10 – 18:30

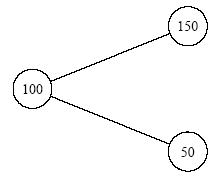
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You may use: Calculator (without alphanumeric capability), pencil, ruler and rubber gum.

General direction: The solution should be well motivated and readable. All notations must

be explained.

Good Luck!!

1. (a) Compute the price of a European call option with strike price *K* = 125 and   
    exercise time *T* = 2 years, using a binomial tree with two trading dates *t1* = 0   
    and *t2* = 1 (your portfolio at time *t3* = 2 is the same as your portfolio at time   
    *t2* = 1) and parameters *s0* = 100, *u* = 1.5, *d* = 0.5, *r* = 0, and *p* = 0.75  
     
   (b) i. Consider a discrete time financial model with dates *t* = 0, 1,…, *T*, a risk free   
    asset with price process *B*, and a stock with price process *S*, i.e.  
     
    *B*(*t*) = price at time *t* of the risk free asset;  
    *S*(*t*) = price at time *t* of the stock.   
     
    Now let *ht* = (*xt, yt*) denote the portfolio which is held from *t* - 1 until *t*, i.e.  
     
    *xt* = number of risk free assets held in the time interval (*t – 1, t*]  
    *yt* = number of stocks held in the time interval (*t – 1, t*]  
     
    What does it mean that a portfolio is self-financing in this model?   
     
    ii. Consider a standard Black-Scholes market, i.e. a market consisting of a risk   
    free asset, *B*, with *P*-dynamics given by:   
     
     and a stock, *S*, with *P*-dynamics given by  
     
      
      
    Here *W* denotes a *P*-Wiener process and *r*, *α* and *σ* are assumed to be   
    constants. Now let *ht* = (*h0t, h1t*) denote the portfolio held at time *t* in this   
    model, i.e.   
     
    *h0t* = number of risk free assets held at time *t*  
    *h1t* = number of stocks held at time *t.*  
     
    What does the self-financing condition look like for this model?   
     
   (c) Consider the standard Black-Scholes model described in (b). For a given   
    portfolio *h* the **relative portfolio** *u* = (*u0*, *u1*) is given by   
     
      
     
    at time *t*. Here *Vh* denotes the value process associated with the portfolio *h*.   
    Note that *u0t + u1t* = 1.   
     
    i. What does the self-financing condition look like in terms of the relative   
    portfolio?  
    ii. Regard the constant relative portfolio *u* = (½, ½) as self-financing and   
    determine the value process associated with it, given that the initial wealth   
    invested in it is *V0 = v*.  
     
   (d) Use the binomial tree in (a) to find a replicating portfolio for the option in (a)   
    and verify that the portfolio is self-financing  
   …...……..………………………………………………….………………….(10p)
2. Below is a picture of a one-period (time points *t* = 0 and *t* = 1) binomial model with parameters *S0* = 100, *u* = 1.5, *d* = 0.5 and *p* = 0.75.   
      
   (a) What are the arbitrage bounds for the interest rate *r*?  
   (b) Given that the price at time *t* = 0 of a European call option with strike price   
    *K* = 108 kr and exercise time *T* = 1 year has been computed to 22 kr, what is the   
    interest rate *r*?  
   ……..………..............…………………………………………………..………….(5p)
3. You are going to buy a European derivative in the Black-Scholes world. From the trading software you get the following data:  
     
    Underlying price = 100.0  
    Risk-free interest rate = 6.0%  
    Option Delta = 0.597866  
    Option Gamma = 0.013659  
    Option Theta = -13.76591  
    Underlying Volatility = 40.0%  
     
   Calculate the price of the derivative.  
   ….............…………………………………………………………………………..(5p)
4. Consider a standard Black-Scholes market, i.e. a market consisting of a risk free asset, *B*, with *P*-dynamics given by:   
     
    and a stock, *S*, with *P*-dynamics given by  
     
      
      
   Here *W* denotes a *P*-Wiener process and *r*, *α* and *σ* are assumed to be constants.   
   (a) Derive the put-call parity in this model.  
   (b) Define the concepts portfolio, value process, relative portfolio and self-financing  
    portfolio.   
   (c) Consider the following relative portfolio  
     
      
     
   Relative portfolios can always be interpreted as relative portfolios of self-financing portfolio strategies. Given that the initial value of the portfolio should be *V0*, which self-financing portfolio strategy does the above relative portfolio correspond to, and what does the value process for this portfolio look like?  
     
   (c) The broker firm F&H has introduced the derivative "the inverse mean" on the   
    market. This contract is specified by two fixed points in time *T0* and *T1*, with   
    *T0* < *T1* . The holder of this contract obtains the sum   
     
      
     
    at time *T1*. Determine the price process П(*t, X*) for *t* < *T0*.  
   ………...…………………………………………………………….…………….(10p)
5. Consider a two-dimensional Black-Scholes market, i.e. a market consisting of a risk free asset, *B*, with *P*-dynamics given by:   
     
    and two stocks, *X* and *Y* with *P*-dynamics given by  
     
      
      
   Here *V* and *W* denotes two independent *P*-Wiener processes and *r*, *α, β, ρ, γ* and *σ* are assumed to be constants.   
     
   Assume that the filtration is the natural filtration generated by the Wiener processes *W* and *V*. Show that this model is free of arbitrage and complete given that *γσ ≠*  0.  
   ……...……………………………………………………….……………….(10p)
6. a) Prove that the Black-Scholes model is complete, i.e. that all contingent claims are   
    reachable.  
   b) Prove that the Black-Scholes model is free of arbitrage  
   …………………………………….…………………………….……………….(10p)

**Formulas:**

* Suppose that there exist processes *X*(**.**, *T*) for every T ≥ 0 and suppose that *Y* is a process defined by:  
    
     
    
  Then we have the following version of Itô's formula  
    
   
* The standard Black-Scholes formula for the price П(*t*) of a European call option with strike price *K* and time of maturity *T* is П (*t*) = *F*(*t*, *S*(*t*)), where  
    
     
    
  Here *N* is the cumulative distribution function for the *N*(0, 1) distribution and  
    
   
* If *N* denotes the cumulative distribution function for the *N*(0; 1) distribution, then  
    
   *N*(-*x*) = 1 - *N*(*x*).

A linear SDE of the form  
  
   
  
where *a* is a constant and *bt* and *σt* are deterministic functions, has the solution  
  
 

